The Higgs particle and higher-dimensional theories

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In spite of the great success of LHC experiments, we do not know whether the discovered “standard model-like” Higgs particle is really what the standard model predicts, or a particle that some new physics has in its low-energy effective theory. Also, the long-standing problems concerning the property of the Higgs and its interactions are still there, and we still do not have any conclusive argument on the origin of the Higgs itself. In this article we focus on higher-dimensional theories as new physics. First we give a brief review of their representative scenarios and closely related 4D scenarios. Among them, we mainly discuss two interesting possibilities of the origin of the Higgs: the Higgs as a gauge boson and the Higgs as a (pseudo) Nambu–Goldstone boson. Next, we argue that theories of new physics are divided into two categories, i.e., theories with normal Higgs interactions and those with anomalous Higgs interactions. Interestingly, both the candidates for the origin of the Higgs mentioned above predict characteristic “anomalous” Higgs interactions, such as the deviation of the Yukawa couplings from the standard model predictions. Such deviations can hopefully be investigated by precision tests of Higgs interactions at the planned ILC experiment. Also discussed is the main decay mode of the Higgs, \( H \rightarrow \gamma \gamma \). Again, theories belonging to different categories are known to predict remarkably different new physics contributions to this important process.

1. Introduction

1.1. What the discovery of the standard model-like Higgs particle means

We have heard of the very impressive results from LHC experiments that a new particle with a mass of 126 (GeV), which behaves like the Higgs particle of the standard model (SM), has been discovered at CERN. So far, the properties of the particle seem to be consistent with the Higgs of the SM, but it would be premature to conclude whether it is really the Higgs of the SM or a standard model-like Higgs particle predicted by some theory of new physics (physics beyond the standard model). In this article we discuss what the discovery of this particle means in the context of higher-dimensional theories with extra dimensions.

1.2. New physics and its “decoupling limit”

Although we are interested in the possibility that the standard model-like Higgs is some particle predicted by new physics, it is at the same time a clear fact that the SM is a very successful theory at low energies (\( E \leq M_W \)). We thus expect that any theory of new physics reduces into the SM at low energies. To be more precise, we can think of a “decoupling limit” in new physics theories. The limit is achieved by sending the typical mass (energy) scale of new physics, say \( M \), to infinity: \( M \rightarrow \infty \). In that limit, various new particles predicted to exist in the new physics with masses of...
\( O(M) \) are expected to “decouple” from the low-energy sector [1], and the theories are expected to reduce into the SM. From a field theoretical point of view, the effects of new particles at low energies are described by operators with mass dimension higher than 4 (from the viewpoint of 4D space-time), “irrelevant operators” whose Wilson coefficients are suppressed by the inverse powers of \( M \) \((1/M^2)\) in the case of operators with mass dimension 6). Thus the effects of new heavy particles are suppressed and they “decouple” from the low-energy effective Lagrangian.

Let us note that, according to the argument based on dimensional analysis, marginal or relevant operators with mass dimensions equal to or less than 4 may be affected by the presence of heavy new particles. The argument of Ref. [1] is that such effects only affect the renormalization procedure and can be absorbed into the bare parameters in the process of renormalization. Let us note, however, that the hierarchy problem—to be more precise, the problem of quadratically divergent quantum correction to the Higgs mass—exactly concerns the relation between the observed Higgs mass and the corresponding bare mass(-squared) parameter. Thus, when we consider the hierarchy problem, the contributions of new heavy particles should be seriously taken into account. This is why superpartners in supersymmetry (SUSY) theory can play an important role in the solution to the hierarchy problem. Also, in a type of higher-dimensional gauge theory, “gauge–Higgs unification (GHU)” [2–7], discussed later, the summation over the contributions of all Kaluza–Klein (KK) modes makes the quantum correction to the Higgs mass-squared finite, thus solving the hierarchy problem [8].

For instance, in the MSSM (minimal supersymmetric standard model), two Higgs doubles \( H_u, H_d \) are introduced to preserve SUSY. The lighter CP-even neutral scalar particle denoted as a linear combination, \( \sqrt{2}[\cos \alpha(\text{Re}H_u^0 - \frac{v_u}{\sqrt{2}}) - \sin \alpha(\text{Re}H_d^0 - \frac{v_d}{\sqrt{2}})] \), is identified with the SM-like Higgs. The angle \( \alpha \) is fixed by the parameters in the Higgs potential and \( v_u, v_d \) are the VEVs of the two Higgs doublets. The couplings of the SM-like Higgs with matter fermions and gauge bosons generally contain the parameters \( \alpha \) and \( \beta \), where \( \beta \) represents the ratio of the VEVs, \( \tan \beta \equiv v_u/v_d \). The decoupling limit of the MSSM is achieved by sending the SUSY breaking mass scale \( M_{\text{SUSY}} \) to infinity. In this limit, it turns out that just a linear combination of two Higgs doublets that alone develops the VEV \( v = \sqrt{v_u^2 + v_d^2} \) becomes the lighter scalar and is identified with the SM-like Higgs, as we naturally expect. Correspondingly, in this limit, \( \alpha \to \beta - \frac{\pi}{2} \) and all interactions are known to reduce to those we expect in the SM, as is anticipated from the decoupling theorem [1]. We therefore conclude that the decoupling limit of the MSSM is just the SM, although, in the MSSM, in contrast to the case of the SM, the Higgs mass is “calculable”, since the quartic coupling of the Higgs potential receives a contribution only from the “D-term” and the Higgs mass at the tree level is more or less equivalent to the weak gauge boson masses.

Now let us turn to the case of higher-dimensional theories. Because of the presence of the extra dimension, we now get an infinite tower of KK modes with higher extra space momenta and therefore higher 4D masses. The non-zero KK modes are new heavy particles. Hence in higher-dimensional theories the decoupling limit is achieved by

\[
M_c = \frac{1}{R} \to \infty ,
\]

(1.1)

where \( R \) is a generic size of the extra dimension, being the radius when the extra space is just a circle \( S^1 \), and \( M_c \) is the corresponding “compactification” mass scale. In this limit, all (non-zero) KK modes, having masses of the order of \( M_c \), are expected to decouple from the low-energy effective Lagrangian.

As a new feature of higher-dimensional gauge theory, it is basically possible to construct a theory in which the Higgs-like particle does not exist in the decoupling limit, i.e. the higgsless model [9]. In this
type of theory the KK zero-mode with vanishing extra space momentum is excluded by imposing a Dirichlet-type boundary condition for the Higgs field along an interval, as the 1D extra space. Thus all KK modes of the Higgs field have masses of $O(M_c)$, while the violation of the unitarity of the scattering amplitude of the (longitudinal components of) weak gauge bosons is remedied at higher energies by the presence of a tower of massive non-zero KK modes of the gauge bosons. The decoupling limit of the higgsless theory does not contain SM-like Higgs.

Now we know that at least an SM-like Higgs has been discovered; the higgsless scenario thus seems to be excluded unless a theory with lower $M_c$ is possible without contradicting various experimental data.

