19 Flavour Symmetry Models after Daya Bay and RENO

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Abstract We discuss the impact of the recent measurements of the lepton mixing angle $\theta_{13}$ by the Daya Bay and RENO reactor experiments on neutrino mass models based on flavour or family symmetry.

19.1 Introduction

It is one of the goals of theories of particle physics beyond the Standard Model to predict quark and lepton masses and mixings, or at least to relate them. While the quark mixing angles are known to all be rather small, by contrast two of the lepton mixing angles, the atmospheric angle $\theta_{23}$ and the solar angle $\theta_{12}$, are identified as being rather large. Until recently the remaining reactor angle $\theta_{13}$ was unmeasured. Recently Daya Bay [1] and RENO [2], collaborations have measured $\sin^2(2\theta_{13}) \approx 0.1$ corresponding to $\theta_{13} \approx 9^\circ$.

![Simple lepton mixing patterns](image)

Figure 19.1: Simple lepton mixing patterns, all involving zero reactor angle and maximal atmospheric angle, and distinguished by solar angles as shown.
From a theoretical or model building point of view, one significance of this measurement is that it excludes the well known tri-bimaximal (TB) lepton mixing pattern shown in Fig. 19.1 in which the atmospheric angle is maximal, the reactor angle vanishes, and the solar mixing angle is approximately $35.3^\circ$. When comparing global fits to TB mixing it is convenient to express the solar, atmospheric and reactor angles in terms of deviation parameters ($s$, $a$ and $r$) from TB mixing$^{[3, 4]}$:

$$
\sin \theta_{12} = \frac{1}{\sqrt{3}}(1 + s), \quad \sin \theta_{23} = \frac{1}{\sqrt{2}}(1 + a), \quad \sin \theta_{13} = \frac{r}{\sqrt{2}}.
$$

(19.1)

For example, the global fit in $^{[5]}$ yields the $1\sigma$ ranges for the TB deviation parameters:

$$
−0.066 \leq s \leq −0.013, \quad −0.146 \leq a \leq −0.094, \quad 0.208 \leq r \leq 0.231,
$$

(19.2)

assuming a normal neutrino mass ordering. As well as showing that TB is excluded by the reactor angle being non-zero, Eq. (19.2) shows a preference for the atmospheric angle to be below its maximal value and also a slight preference for the solar angle to be below its tri-maximal value. An interesting possibility consistent with the data is Tri-bimaximal-Cabibbo (TBC) mixing$^{[6]}$ with $a = s = 0$ and $r$ set equal to the Wolfenstein parameter $\lambda$, corresponding to $\theta_{13} = 9.2^\circ$.

As a result of the rapidly changing landscape of neutrino mixing parameters, many models based on discrete family symmetry which were proposed initially to account for TB mixing are now either excluded, or have been subjected to modification$^{[7]}$. This not only applies to TB mixing but also to other simple lepton mixing patterns as shown in Fig. 19.1, including bi-maximal (BM) and Golden ratio (GR). All these simple mixing patterns can all be enforced by an underlying symmetry, as we will shortly discuss. The fact that they are all excluded therefore calls into question the symmetry approach. However it is worth noting at the outset that simple variants of TB mixing are still viable such as those shown in Fig 19.2 and they may also arise from family symmetry, as we shall discuss later.

Some authors regard the large reactor angle as signalling an anarchical neutrino mass matrix$^{[8]}$. The basic choice facing theorists following Daya Bay and RENO is therefore: symmetry vs anarchy, as shown in the left panel of Fig. 19.3. In this talk we shall continue to follow the symmetry approach, based on family symmetries such as those shown in the right panel of Fig. 19.3, where the family symmetry may be implemented either directly or indirectly as also indicated in the left panel, where this classification was introduced in$^{[9]}$.

It is worth recalling the situation before the measurement of the reactor angle. For example, let us consider TB mixing. In this case, simple finite family symmetries such as $A_4$ and $S_4$ were capable of embedding the Klein symmetry of the TB neutrino mass matrix. For example $S_4$ contains the Klein generators $S$, $U$, together with $T$ enforcing the diagonality of the charged lepton mass matrix in this basis. In the direct approach to models of TB mixing, the family symmetry $G_F$ is broken by flavons such that the $S$, $U$ preserving flavons $\phi_{S,U}$ only appear in the neutrino sector, while the $T$ preserving flavon $\phi_T$ only appears in the charged lepton sector as shown in the left half of Fig 19.4. Similar arguments may be applied to account for the other simple mixing patterns, where BM mixing can also emerge from $S_4$, while GR mixing may arise from $A_5$. All these possibilities BM, TB, GR involve zero reactor angle and maximal atmospheric angle due to the 2-3 symmetry enforced by the $U$ generator of the Klein symmetry.
Figure 19.2: Simple variants of TB mixing, namely: tri-bimaximal-reactor (TBR) mixing; tri-maximal mixing with first column of TB form (TM1); tri-maximal mixing with second column of TB form (TM2). The distinctive atmospheric sum rules are indicated in the notation of Eq. 19.1.

