MULTI-REGGE PROCESSES AND THE POMERANCHUK SINGULARITY IN NON ABELIAN GAUGE THEORIES

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My talk is based on the results obtained by V.S.Fadin, E.A.Kuraev and myself. I would like to tell you about our calculations of the high energy asymptotics in non-Abelian gauge theories. The papers of other authors on the subject are enumerated in ref.1. Our final expression for the elastic scattering amplitudes is obtained as a generalization of our results valid up to the eight order of perturbation theory. The result of the tenth order calculation by B.M.McCoy and T.T.Wu coincides with the corresponding term in the perturbative expansion of our final expression and therefore it supports indirectly our conjecture of a multi-Regge form of production amplitudes.

To study the Pomeranchuk singularity at $\hat{s} = 1$ in perturbation theory one is obliged to involve vector particles. Renormalizable field models containing vector particles are based on gauge theories. In the simplest gauge theory quantum electrodynamics (QED) - the leading $\hat{s}$ plane singularity turns out to be a fixed branch point at $\hat{s} = 1 + \frac{11}{32} \pi \hat{A}^2$. The violation of the Froissart bound here is a result of the fact that the photon is not reggeized in QED. More realistic models for the strong interaction may be based on the Yang-Mills fields with the Higgs phenomenon. In these theories the vector boson is reggeized as it was shown by many authors by studying several orders of the perturbation theory. The fermion is also reggeized - see ref. 2.

Therefore the main reason for the violation of the Froissart bound in the leading logarithmic approximation is absent. But our result is a disappointing one: the leading plane singularity turns out to be to the right of the point $\hat{s} = 1$. 

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2. The field model we have used contains an isotriplet of vector fields with mass $M$ and a scalar isosinglet $\Phi$. This model is obtained owing to the Higgs phenomenon from the theory with the Yang-Mills fields interacting with a doublet of scalar particles with negative $m^2$.

We have calculated the asymptotics of elastic scattering amplitudes in different channels in the kinematical region:

$$s \geq M^2, \quad -t \sim M^2$$  \hspace{1cm} (Ia)

provided that

$$q^2 \ll 1, \quad \frac{q^2}{M^2} \ll 1. \hspace{1cm} (Ib)$$

The calculation method is based on using the dispersion relations in $s$- and $t$-channels and unitarity conditions. Therefore it is necessary to know inelastic amplitudes $A_{2 \rightarrow 2+n}$ to find the elastic one. We have verified in lowest orders of the perturbation theory that in the multi-Regge kinematical region the following expression holds for inelastic vector-vector scattering amplitudes:

$$A_{2 \rightarrow 2+n} = \sum_{R=2,0} \left( \begin{array}{c} c_1 \\ 2z_1 \\ m_1 \\ \end{array} \right) \left( \frac{t}{\Delta} \right)^{-1} \gamma_{c_1} \gamma_{c_2} \cdots \gamma_{c_n}$$

$$\left( \begin{array}{c} n \rightarrow m \\ \Delta \\ \end{array} \right)$$

$$\frac{t^n - m^n}{t - m},$$

where $\Delta$ is the squared sum of the energies of the produced particles in their c.m. system, $c_1, c_2, \ldots, c_n$ are the isotopic states of the virtual vector bosons with mass $\sqrt{t}$, and

$$\gamma(t) = 1 + \frac{q^2}{2M^2} (t - M^2) \int \frac{d^2k}{(k^2 + m^2)} \left[ k^2 - (k - k')^2 - m^2 \right]$$

For $n = 0$ the relation (2) demonstrates the reggeization of the vector boson ($\alpha(M^2) = 1$). For $n > 1$ we checked the relation (2) in the Born approximation and by calculating first radiative corrections. In Eq. (2) the vertex functions $\Gamma$ and $\gamma$ have the following form:

$$\Gamma_{a \rightarrow 2} = \frac{1}{2} \delta_{a, c_1, c_2, \ldots, c_n} = \delta_{c_1} \delta_{c_2} \cdots \delta_{c_n}$$

$$\gamma_{c_1} \gamma_{c_2} \cdots \gamma_{c_n}$$

In the case of scalar particle production we have

$$\gamma_{c_1} \gamma_{c_2} \cdots \gamma_{c_n} = \left\{ \begin{array}{ll} -t, & \frac{1}{2}, \lambda = 1, 2, \ldots, n \\ -\frac{1}{2}, \lambda = 3, 4, \ldots, n \end{array} \right\} \delta_{c_1} \delta_{c_2} \cdots \delta_{c_n}$$