Also ruled out is the theory where, even though the decoupling limit is the SM, the modification of the Higgs sector (Higgs interaction) is so huge that it already contradicts reality. The GHU model with H-parity [10,11] is probably such a theory. H-parity is a discrete symmetry under which, among SM fields, only the Higgs field has odd parity while the other fields have even parities. This symmetry exists only when the weak scale and the compactification scale are comparable:

$$x \equiv \frac{g_4}{2} \pi R v = \pi \frac{M_W}{M_c} = \frac{\pi}{2},$$

where $g_4$ is the 4D gauge coupling constant. Note that, even though $M_c$ is near the weak scale, the masses of non-zero KK modes can be sufficiently large due to the presence of the warp factor when the theory is formulated on the Randall–Sundrum space-time [12–17]. Just as the R-parity implies in the MSSM, the H-parity implies that a single Higgs particle cannot decay into ordinary SM particles, thus making the Higgs stable. As the recently discovered SM-like Higgs particle decays into 2 photons, this interesting scenario seems to be inconsistent with the data.

There still remain other interesting higher-dimensional scenarios. It should be noticed that some of these scenarios are not only consistent with the data, having the SM at their decoupling limits, but they also predict some characteristic deviations from the SM predictions, as we will see below.

Even though the properties of the SM-like Higgs particle seem to be consistent with that of the SM Higgs, we still have a strong motivation to investigate further the properties of the Higgs. Namely, in the SM, in clear contrast to the sector of gauge interaction, the Higgs sector is mysterious, having several long-standing problems, such as the hierarchy problem, as already mentioned, and the problem of many arbitrary parameters in the sector of Higgs interactions, such as Yukawa couplings. We still do not understand the origin of the generation-dependent hierarchical fermion masses and generation (flavor) mixings. These problems all come from the fact that, in contrast to the case of the gauge interactions, there is no guiding principle (symmetry?) to restrict the interactions of the Higgs. Let us note that, even though many of the new physics theories have been proposed mainly in order to solve the hierarchy problem, not all of them have mechanisms to restrict the Yukawa coupling. For instance, in the MSSM there is no principle to restrict the couplings (except for gauge invariance) and the Yukawa couplings are free parameters to start with.

These problems suggest that we have not (fully) understood the origin of the Higgs itself. From such points of view, it will be of crucial importance to study how new physics theories predict the Higgs interactions and to perform precision tests of those interactions. As mentioned above, some higher-dimensional theories and also some closely related 4D theories make very specific predictions concerning the Higgs interactions, i.e. “anomalous Higgs interactions”.

Such precision tests will not be easy to perform at the LHC experiment and are expected to be done at the planned ILC experiment.
2. Brief review of higher-dimensional theories

We now summarize very briefly some of the representative scenarios of new physics based on higher-dimensional theories. Also discussed are 4D scenarios closely related to higher-dimensional gauge theory, i.e. the scenarios in which Higgs is a (pseudo) Nambu–Goldstone (NG) boson due to some global symmetries. Since solving the hierarchy problem has been the main motivation for many new physics theories, we categorize the scenarios depending on their attitudes concerning the hierarchy problem.

2.1. Theory without a solution to the hierarchy problem

The representative and popular scenario of this type is the so-called “universal extra dimension (UED)”. It is a straightforward extension of the SM to its higher-dimensional bulk version [18]. All SM particles are allowed to propagate universally in the higher-dimensional bulk space. The extra dimension is assumed to be an orbifold torus divided by its discrete symmetry, such as $S^1/Z_2$ or $T^2/Z_4$, in order to realize a chiral (left–right asymmetric) theory needed to incorporate the SM. The theory has a discrete symmetry called KK-parity, under which particles of even (odd) KK numbers have even (odd) KK-parities. Thus the lightest first KK mode, having odd KK-parity, cannot decay into ordinary SM particles with even KK-parities and thus becomes stable, just like the LSP in the MSSM with R-parity; it therefore is a candidate for the dark matter. This KK-parity makes the production of a single first KK excited state impossible at ILC. On the other hand, the production of a second KK excited state with even KK-parity is possible through the KK-number violating processes induced by the quantum loop effects [19].

2.2. Theories with a mechanism to solve the hierarchy problem not invoking any symmetry

We now discuss the scenarios formulated in higher-dimensional space-time that aim to solve the hierarchy problem, namely, to explain naturally the hierarchy between the Planck scale $M_{\text{pl}}$ and $M_W$. The scenarios we discuss are the ADD model with large extra dimension [20] and the Randall–Sundrum model with warped extra dimension [21]. In both scenarios, the SM particles live in a 4D space-time (a brane), not an extra dimension, and only gravity propagates in the bulk. This property leads to new types of solutions to the hierarchy problem, not seen in the theories in ordinary 4D space-time. The mechanism to solve the hierarchy problem does not invoke any symmetry.

Let us first discuss the ADD model. When Einstein proposed his unified theory of gravity and electromagnetism along the idea of Kaluza and Klein, the extra dimension was assumed to be small, i.e. $M_c \simeq M_{\text{pl}}$: a “small extra dimension”. The basic reason for this is that, in the unified field theory, electromagnetic interaction originates from gravity interaction. Thus the electric charge $e$ is inevitably proportional to the (square root of) the Newton constant $G_N$. On the other hand, the source of the gravity is the energy-momentum and the electric charge should also be proportional to the extra-dimensional momentum of the order of $M_c$. In this way, we obtain the relation

\[ e = \frac{4\pi G_N}{R} \rightarrow R = \frac{4\pi G_N}{e} \approx 4 \times 10^{-32} \text{ (cm)}. \]  

In the ADD model, all the SM particles, including the photon, are assumed to live in the brane, and the size of the extra dimension is free from the constraint coming from the value of the electric charge. In this scenario, the original higher-dimensional Planck scale $M_{\text{pl}}^{(0)}$ is of the order of the weak scale $M_{\text{pl}}^{(0)} \sim M_W$, and, to begin with, there is no hierarchy between the Planck scale and the weak scale. Too small a Planck scale may lead to an unacceptably strong gravitational force. This potential
difficulty is solved by assuming a large extra dimension. Namely, the “gravitational flux” is spread toward the direction of the large extra dimension, thus recovering ordinary very weak 4D gravity. Assuming the presence of an $n$-dimensional extra dimension, the 4D Planck scale $M_{pl}$ is given as

$$M_{pl}^2 \sim M_{pl}^{(0)2+n} \cdot R^n.$$  

Assuming that $M_{pl}^{(0)} \sim 1$ (TeV), we get $R \sim 0.1$ (mm) for $n = 2$ (for $n = 1$ the obtained $R$ is unacceptably large). This is the scenario of the “large extra dimension”.

While the gravity at distances larger than the compactification size reduces to ordinary Newtonian gravity, at shorter distances, the gravity recovers the original higher-dimensional one whose mass scale is not $M_{pl}$ but $M_{pl}^{(0)} \sim M_W$. Thus the most stringent bound for the ADD model comes from the data from LHC. A search for the anomalous jet + missing $E_T$ events at CMS has put a bound $M_{pl}^{(0)} > 3.0–5.0$ (TeV) for $n = 2–6$ (for higher $n$ the bound is less stringent) [22].