Figure 19.3: Left panel shows the simple choice facing theorists after Daya Bay and RENO. The right panel shows some possible family symmetries.

Alternatively, in the indirect approach, the family symmetry \( G_F \) is completely broken by three flavons whose VEVs are aligned along the columns of the TB mixing matrix, but which appear quadratically in the neutrino sector, as shown in the right half of Fig. 19.4.

Following Daya Bay and RENO, the possible strategies for direct models are as shown in Fig. 19.5. For the smaller groups such as \( A_4, S_4, A_5 \), on the left-hand part of Fig. 19.5, which all predict zero reactor angle at the leading order (LO), the possible options are: break the \( T \) generator by invoking charged lepton corrections; break the \( U \) generator by some special higher order (HO) corrections, leaving the \( S \) generator in tact, leading to special mixing patterns such as tri-maximal mixing; or break both \( S, U \) by general HO corrections, leading to a generally unpredictable scheme. We now consider each possibility in turn.

The case where only the \( T \) generator is broken by a non-diagonal charged lepton mass matrix
Figure 19.4: The direct vs indirect approach to family symmetry models of TB mixing before Daya Bay and RENO.

Figure 19.5: Possible strategies for direct models after Daya Bay and RENO.

leads to solar sum rules involving $\cos \delta$ as follows \cite{10, 11}:

\begin{align*}
BM : \quad \theta_{12} &\approx 45^\circ + \theta_{13} \cos \delta \\
TB : \quad \theta_{12} &\approx 35.26^\circ + \theta_{13} \cos \delta \\
GR : \quad \theta_{12} &\approx 31.7^\circ + \theta_{13} \cos \delta.
\end{align*}

(19.3)

Since $\theta_{13} \approx 9^\circ$ the requirement of a solar angle $\theta_{12} \approx 34^\circ$ leads to a distinctive prediction for $\cos \delta$ in each case. The basic assumption is that the charged lepton correction is dominated
by Cabibbo-like (1,2) mixing with a charged lepton mixing angle equal to the Cabibbo angle, giving $\theta_{13} = \lambda/\sqrt{2}$ as in TBC mixing [6].

The case where only the $U$ generator is broken (with $S, T$ preserved) can lead to a simple pattern of mixing, namely TM2 mixing, with examples of such models given in [12–14]. Other TB variants in Fig. 19.2 can also arise from the indirect approach as shown in Fig. 19.6. For example, TBR and TM1 mixing can arise from different kinds of sequential dominance (SD) with the alignments shown in Fig. 19.6. CSD2 yields TM1 mixing as shown in Fig. 19.2 [15]. PCSD yields TBR mixing as shown in Fig. 19.2 [16, 17]. If we set $r = \lambda$ then special case corresponds to TBC mixing [6].

Finally, if a larger family symmetry such as $\Delta(96)$ is assumed, as in the right-hand part of Fig. 19.5, then it is possible to have a different kind of Klein symmetry at the LO which already gives a reactor angle of $\theta_{13} \approx 12^\circ$, together with $\theta_{12} \approx \theta_{23} \approx 36^\circ$, closer to the desired value. However, in the framework of a GUT model, modest charged lepton corrections of about 3° can correct these angles to acceptable values of $\theta_{13} \approx 9.6^\circ$, together with $\theta_{12} \approx 33^\circ$ and $\theta_{23} \approx 37^\circ$ [18].

**Indirect Models (general strategy)**

Starting point is type I see-saw

$\nu_{\text{L}} = \left( \begin{array}{ccc} A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \\ A_3 & B_3 & C_3 \end{array} \right) \ M_{\text{CKM}} = \left( \begin{array}{ccc} M_1 & 0 & 0 \\ 0 & M_2 & 0 \\ 0 & 0 & M_3 \end{array} \right)$

$m^\nu = \frac{AA^T}{M_1} + \frac{BB^T}{M_2} + \frac{CC^T}{M_3}$

$A^T = (A_1, A_2, A_3) \ B^T = (B_1, B_2, B_3)$

Construct the columns $A$, $B$, $C$ from flavon fields $G_f$ yields special vacuum alignments, for example:

- $(A, B, C)$ proportional to columns of PMNS called Fermi Dominance (FD)
- $AA^T/M_1 < BB^T/M_2 < CC^T/M_3$ called Sequential Dominance (SD)
- SD with $B = (1, 1, -1)$ and $C = (0, 1, 1)$ called Constrained SD gives TB mixing
- SD with $B = (1, 1, -1)$ and $C = (0, 1, 1)$ called Partially SD gives TBR/C mixing
- SD with $B = (1, 2, 0)$ and $C = (0, 1, 1)$ called CSD2 gives TM1 mixing

Figure 19.6: Possible strategies for indirect models before/after Daya Bay and RENO. CSD yields TB mixing. CSD2 yields TM1 mixing as shown in Fig. 19.2. PCSD yields TBR mixing as shown in Fig. 19.2. If we set $r = \lambda$ then special case corresponds to TBC mixing.

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Bibliography