The Binding Energy of Diquark-Antidiquark System in Nanometer and Subnanometer Scales

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Abstract
In this paper, using Monte Carlo Fortran code, we have obtained the binding energies for three different systems of diquark–antidiquark in distances from 0.01 to 15 nm. In [0.1 - 15] nm interval, we made use of Coulomb potential because in this interval, strong interaction is negligible. We have compared the binding energies of the systems with one another. The results of these comparisons were close to our anticipations. We also obtained the binding energy of one of the systems in the interval below 1 fm, where diquark-antidiquark systems comprise a tetraquark and the potential is of strong interaction type. Because of weak Coulomb interaction, strong interaction has been used as the basis of the calculations. The binding energy resulted is consistent with the existing references.

1. Introduction
It is apparent that the concept of diquark (two-quark system) is essential for understanding the hadron structure and particle interactions at high energies. According to the quark model [1], hadrons are composed of quarks. A quark and an antiquark form a meson in a bound state, and a combination of three quarks (in a bound state) results in the formation of a baryon. However, hadrons may be made up of particles other than quarks. Diquarks (two quarks in bound states) may also be the constituent elements of hadrons. This view has recently been a subject of interest [2-4]. The diquark model began to develop following the emergence of the quark model. The diquark model states that two quarks combine to form a color-antitriplet in a bound state. Gell-Mann initially proposed that diquarks could exist [5]. Studies on the composition of baryons are also indicative of the existence of diquarks [6-10]. Furthermore, mesons have been supposed as tetraquarks with a diquark plus an antidiquark [11-14]. In fact,
realization of a diquark as a pseudo-particle or just a roughly bound state is still under dispute. Diquarks have also been regarded as particles which are the major elements inside hadrons. These diquarks should have quantum numbers and definite masses if they are to exist as real stable particles.

Based on the current knowledge of QCD (quantum chromodynamics), diquarks’ correlations may result from the following factors:
1) quarks’ radial or spin excitement
2) spin-dependent interaction between two quarks
3) quarks’ mass differences.

Diquark substructure affects the static features of baryons and their decay mechanism.

Diquarks are important for understanding the hadron structure and high-energy particle reactions. Diquarks also play a role in hadron production in hadron-initiated reactions, in deep-inelastic lepton scattering by hadrons, and in reactions [15].

Diquarks have been known as the building blocks of baryons [16,17]. Some mesons also contain diquarks in their structure, such as light scalar mesons and some charmed mesons [18,19].

For convenience, a diquark is regarded as a point particle with the quantum number of two quarks. This simplified concept often leads to predictions which are consistent with the experiments [15].

More generally, a diquark is any system of two quarks considered collectively. For example, a two-quark correlation in a hadron containing more than one quark is a diquark system. A diquark in its ground state has positive parity and may be an axial (spin1) or a scalar (spin0) vector.

Regular hadron spectra have both the real part and the imaginary part which contributes to the diquark stability. It means that the lifetime of the diquark might be finite [20]. A diquark system may decay into two quarks under different conditions such as through absorbing gluons. Heavy diquarks made up of heavy quarks have a longer lifetime since they are more resistant against decay.

Diquarks could be classified into three categories by their masses: light-light, heavy-heavy, and light-heavy. Light quarks include d, u, and s, yet c and b are considered heavy. This paper will investigate the binding energies of the first group.

2. Diquark Composition
It must be first noted that since light quarks are in the flavor representation, they may form either flavor sextets or antitriplets:

\[ Q = qq : \ 3_f \otimes \bar{3}_f = 6_f \oplus 3_f \ SU(3)_f, \ (1) \]

where \( Q \) stands for a diquark. The sextet \( 6_f \) is symmetric under flavor exchange of the two quarks, while the antitriplet \( \bar{3}_f \) is antisymmetric under this exchange. For SU(4) flavor, which includes the charm quark, the diquarks form the representation

\[ Q = qq : \ 4_f \otimes 4_f = 10_f \oplus 6_f^c \ SU(4)_f, \ (2) \]

where is symmetric and is antisymmetric under flavor exchange. The diquark building blocks lie in the antisymmetric [21].