This scenario, however, faces its own new hierarchy problem: the size of the extra dimension 0.1 (mm) for $n = 2$ means $M_c = 2 \times 10^{-2}$ (eV), which implies a new hierarchy $M_c/M_W \sim 10^{-13}$!

Randall and Sundrum pointed out that the new hierarchy problem in the ADD model can be solved once we allow that the bulk space-time is a curved one [21]. They assume that 5D space-time is anti-de Sitter with a negative cosmological constant, while the extra space is compactified on an orbifold $S^1/Z_2$. The solution to the Einstein equation has a “warp factor”:

$$g_{MN} = \begin{pmatrix} e^{-2\kappa|y|} \eta_{\mu\nu} & 0 \\ 0 & 1 \end{pmatrix},$$

where, in the warp factor $e^{-2\kappa|y|}$, $\kappa$ is a parameter of the theory of $O(M_{pl})$ and $y$ is the extra space coordinate. The warp factor may be understood as the factor of “space-like inflation”. In fact it mimics the factor $e^{Ht}$ ($H$: Hubble constant) in the inflationary universe caused by a positive cosmological constant. The sign difference of the cosmological constant may be attributed to the sign difference of the time and space components of the metric tensor. The absolute value $|y|$ in the warp factor is due to the orbifolding of the extra space $S^1/Z_2$.

Assuming that the SM particles all live in the 4D brane at $y = \pi R$ (“visible brane”), the hierarchy between the Planck scale and the weak scale is naturally realized by the warp factor without any hierarchy between the input parameters of the theory:

$$\frac{M_W}{M_{pl}} \sim e^{-\kappa \pi r_c}$$

where $r_c$ is the radius of the circle.

The theory predicts the presence of graviton non-zero KK modes, whose masses are spaced by $O$(TeV) and whose gravity couplings are suppressed not by $M_{pl}$ but by $O$(TeV). Therefore, such new KK gravitons can be searched for at LHC as the resonance states in the $l^+l^−$ and $\gamma\gamma$ final states. The current lower bound on the KK graviton mass ranges from 0.9 (TeV) to 2.1 (TeV) depending on the parameter of the theory [23,24].

Incidentally, how can this Randall–Sundrum model solve the hierarchy problem at the quantum level, i.e. the problem of quadratic divergence in the quantum correction to the Higgs mass, without relying on some symmetry? Though an explicit argument cannot be found in the literature, the quadratic divergence will also be accompanied by the warp factor after the renormalization of the Higgs field as $A^2 e^{-2\kappa \pi r_c}$ and the divergence will be harmless.
2.3. Theories with a mechanism to solve the hierarchy problem invoking some symmetry

The hierarchy problem concerns a very small number. The smallness of the physical observable can be “naturally” preserved under its quantum correction provided that some symmetry is enhanced in the action of the theory when that observable is switched off. This is because, in the case in which the condition is met, even if the observable is induced at quantum level, it should be inevitably proportional to that small number, such that the correction goes away at the limit of exact symmetry.

We now discuss a few representative scenarios of this sort in the context of higher-dimensional theories. Some of them can actually be formulated in 4D space-time, but are closely related to a kind of higher-dimensional gauge theory.

2.3.1. Gauge–Higgs unification. We first discuss the scenario of “gauge–Higgs unification (GHU)”. Einstein attempted to unify the gravity and electromagnetic interactions known at that time mediated by bosons with spin \( s = 2, 1 \) in the framework of 5D gravity theory. We now know that there is also Higgs interaction. So it is natural to expect that gauge and Higgs interactions mediated by \( s = 1, 0 \) bosons are unified in the framework of higher-dimensional gauge theories. For instance, in the simplest 5D U(1) gauge theory, the gauge field \( A_M \) is decomposed into

\[
A_M = (A_\mu, A_y)
\]

(2.5)

where \( A_\mu \) corresponds to the 4D gauge field (and its non-zero KK partners), while the extra space component \( A_y \)—to be more precise, its KK zero mode, behaving as a 4D scalar—is identified with the (SM-like) Higgs field. This is the GHU scenario, the idea of which is not new [2–7]. In particular, Hosotani proposed a mechanism of dynamical spontaneous gauge symmetry breaking due to the VEV of \( A_y \) for the non-Abelian case, the “Hosotani mechanism” [5–7].

One pleasing aspect of this scenario is that it provides a new avenue for the solution to the hierarchy problem by virtue of higher-dimensional local gauge symmetry under which \( A_y \) transforms inhomogeneously: \( A_y \rightarrow A_y + \partial_y \lambda, \lambda \) being a \( y \)-dependent gauge parameter [8]. We know that a photon never acquires mass, even at the quantum level, since the local mass-squared operator \( m_A^2 A_\mu A^\mu \) is forbidden by local gauge symmetry. In the same way, the Higgs (the zero-mode of \( A_y \)) never has a local mass-squared operator, thus eliminating the quadratic divergence. What was stressed in Ref. [8] was the importance of the summation over all KK modes at the quantum correction of the Higgs mass. It is frequently argued that, when we consider low-energy effective theories, only KK zero-modes should be taken into account, probably relying on the wisdom of the decoupling theorem [1]. However, as has already been pointed out in the introduction, when we consider the hierarchy problem, heavy new particles play important roles. Also, we should note that a momentum cutoff spoils local gauge invariance. If we truncate the KK modes at some level, it is equivalent to the cutoff of the extra-dimensional momentum. This is why the summation over all KK modes is crucial to get the finite quantum correction to the Higgs mass.

Actually, the Higgs acquires a finite mass at the quantum level. This is because the zero-mode of \( A_y \), i.e. the Higgs, has a physical interpretation as an Aharonov–Bohm (AB) phase or the phase of a Wilson loop. Let us note that the VEV of the Higgs in this scenario is nothing but a constant gauge field, which gives a vanishing field strength and therefore seems to be just a pure gauge configuration. However, in the case in which the extra dimension is a non-simply-connected space like a circle \( S^1 \), the constant gauge field can be interpreted as a component of the vector potential generated by the magnetic flux, penetrating inside the circle. Thus the Higgs field is not a pure gauge but has a physical meaning as the AB phase or the phase of the Wilson loop \( W \). At the quantum level, the
Higgs potential is induced as (the real part of) the polynomial of $W$. Note that $W = P(e^{i \frac{\phi}{2} \oint A_i dy})$ is of course a gauge invariant but global (not local) operator, which has nothing to do with UV divergence. This is why we get a finite but non-vanishing Higgs mass in GHU, which disappears at the “decompactification limit $R \to \infty$”, where the Wilson loop is trivial. The situation is very similar to the case of finite temperature field theory. At finite temperature the Coulomb potential of a photon is known to have a mass of $O(T)$ ($T$: temperature). The mass disappears as $T \to 0$, corresponding to the decompactification limit.