$$\delta_{c_1} \delta_{c_2} \cdots \delta_{c_n} = \left\{ \begin{array}{ll} -t, & \frac{1}{2}, \lambda = 1, 2, \ldots, n \\ -\frac{1}{2}, \lambda = 3, 4, \ldots, n \end{array} \right\} \delta_{c_1} \delta_{c_2} \cdots \delta_{c_n}$$

The multi-Regge region

$$s_{i+1} \geq M^2, \quad \gamma_{s_{i+1}} \sim s_i, \quad q_i \sim M$$

gives the main contribution to the $s$ and $t$ channel discontinuities corresponding to $(n + 2)$ particle intermediate states. By using the dispersion relation in $s$ channel and by summing contributions from all intermediate states we obtain:

$$\sum_{n \rightarrow m} \sum_{a=1}^{n} \int \frac{d^2k}{(k^2 + m^2)} \left[ k^2 - (k - k')^2 - m^2 \right]$$

$$\delta_{\lambda_1 \lambda_2} C_{\lambda_1} C_{\lambda_2} + \delta_{\lambda_3 \lambda_4} C_{\lambda_3} C_{\lambda_4}$$

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$B_\lambda = -\frac{s}{2} + \frac{s}{2} \left( \lambda + 1 \right)$

where $\lambda$ is the total isospin in the $t$ channel and $t$ is the total isospin in the channel $t$. The Clebsch-Gordan coefficients $\epsilon_{\lambda_1 \lambda_2 \lambda_3 \lambda_4}$ satisfy the following equation:

$$\sum_{\lambda_1 \lambda_2 \lambda_3 \lambda_4} \epsilon_{\lambda_1 \lambda_2 \lambda_3 \lambda_4} (k_1, k_2, k_3, k_4) = \frac{1}{2} \left[ (k_1, k_2, k_3, k_4) \right]$$

$$\delta_{\lambda_1 \lambda_2} C_{\lambda_1} C_{\lambda_2} + \delta_{\lambda_3 \lambda_4} C_{\lambda_3} C_{\lambda_4}$$

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$$\delta_{\lambda_1 \lambda_2} C_{\lambda_1} C_{\lambda_2} + \delta_{\lambda_3 \lambda_4} C_{\lambda_3} C_{\lambda_4}$$

and

$$\delta_{\lambda_1 \lambda_2} C_{\lambda_1} C_{\lambda_2} + \delta_{\lambda_3 \lambda_4} C_{\lambda_3} C_{\lambda_4}$$

The calculation of these contributions requires the knowledge of the inelastic amplitudes $A_{2 \rightarrow 2+n}$.
The solution of this equation for $\mathcal{T} = 1$ is

$$\begin{aligned} \xi_j(k, \zeta, \mu) \bigg|_{\mathcal{T} = 1} = -\frac{\Delta \gamma^2}{(q^2 + m^2)(\Delta + 1 - \gamma^2(q^2))} \end{aligned}$$

which shows that the assumption (2) is self-consistent: we have a bootstrap scheme in which a multi-Regge equation gives in the $j$ plane the assumed Regge behaviour.

For the cases $\mathcal{T} = \mathcal{C}$ and $\mathcal{T} = \mathcal{R}$, Eq. (5) leads to the $j$ plane cut singularities resulting from two reggeised vector meson exchange. In the vacuum channel $\mathcal{T} = 0$ the equation can be solved exactly in the region $\mathcal{K}_j \gg M^2$. The leading $j$ plane singularity turns out to be a square root branch point at $\frac{j}{\mathcal{K}} = 1 + \frac{2\gamma \Delta}{\mathcal{K}} \ln \frac{\mathcal{K}}{2 \pi}$, (In the general case of the gauge group $SU(N)$ the branch point is located at $\frac{j}{\mathcal{K}} = 1 + \frac{2\gamma \Delta}{\mathcal{K}} \ln \frac{\mathcal{K}}{2 \pi}$).