3. Binding Energies
We used Monte Carlo Fortran code, which can solve Lippman-Schowinger equation for two-particle bound systems. The input data for this program include diquark-antidiquark average masses (an antidiquark is equivalent to a diquark with an opposite charge and strangeness), an
arbitrary initial energy, potential, and radius cut (r_cutoff) where -cutoff is a quantity at which the potential tends to zero [22]. The potential root gives us the r_cutoff.

Table 1 presents the masses and form factor parameters of diquarks.

Table 1. Masses and form factor parameters of the diquarks

<table>
<thead>
<tr>
<th>Quark Content</th>
<th>Diquark Type</th>
<th>M(Mev)[5]</th>
<th>Q[6]</th>
</tr>
</thead>
<tbody>
<tr>
<td>[c,u]</td>
<td>Scalar</td>
<td>1973</td>
<td>4/3</td>
</tr>
<tr>
<td>[c,d]</td>
<td>Vector</td>
<td>1973</td>
<td>1/3</td>
</tr>
<tr>
<td>{c,u}</td>
<td>Vector Axial</td>
<td>2036</td>
<td>4/3</td>
</tr>
<tr>
<td></td>
<td>Vector</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The constituent quarks’ masses are [23].

\[ m_c = 0.5 \text{ GeV} \quad m_u = m_d = 0.33 \text{ GeV} \]

\[ m_e = 1.55 \text{ GeV} \]

Strong interaction potential can leave its effects in distances up to 1 fm. Since we want to calculate the diquark-antidiquark energy at 0.1 to 15 nm interval, the interaction potential will be only Coulomb potential. The potential used is of the following form:

\[ V(r) = \frac{e^2 q q' \hbar c}{4\pi r} + \frac{e^2 q q' \hbar c}{4\pi r_{cutoff}} \quad (3) \]

where has been used for the coordination of dimensions. are the electrical charges of diquark and antidiquark, respectively. A distinct feature of the potential is the parameter \( e \), which (in natural units) is related to the fine-structure constant \( \alpha \) by [24].

Because we wanted to determine r_cutoff arbitrarily, we added the second term to Coulomb potential so that the potential could change proportionally. We changed r_cutoff within 0.1 to 15 nm interval.

First, we assigned 0.1 to r_cutoff and change the other input data and ran the program. We kept changing r_cutoff 0.1 by 0.1 until we arrived at a suitable binding energy as the program output. We carried out these operations for three systems Fig. 1 and 2 illustrate the binding energy in terms of r_cutoff.

Fig. 1. Variation of Binding Energy with r_cutoff Variation.

4. Investigation of [c, u] [c, \bar{u}] System’s Binding Energy at [0.1, 1] fm Interval

We studied system’s binding energy at 0.1 to 1 fm interval. The potential used for this purpose was the strong interaction potential of the following form:

\[ V(r) = \frac{-4 \alpha \hbar c}{3 r} + \frac{\hbar c \alpha_s}{r_{cutoff}} \quad (4) \]
Variation 

R_cutoff is assigned the values between 0.1 and 1 fm. stands for strong interaction constant at high energies [24]. Based on the suitable energies, figure 3 is derived:

Figure 3: Variation of Binding Energy with R_cutoff Variation.

At the distance below 0.1 fm, diquark-antidiquark form a tetraquark since the figure shows minimum r at this point, where the binding energy is -247 Mev. This energy remains constant at 0.1 to 0.5 fm interval. This tetraquark’s Binding energy is -247 Mev. The mass of is 3699 Mev based on the following equation:

\[ (5) \]

This equation is similar to meson mass equation [25]:

\[ (6) \]

as we assume diquark and antidiquark system as point particles, our result (3699 Mev) is in agreement with [23].

5. Data Analysis and Conclusion

The figures for and are only slightly different at the distance below 1 fm. This difference is due to the difference between mass and spin. In other words, it is the spin difference that leads to mass difference. As the distance exceeds 1 fm, the two figures overlap even more. Therefore, at distances over 1 fm, the spin effect is trivial and thus negligible. Figure 2 shows that the two systems have significantly different energies at below 1 nm distance. However, as the distance increases, figures 1 and 2 tend to present the same quantity, and their energy differences decrease. This phenomenon is because of their different charges.

Acknowledgments

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References