Another pleasing aspect of GHU is that it may shed some light on the problem of arbitrary Yukawa couplings, for instance. In this scenario the Higgs is originally a gauge field. Therefore Yukawa coupling is gauge coupling to start with and, if we succeed in constructing a realistic model, the Yukawa couplings are expected to be constrained by the gauge principle. On the other hand, it is a non-trivial question how the hierarchical fermion masses are realized starting from the gauge coupling, which is universal for all generations. Fortunately, as a new feature of higher-dimensional gauge theories, when orbifold $S^1/Z_2$ is adopted as the extra dimension, the so-called $Z_2$-odd bulk mass term of the form

$$-\epsilon(y) M \bar{\psi} \psi$$

is allowed. Here $\epsilon(y)$ is a sign function ($\pm 1$ depending on the sign of $y$), which mimics the kink-like configuration of some scalar field and causes the localization of Weyl fermions at two different fixed points of the orbifold depending on its chirality: the mode functions of the KK zero-mode of right- and left-handed fermions behave as $\propto e^{-M|y|}$, $\propto e^{-M|y-\pi R|}$ ($R$ is the radius of $S^1$). Since the Yukawa coupling is the overlap integral of these mode functions of different chiralities, we eventually get the exponentially suppressed fermion masses behaving as ($M_i$ being flavor-dependent bulk masses)

$$\sim M_W (\pi R M_i) e^{-\pi R M_i} (R: \text{the radius of } S^1)$$

for lighter (1st and 2nd) generations. We thus have a mechanism to realize the hierarchical fermion masses without any hierarchical structure of the parameters $M_i$. It is interesting to note that, if we plot the logarithm of observed quark masses as a function of generation number, they align along a straight line, roughly speaking. If we take this seriously, this suggests that the quark masses were all equal before the exponential suppression, which is exactly what GHU implies starting from the universal Yukawa couplings. A study pursuing this line of argument is now ongoing.

For the scenario of GHU to be viable, we need to construct a minimal model based on this scenario, just as the MSSM in the case of SUSY. In this attempt, one non-trivial feature arises. In the case of the MSSM, the SM was just made supersymmetric with the same gauge group SU(2) $\times$ U(1) for the electroweak sector. In the case of GHU, however, this gauge group should be inevitably extended. This is because in this scenario the Higgs is originally a gauge field and therefore belongs to an adjoint representation of the gauge group, while, as is well known, the Higgs in the SM is an SU(2) doublet, i.e. a fundamental representation of SU(2). The breakthrough of this problem is to extend the gauge group a little. The simplest choice is to adopt SU(3) as the electroweak sector. A minimal SU(3) GHU (electroweak) model has been discussed [25,26].

The minimal model is formulated in 5D space-time with an orbifold $S^1/Z_2$ as its extra dimension. The orbifold is defined by the identification of two points on the circle connected by $Z_2$ transformation,

$$Z_2 : y \to -y,$$
where \( y \) is the coordinate along \( S^1 \). We have two fixed points, \( y = 0, \pi R \), which are invariant under the transformation. The main motive for adopting the orbifold, not just a manifold like \( S^1 \), is the realization of chiral theory by “orbifolding”. By identification under \( Z_2 \), the degree of freedom of the extra space points becomes one half and, correspondingly, either the right or left Weyl fermion survives as the fermion zero mode.

Another merit of adopting the orbifold is that, by assigning different \( Z_2 \)-parities for each element of the irreducible representation of SU(3), the gauge symmetry can be explicitly broken by the orbifolding [27], as is explained now.

To realize the mechanism, we assign the \( Z_2 \)-parities for the elements of the SU(3) triplet representation as follows:

\[
\Psi(-y) = -P \gamma^5 \Psi(y)
\]

where the triplet \( \psi \) contains the quark fields

\[
\psi = \begin{pmatrix} u_L \\ d_L \\ d_R \end{pmatrix}
\]

(2.10)

The matrix \( P \) represents the \( Z_2 \)-parities of the elements of the triplet. Let us note that, as is seen in (2.9), \( Z_2 \) has an aspect of chiral transformation for fermions. Thus the zero-mode, having even \( Z_2 \)-parity, of the upper two elements of \( \psi \) is a left-handed fermion, while the zero-mode of the lowest element is right-handed. Thus a chiral theory needed to accommodate the SM is realized. Now it is clear that an ordinary bulk mass term of the form \( M \bar{\psi} \psi \) is not allowed, since \( Z_2 \) contains a chiral transformation. This is why the \( Z_2 \)-odd bulk mass term (2.6) is introduced to be consistent with the orbifolding.

The KK zero-mode of the 4D gauge boson sector is given as

\[
A_{\mu}^{(0)} = \frac{1}{2} \begin{pmatrix} W_\mu^3 + \frac{B_\mu}{\sqrt{3}} & \sqrt{2} W_\mu^+ & 0 \\ \sqrt{2} W_\mu^- & -W_\mu^3 + \frac{B_\mu}{\sqrt{3}} & 0 \\ 0 & 0 & -2 \frac{B_\mu}{\sqrt{3}} \end{pmatrix}
\]

(2.11)

Namely, only the gauge bosons connecting the elements of \( \psi \) with the same \( Z_2 \)-parities have even \( Z_2 \)-parity and therefore the zero-modes. It is now clear that the zero-mode sector of the gauge bosons is exactly what we need in the SM. In this way, SU(3) gauge symmetry is broken into SU(2) \( \times \) U(1) by the orbifolding [27].

The zero-mode sector of the 4D scalar is given as

\[
A_y^{(0)} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & \phi^+ \\ 0 & 0 & \phi^0 \\ \phi^- & \phi^0 & 0 \end{pmatrix}
\]

(2.12)

We find that, this time, only the “broken generator” part \( G/H \) (\( G = SU(3), H = SU(2) \times U(1) \)) has zero-modes. This is because \( A_\mu \) and \( A_y \) should have opposite \( Z_2 \)-parities, just as \( x^\mu \) and \( y \) have. The “off-diagonal” elements of (2.12) just correspond to the SU(2) doublet of the Higgs field in the SM.
Thus by orbifolding we just get both the necessary gauge fields and the Higgs doublet of the SM, and nothing else. This seems to suggest that the adjoint representation of SU(3) has been prepared in order to accommodate the gauge–Higgs sector of the SM: \( 8 \rightarrow 3 + 1 + 2 \times 2 \).

### 2.3.2. Higgs as a pseudo Nambu–Goldstone boson.

So far, we have discussed supersymmetry and gauge symmetry (the Higgs as a gauge boson in the case of GHU) as possible symmetries for solving the hierarchy problem. There remains a third possibility, i.e. global symmetry: the Higgs as a (pseudo) Nambu–Goldstone (NG) boson. A representative scenario based on this idea is the “little Higgs (LH)”.

Quite interestingly, there seems to be a close relation between the GHU and LH scenarios, although the LH is a theory in 4D space-time.

There are a few pieces of “circumstantial evidence” to imply such a close relation:

1. In both scenarios, the gauge group of the SM is enlarged to some larger group \( G \), which is broken to some subgroup \( H \), and the Higgs is identified with the NG boson of \( G/H \) in the case of LH, and \( A_y \) of \( G/H \) in GHU, as we have seen.
2. The coupling of \( A_y \) to fermions in 5D GHU takes the form \( g \bar{\psi}(i\gamma_5)\psi \cdot A_y \), which mimics the pseudo scalar coupling of NG bosons, such as pions.
3. Both have shift symmetries,
\[
A_y \rightarrow A_y + \partial_y \lambda \quad \text{(for GHU)}, \quad G \rightarrow G + \text{const.} \quad \text{(for LH), (2.13)}
\]
where the former transformation is a higher-dimensional local gauge transformation and the latter is the transformation of some global symmetry.