Thus, the total cross section in the non-Abelian gauge theory increases as a power of $\mathcal{T}$ although the cross sections for the production of any finite number of particles decrease rapidly with energy owing to the multi-Regge form of inelastic amplitudes (2). The reason for the violation of the Froissart theorem is that the $\mathcal{T}$ channel elastic unitarity is not fulfilled in the leading logarithmic approximation (in Eq. (2) the contribution of the vacuum $t$-channel state should be taken into account when $\mathcal{K}_j \gg M^2$).

There is an interesting phenomenon valid in the leading logarithmic approximation. It can be shown that at large $\mathcal{K}_j \gg \frac{m^2}{\mathcal{T}}$ the essential region of integration over $\mathcal{K}_j$ grows with energy. If one modifies Eq. (7) by introducing an invariant charge $\frac{2}{\mathcal{T}} \Delta \gamma^2 \left( \frac{\mathcal{K}_j}{m^2} \right)^{\frac{2}{\mathcal{T}}} \left( \frac{\mathcal{K}_j}{m^2} \right)^{\frac{2}{\mathcal{T}}} \mathcal{K}_j$ instead of the renormalized charge $\gamma^2$ it results in the decay of the fixed branch point into moving poles (their total number is of the order of $\frac{\mathcal{K}_j}{m^2}$) which, unfortunately, remain to the right of $\frac{j}{\mathcal{K}} = 1$.

It should be stressed that in our leading logarithmic approximation the vacuum singularities emerge as bound stated on only two Reggeons with $\mathcal{T} = 1$. One can hope, however, that an appropriate Reggeon field theory similar to the Gribov calculus with Reggeon vertices which can be computed by perturbative methods would lead to a selfconsistent theory compatible with the Froissart restriction.

References

5. Y.S. Fadin, V.S. Sherman, JETP, Pis'ma, 23, 599 (1976).
MULTIPARTICLE REGGE POLES AND THEIR TRAJECTORIES

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This talk is based on the three papers by A.G. Sedrakian and author /1-2/.

It became usual to relate to the reggeon a sum of ladder diagrams though the $\frac{g}{\alpha s}$ theory gives its intercept near $\gamma = 1$.

In this connection it always seemed attractive to provide the positive intercept by the inclusion of multiparticle states in the t-channel. Well known Mandelstam diagrams corresponding to multi-particle states in t-channel and giving the branch points in j-plane, of course, are not enough, as they refer to the multiparticle configurations where only "two-body bound" states are present. However, in the theory with attraction it is possible the formation of many-particle bound states. This problem was first considered in the field theory by McCoy and Wu /4/. They showed, that the contribution of fig.la type diagrams to the asymptotics dominates over the contribution of fig.lb type Mandelstam diagrams corresponding to the reggeon-particle cut.

The detailed analysis shows that any line from $n_1$ group can be connected with any line of the $n_2$ group, giving thus the additional factor $g^2 \frac{3}{m_2^2}$. The connections between one group lines are not essential giving the factor $\frac{g^2}{3}$. Leading logarithmic asymptotics of the diagram of an elastic scattering with $n=n_1+n_2$ particles in the t-channel and horizontal lines has a form ($q_\perp$ is the transverse momentum):

$$\text{Fig. 2}$$

- First, consider small $q_\perp$ values ($|q_\perp| < m$). Summing all the topologically different diagrams of the 2($l+n$) order obtained by means of the rearrangement of horizontal lines as well as of all the possible crossings of vertical lines both within $n_1$ and $n_2$ group and the crossings of $n_1$ group lines with $n_2$ group ones, and summing over $L$ we find the following asymptotics (for $q_\perp = 0$):

$$\frac{-i (g^2/m)^{n+1}}{(l+2m_4)} g^2 \left(\frac{3}{m} \right) \left(\frac{3}{m} \right)^m \left(\frac{2}{g} \right)^{m-1} \left(\frac{1}{g_{\alpha s}} \right)^{m_2} \times \left(\frac{3}{m^2} \right)^{l} \delta(n_1, n_2).$$

We generalise here this important result for the case of an arbitrary number of particles in t-channel.

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We generalise here this important result for the case of an arbitrary number of particles in t-channel.

Let there be $n_1+n_2$ particles in t-channel, of which $n_1$ emit and $n_2$ absorb an arbitrary number of particles. Fig.2 gives the example of the considered diagrams.