Actually, the LH scenario was inspired by the so-called “dimensional deconstruction” scenario, which may be regarded as a “latticized” GHU, as we will see below. So, once we make the scenario of dimensional deconstruction into the bridge between GHU and LH, their mutual relation becomes more solid.

One may wonder how global and local symmetries can be “closely related”. The point is that in the LH and also in dimensional deconstruction there is a repetition of a global symmetry, \( SU(m) \times SU(m) \times \cdots \times SU(m) = (SU(m))^N \) with gauge parameters \( \lambda_i \) \( (i = 1, 2, \ldots N) \). On the other hand, in the GHU, higher-dimensional local gauge symmetry is described by a gauge parameter \( \lambda(y) \). If we treat the integer \( i \) of \( \lambda_i \) as a “discretized extra space coordinate”, it can be identified. This argument suggests that dimensional deconstruction (and also LH) may be understood, roughly speaking, as a sort of “latticized” GHU.

### Dimensional deconstruction

Before going into the LH scenario, we briefly discuss the scenario of dimensional deconstruction \[28,29\], and its close relationship with the GHU.

The scenario has the following remarkable features:

1. The Higgs is a pseudo NG boson, a bound state of fermions, just as pions in QCD.
2. There is a repetition of gauge symmetries: \( (G \times G_s)^N \) \( (G = SU(m), G_s = SU(n)) \).
3. The quantum correction to the Higgs mass is finite (for \( N \geq 3 \)) without relying on SUSY.

The model has \( N \) pairs of (say, left-handed) Weyl fermions as matter fields with the bi-fundamental representations of
\[
(m, \bar{n}) \quad (SU_i(m), SU_i(n)) \]
\[
(\bar{m}, n) \quad (SU_{i+1}(m), SU_i(n)) \quad (i = 1 - N), \quad (2.14)
\]
where periodic boundary conditions \( \text{SU}_{N+1}(m) = \text{SU}_1(m) \) etc. are imposed. The structure of the theory is represented by a so-called “moose diagram”, shown in Fig. 1, where each oriented line corresponds to a Weyl fermion with either the \((m, \bar{n})\) or \((\bar{m}, n)\) representation, depending on whether the blob denoting the gauge group \( G_s \) is to the right or left of the line.

Both \( \text{SU}(m) \) and \( \text{SU}(n) \) have asymptotically free gauge symmetry with typical mass scales \( \Lambda_1, \Lambda_1^s \). Assuming \( \Lambda_1^s \ll \Lambda_1 \), the \( \text{SU}(n) \) interaction becomes strong at higher energy, thus forming a bound state of these Weyl fermions just like the hadrons in QCD. At lower energies, \( E \leq \Lambda_1^s \), the effective low-energy theory is described by pseudo scalars, à la the pion \( \pi \), as (pseudo) NG bosons due to the spontaneous breaking of the chiral symmetry \( \text{SU}(m) \times \text{SU}_{i+1}(m) \):

\[
U_i = \exp \left( \frac{i \pi \theta}{f} \right) (T_a : \text{generators of } \text{SU}(m)),
\]

where \( f \) corresponds to the pion decay constant. The non-linear realization \( U_i \) behaves as a bi-fundamental representation of the chiral symmetry, i.e. \((m, \bar{m})\) under \( (\text{SU}_i(m), \text{SU}_{i+1}(m)) \).

Actually, \( \text{SU}(m) \) has been gauged and the effective action is a gauged non-linear sigma model:

\[
S = \int d^4x \left( -\frac{1}{2g^2} \sum_{j=1}^{N} \text{tr}(F_{\mu\nu}^j)^2 + f^2 \sum_{j=1}^{N} \text{tr}[(D_{\mu}U_j)^{\dagger}(D^\mu U_j)] \right),
\]

with the covariant derivative

\[
D_{\mu}U_j = \partial_{\mu}U_j - iA^j_{\mu}U_j + iU_jA^{j+1}_{\mu}. \tag{2.17}
\]

Here \( A^j_{\mu} \) are gauge bosons of \( \text{SU}(m) \) and \( F_{\mu\nu}^j \) are their field strengths.

This is nothing but 5D \( \text{SU}(m) \) pure non-Abelian gauge theory, with extra space being latticized: \( U_j \) may be regarded as the link variable (the “Wilson line” along the extra dimension). Indeed, in the simplified Abelian case

\[
D_{\mu}U_j \rightarrow i[\partial_{\mu}\pi - gf(A^{j+1}_{\mu} - A^j_{\mu})]U_j, \tag{2.18}
\]

and identifying the lattice spacing \( a \) of the extra dimension as

\[
a = \frac{1}{gf}, \tag{2.19}
\]

\( D_{\mu}U_j \) just corresponds to the field strength \( F_{\mu\nu} \) in the GHU:

\[
D^\mu U_j \rightarrow \partial_{\mu}A_y - \partial_y A_{\mu} = F_{\mu\nu} (\pi \equiv A_y). \tag{2.20}
\]

Thus, the extra dimension has been constructed by the dynamics in 4D space-time. It should be noticed that above \( \Lambda_s \), the theory recovers to original renormalizable theory, in clear contrast to the non-renormalizable higher-dimensional gauge theory, which the GHU is formulated on.
The theory has gauge symmetry \((\text{SU}(m))^N\) that is spontaneously broken into a single \(\text{SU}(m)\):

\[
\langle U_i \rangle = 1 \rightarrow g_i \langle U_i \rangle g_i^\dagger = \langle U_i \rangle, \quad \text{only when } g_i = g_{i+1},
\]

where \(g_i\) is an element of \(\text{SU}_i(m)\). Thus, among \(U_i\) \((i = 1, 2, \ldots, N)\), \(N - 1\) pieces are “eaten” by the Higgs mechanism, and there remains only one physical (pseudo) NG boson, which is identified as the Higgs. The remaining Higgs,

\[
\text{Tr}(U_1 U_2 \cdots U_N),
\]

is invariant under the gauge transformation, \(U_1 U_2 \cdots U_N \rightarrow g_1(U_1 U_2 \cdots U_N)g_1^\dagger (g_{N+1} = g_1)\), and therefore cannot be “gauged away” by taking the unitary gauge.

In the Abelian case, \(U_1 U_2 \cdots U_N = e^{i(\pi_1 + \cdots + \pi_N)} (\phi = \sqrt{N} \pi_1 + \cdots + \pi_N)\), and the field \(\phi\) just corresponds to the zero-mode of \(A_y\), or the phase of the Wilson loop, in GHU: \(W = e^{i\frac{\pi}{N} A_y dy}\).

The field \(\phi\) is a pseudo NG boson. In fact, at the quantum level, its potential is induced:

\[
V(\phi) = -\frac{9}{4\pi^2} f^4 \sum_{n=1}^{\infty} \frac{\cos \left( \frac{2\pi \sqrt{N} \phi}{f} \right)}{n(n^2N^2 - 1)(n^2N^2 - 4)} + \text{constant}.
\]

This potential is known to be finite for \(N \geq 3\), which can be easily shown by using relations such as \(\sum_n \cos \left( \frac{2\pi n}{N} \right) = 0 \quad (-\frac{N}{2} < n < \frac{N}{2})\). Let us note that such summation over all possible \(n\) just corresponds to the summation over all KK modes in the GHU, which led to the finite Higgs mass [8]. Thus we may also understand that the scenario of dimensional deconstruction provides a very reasonable regularization scheme for the KK mode sum: we do not have to take the sum over an infinite number of KK modes; instead, just a few modes are enough to guarantee the finiteness of the Higgs mass.

The potential reduces to that in GHU [5–8,25,26] in the limit \(N \rightarrow \infty\):

\[
V(A_y) = \frac{9}{4\pi^2} \frac{1}{(2\pi R)^4} \sum_{n=1}^{\infty} \frac{\cos(n g A_y 2\pi R)}{n^5}.
\]

Little Higgs

The purpose of the little Higgs (LH) scenario is to construct a 4D theory including SM, where the Higgs is a pseudo NG boson, while the quadratic divergence of the quantum correction to the Higgs mass cancels out without relying on SUSY. Though LH is a scenario inspired by dimensional deconstruction, in this approach the NG boson need not be a bound state of fermions. So the remnant of the higher-dimensional theory is not apparent. Nevertheless, there still remains some close relation between the LH and GHU scenarios, as has already been mentioned in this subsection. (The correspondence may be rigorously argued once we utilize AdS–CFT correspondence.)

There are various versions of LH models, but here we discuss “the simplest little Higgs” [30] in order to understand the key ingredient of the scenario, “collective breaking”.

Interestingly, in the LH the gauge symmetry should also be enlarged. This is basically because the scenario needs a global symmetry that is larger than \(SU(2) \times U(1)\), so that even after the Higgs mechanism physical NG bosons remain. Thus, let us consider the simplest \(SU(3)\) model with triplet...
scalar $\phi$. (The model is eventually extended to that with additional $U(1)_X$. However, for simplicity, here we ignore it.) The VEV

$$\langle \phi \rangle = \begin{pmatrix} 0 \\ 0 \\ f \end{pmatrix}$$  \hspace{1cm} (2.25)

causes a spontaneous breaking of global symmetry $SU(3) \rightarrow SU(2)$, and the resultant NG bosons are written as

$$\pi = \begin{pmatrix} -\frac{\eta}{2} & 0 & \phi^+ \\ 0 & -\frac{\eta}{2} & \phi^0 \\ \phi^- & \phi^{0*} & \eta \end{pmatrix},$$  \hspace{1cm} (2.26)

where $h = (\phi^+, \phi^0)^f$ is identified with the Higgs doublet. In this model, however, $h$ and $\eta$ are all absorbed by massive gauge bosons by the Higgs mechanism in the process of spontaneous breakdown of $SU(3) \rightarrow SU(2)$, and no physical Higgs remains.

Then we enlarge the model a little and introduce two copies of triplet scalars $\phi_1$ and $\phi_2$, and introduce covariant derivatives $D_\mu$ with the same $SU(3)$ gauge bosons $A_\mu$ for both:

$$L = |D_\mu \phi_1|^2 + |D_\mu \phi_2|^2 \ (D_\mu = \partial_\mu - igA_\mu)$$  \hspace{1cm} (2.27)

where two triplet scalars are non-linearly realized:

$$\phi_1 = \exp{\frac{i\pi}{f}} \begin{pmatrix} 0 \\ f \end{pmatrix}, \quad \phi_2 = \exp{\frac{i\pi}{f}} \begin{pmatrix} 0 \\ f \end{pmatrix}.$$  \hspace{1cm} (2.28)

For simplicity the decay constants of $\phi_{1,2}$ are taken to be the same: $f$.

Each sector of the two scalars $\phi_{1,2}$ has its own global $SU(3)$ symmetry. But, once gauge interaction is switched on, the global symmetry is explicitly broken to the “diagonal” $SU(3)$: When each of $\phi_1$ and $\phi_2$ transforms as

$$\phi_1 \rightarrow U_1 \phi_1, \quad \phi_2 \rightarrow U_2 \phi_2,$$  \hspace{1cm} (2.29)

the gauge fields should transform as

$$A_\mu \rightarrow U_1 A_\mu U_1^\dagger, \quad A_\mu \rightarrow U_2 A_\mu U_2^\dagger.$$  \hspace{1cm} (2.30)

Since one gauge field $A_\mu$ couples with both triplets, for consistency it is necessary that $U_1 = U_2$.

The NG boson corresponding to this diagonal $SU(3)$ remains exactly massless, but is “eaten” by the Higgs mechanism. The orthogonal one, in particular its doublet component $h$, is identified with the Higgs and acquires a mass, since the relevant global symmetry is explicitly broken by the presence of the gauge interaction.

Let us note that, to break the global symmetry explicitly by the gauge interaction, $SU(3)^2 \rightarrow SU(3)$, gauge couplings of $A_\mu$ to both $U_1$ and $U_2$ are necessary. Namely, the gauge couplings with both triplets collectively break the global symmetry. This is what “collective breaking” means. The physically remaining Higgs is not a real NG boson but a pseudo NG boson, acquiring a mass due to the breaking of the global symmetry. The collective breaking in turn means that, even if the Higgs acquires a mass at quantum level, it should be induced by Feynman diagrams where the gauge interactions with both triplets are included. Thus the diagram leading to the quadratic divergence of the Higgs mass is not allowed (at least at the one-loop level) and the Higgs does not suffer from the
quadratically divergent quantum correction to the Higgs mass. This is the mechanism of collective breaking in order to remove the quadratic divergence.

Let us note that, in order for the collective breaking to work, the repetition of fields is essential, which seems to correspond to the presence of KK modes in GHU. Also, the quadratic divergence of the Higgs mass disappears as a result of the repetition of fields in the LH model, just as in the case of GHU, where the sum over all KK modes provides a finite Higgs mass [8].

2.4. Close relation between GHU and superstring theory

The GHU also has a very close relation to the superstring theory. In fact, the (bosonic part of the) point particle limit of open superstring theory, i.e. 10D SUSY Yang–Mills theory, may be understood as a sort of GHU.

Pure SUSY Yang–Mills theory with a gauge multiplet alone is possible only for specific space-time dimensionality \( D = 3, 4, 6, \) and \( 10 \), just because the matching of physical degrees of freedom of the gauge boson and the gauge fermion is realized in these dimensions:

\[
D - 2 = r \ 2^{\frac{D-2}{2}} \ (r = \frac{1}{2} \text{ for Majorana–Weyl}).
\] (2.31)

This means that the 10D SUSY Yang–Mills theory (with \( r = \frac{1}{2} \)) is formulated as a pure SUSY Yang–Mills theory without any need to introduce an additional matter field. Therefore, the only possible origin of the Higgs field is the gauge field, or to be more precise the extra-dimensional component of the 10D gauge field. This is nothing but GHU.

Although it is usually argued that, in supersymmetric theory, the hierarchy problem of quadratic divergence is solved by SUSY, if the theory is regarded as that inspired by superstring, the hierarchy problem may be solved by the mechanism in the GHU as well. In fact, if the mode sum over all KK modes is performed, the quantum correction to the Higgs mass is finite even after the SUSY breaking.

3. Normal vs. anomalous Higgs interactions

Now that a new particle, which behaves like the Higgs particle in the standard model (SM), has been discovered at CERN, only the theory with the SM as its decoupling limit should be acceptable as the theory of new physics. In order to determine which type of theory we should pursue among the remaining candidates, a precision test of the Higgs interaction is quite important. This may not be easy at LHC and we hope that it can be performed at the planned ILC experiment.

Interestingly, representative new physics theories show some sort of deviation of Higgs interactions from those predicted by the SM. For instance, in the MSSM, the couplings of the Higgs interactions generally depend on the parameters \( \beta \) and \( \alpha \). For instance, the ratios of the Yukawa coupling of the SM-like Higgs to that of the SM for the third generation are given as

\[
f_r^{(MSSM)} \frac{f_r^{(SM)}}{f_r^{(SM)}} = \frac{\cos \alpha}{\sin \beta}.
\] (3.1)

\[
f_b^{(MSSM)} \frac{f_b^{(SM)}}{f_b^{(SM)}} = \frac{f_r^{(MSSM)}}{f_r^{(SM)}} = -\frac{\sin \alpha}{\cos \beta}.
\] (3.2)

Although the Yukawa couplings generally deviate from the predictions of the SM, the deviation is universal for all generations. This is an important prediction of the MSSM. It should also be noted that, at the decoupling limit \( \alpha \rightarrow \beta - \frac{\pi}{2} \), the deviations just go away, as we anticipated.

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Let us now discuss higher-dimensional theories. We will see that in the gauge–Higgs unification (GHU) and also in the related theories (dimensional deconstruction, little Higgs (LH) and probably superstring) we expect really “anomalous” Higgs interactions, qualitatively different from those in the SM and MSSM, coming from the very fact that the Higgs is not a scalar particle, but is originally a gauge field or a pseudo NG boson.

From the viewpoint of the precision test of the Higgs interactions, it seems to be quite interesting that new physics theories are divided into two categories concerning the Higgs interactions, i.e. theories with normal Higgs interactions and theories with anomalous Higgs interactions. We discuss these two categories separately below.

3.1. Theory with normal Higgs interactions

The higher-dimensional new physics theory of this type that we discuss here is the scenario of the universal extra dimension (UED) [18]. This theory is obtained by simply making the SM higher-dimensional, although to get a chiral theory the extra dimension is assumed to be orbifold, not just a circle or sphere. In particular, its Higgs sector is just as in the SM: no additional Higgs doublet is introduced, in contrast to the case of the MSSM, and the Higgs is just an elementary scalar field even in the bulk, in clear contrast to the case of GHU or LH. Thus the KK zero mode of the Higgs field has just the same Yukawa couplings as those in the SM.

3.2. Theories with anomalous Higgs interactions

GHU

We first discuss gauge–Higgs unification (GHU), as a typical example of the new physics theory with anomalous Higgs interactions. In GHU (we work in 5D space-time), the SM-like Higgs $H$ is the zero-mode of $A_y, A_y^{(0)}$, and has a physical meaning as the Wilson loop phase (the AB phase), coming from the fact that the circle is a non-simply-connected space, as was discussed in the previous section:

$$W = e^{ig_4 RA_y^{(0)}}$$

(3.3)

where $R$ is the radius of the circle and, in the line integral along the circle, only the zero mode contribution remains.

The fact that the Higgs should be regarded as an AB phase (or Wilson loop phase) as seen in (3.3) leads to the anomalous Higgs interactions.

To see this, we start from a general argument on the fermion masses and Yukawa couplings that, in any gauge theories with spontaneous gauge symmetry breaking, the fermion mass term is written as

$$m(v)\bar{\psi}\psi,$$

(3.4)

where $m(v)$ is a function, say the “mass function”, of the VEV $v = \langle H \rangle$. The interaction of the physical Higgs field $h$ with the fermion is expected to be provided by a replacement $v \rightarrow v + h$, since the Higgs is the field to denote the shift of $H$ from its VEV. Thus, Yukawa coupling $f$ of $h$ with the (KK zero mode of the) fermion is expected to be given by

$$f = \frac{d m(v)}{dv}.$$  

(3.5)

This prescription works perfectly in the case of the SM, where $m(v)$ is a linear function of $v$, $m(v) = f v$. In the case of GHU, the story is a little more complicated. Even if we consider only one flavor,
the fermion still has an infinite number of KK modes. Thus the mass term and Yukawa couplings are written in the form of matrices in the base of KK modes. Provided that the VEV $v$ is fixed, each KK mode should be given as an eigenvector of the mass matrix determined by $v$. Thus in the base of KK modes the mass matrix (the matrix form of the mass terms) is diagonalized. The unusual thing, however, is that the matrix denoting the Yukawa couplings is generally non-diagonal even in the base of the mass eigenstates [31]. This characteristic feature of the GHU is closely related to the fact that the mass function $m(v)$ of KK zero mode, for instance, can be a non-linear function, such as trigonometric function, in general reflecting that the Higgs is a phase, as we will see below. Thus, to be precise, the prescription given in (3.5) is only for the diagonal elements of the Yukawa coupling matrix (valid for all KK modes, incidentally).

Since in the GHU the Higgs should be understood as a phase (AB phase), we expect that all physical observables have periodicity in the Higgs field $H$:

$$v \rightarrow v + \frac{2}{g_4 R} (g_4 : 4D \text{ gauge coupling}). \quad (3.6)$$

In fact, we find for the zero mode quarks of light (1st and 2nd) generations (together with the $b$ quark), the quark masses are exponentially suppressed as in (2.7), but in addition, their dependence on the VEV $v$ is approximately a trigonometric function:

$$m(v) \propto \sin \left( \frac{g_4}{2} \pi R v \right), \quad (3.7)$$

which is non-linear in $v$. Then, the prescription (3.5) yields

$$f \propto \cos \left( \frac{g_4}{2} \pi R v \right). \quad (3.8)$$

It is a remarkable fact that the Yukawa coupling even vanishes for a specific value of the VEV,

$$x \equiv \frac{g_4 \pi R v}{2} = \frac{\pi}{2}, \quad (3.9)$$

as was first claimed by Hosotani et al. [10, 11].

Although such a drastic case does not seem to be consistent with the recent data from the LHC experiments, it is still possible that GHU predicts anomalous Yukawa couplings for light quarks for a general value of $x$. In fact, the ratio of the Yukawa coupling predicted by GHU to that predicted by the SM for light quarks is known to be very well approximated by an analytic formula [31],

$$\frac{f_{\text{GHU}}}{f_{\text{SM}}} \simeq x \cot{x}. \quad (3.10)$$

Note that, at the “decoupling limit”,

$$x = \frac{g_4}{2} \pi R \ll 1 \leftrightarrow M_W \ll \frac{1}{R}, \quad (3.11)$$

the SM prediction is recovered, as is easily seen in (3.10).

Actually, when the bulk mass $M_f$ is switched off, the mass function $m(v)$ becomes a linear function of $x$, as is seen in Fig. 2(a), just as in the SM, so the Yukawa coupling does not deviate from the SM prediction for $x < \frac{\pi}{2}$. In this case, the periodicity of the mass eigenvalue is realized by a level crossing of the zero mode with the first KK mode at $x = \frac{\pi}{2}$ (see Fig. 2(a)). Namely, at $x = \frac{\pi}{2}$, the zero mode is replaced by the first KK mode whose mass eigenvalue decreases as $x$ increases. For $M_f = 0$, however, mixing between these two modes will not appear because of the conservation of the (absolute value of the) extra space component of the momentum. If we switch on the bulk mass $M_f$, the translational
invariance along the extra space is violated by the presence of the bulk mass term (2.6), thus causing mixing between the two modes. Due to the mixing, the degeneracy of the two mass eigenvalues at the level crossing is lifted and a deviation of the mass function from the linearity appears, leading to the trigonometric function for sufficiently large bulk mass $M_i$, as is seen in Fig. 2(b).

Thus, even though the original seed of the anomalous Higgs interaction is the periodicity characteristic of GHU, more directly it may be attributed to the violation of translational invariance in the extra dimension. From this point of view, it may be worth noting that on the Randall–Sundrum background, the translational invariance is always (universally) violated by the presence of the warp factor $e^{-\kappa|y|}$. This should be the reason why, on the Randall–Sundrum background, the Higgs interactions with massive gauge bosons, $W$ and $Z$, are also anomalous [32,33], while on the flat space-time it is “almost normal” (normal for $x < \pi/2$), as in the gauge–Higgs sector there is no parameter violating translational invariance in the case of flat space-time [31].

From a field theoretical point of view, the reason why we get such non-linearity in the mass function like (3.7) is that, due to the non-diagonal Yukawa couplings mentioned earlier, there appear mixings between the zero mode and non-zero modes, which leads to higher-mass-dimensional operators at tree level of the form $h^{2n+1}\bar{\psi}\psi$ ($n = 1, 2, \ldots$) (just like the dimension 5 operator in the see-saw mechanism). The coefficients of these higher-mass-dimensional operators are suppressed by the inverse powers of $M_c = \frac{1}{R}$, and as we have already seen, the effects of such “irrelevant” operators disappear at the decoupling limit $M_c \rightarrow \infty$.

### 3.3. Dimensional deconstruction

As we have seen in the previous section, though it is formulated on the ordinary 4D space-time, the scenario of dimensional deconstruction may be understood as a “latticized” GHU. We thus expect that the anomalous Higgs interaction is also expected in this scenario. To be more specific, in this case, even if we adopt just periodic boundary conditions for fields like $A^{N+1}_\mu = A^1_\mu$, corresponding to the choice of a circle, not an orbifold, as the extra dimension in GHU, still the latticizing itself breaks the (continuous) translational invariance, which should lead to anomalous Higgs interactions. In fact, the mass eigenvalues in this scenario generally behave as trigonometric functions:

$$m_n(v) = \frac{2}{a} \sin \left( \frac{n\pi}{N} + \frac{g_4a}{4} \right) \quad (a : \text{lattice spacing}).$$  (3.12)
which are basically the same as the eigenfrequencies of a system of springs and balls. As can easily be seen, (3.12) reduces to linear functions of \( v \) in the continuum limit \( a \to 0, N \to \infty \), keeping the circumference of the circle \( L \equiv Na \) a constant; \( m_n(v) \to \frac{2\pi}{L} n + \frac{\pi v}{L} \). Work on the anomalous interactions in dimensional deconstruction pursuing this line of argument is now in progress (N. Kurahashi et al., work in progress).

3.4. Little Higgs

We have also argued that the scenario of little Higgs (LH) has a close relationship with GHU. Just as in the case of GHU, the Higgs is non-linearly realized,

\[
U = \exp \frac{H}{\pi},
\]

and therefore periodicity and non-linearity concerning the Higgs field seem to arise very naturally. Thus we again expect anomalous Higgs interactions in the scenario of LH, though more complete analyses are clearly desirable in this case.

4. \( H \to \gamma \gamma \)

\( H \to \gamma \gamma \) is the clean decay mode of the Higgs, important for the identification of the Higgs particle. This di-photonic decay is induced at the quantum level since the photon is a massless particle, and is a very good testing ground for new physics. It is also sensitive to the Higgs interactions with fermions and gauge bosons running inside the loop diagrams, thus providing useful information on the properties of Higgs interactions, whether they are normal or anomalous, as was discussed in some detail in the previous section. The gluon fusion process \( gg \to H \), a main process of Higgs production, is also induced at the quantum level and has some similarity to the photonic Higgs decay, though gauge bosons do not contribute in this case.

In this section we focus on two higher-dimensional new physics scenarios with considerably different properties, as we have already seen in the previous sections: UED and GHU. We will pay a great deal of attention to the differences in the predictions these two scenarios make about the photonic decay.

Recently, there has been an interesting claim that the contribution of the non-zero KK modes of the top quark to the decay amplitude of \( H \to \gamma \gamma \) has an opposite sign to that of the top quark in the SM and to that of the KK modes of the top quark in UED [34]. The origin of such a qualitative difference may be rather easily understood once we rely on the operator analysis. Let us assume \( M_c = 1/R \gg v \). At first glance, the relevant operator for the photonic decay seems to be \( hF_{\mu\nu}F^{\mu\nu} \), with \( F_{\mu\nu} \) being the field strength of the photon. If this is the case, the coefficient should be divergent in the process of the KK mode sum, since the mass dimension of the operator is 5. But this operator is not gauge invariant under SU(2) \( L \times U(1)_Y \). So, actually the decay amplitude is dominated by a gauge invariant operator of mass dimension 6 written in terms of the Higgs doublet \( \phi \):

\[
\phi^\dagger \phi F_{\mu\nu}F^{\mu\nu}.
\]

(4.1)

Note that, when one of the Higgs doublets is replaced by its VEV, this yields the dimension 5 operator mentioned above. Now the coefficient of the operator (4.1) is finite, at least for 5D space-time.

Also note that the operator in (4.1) comes from the diagram shown in Fig. 3, obtained by inserting the Higgs doublet \( \phi \) twice to the self-energy diagram of the photon, that is responsible for the, quantum correction to the operator \( F_{\mu\nu}F^{\mu\nu} \). So, by utilizing the background field method, the Wilson coefficient of the operator (4.1) is obtainable just by calculating the self-energy diagram with the
Fig. 3. A diagram contributing to the operator that describes the Higgs decay.

top quark (or its KK modes) inside the loop, having masses given by the VEV $v$, and finally taking the second derivative with respect to $v$. Since the coefficient of the self-energy diagram depends logarithmically on the mass of the virtual state, the coefficient of (4.1) is given as

$$C \propto \frac{d^2}{dv^2} \log m_n^2 \bigg|_{v=0},$$

where $m_n$ are the mass eigenvalues of the $n$th KK mode.

Now we are ready to discuss the differences between the predictions of UED and GHU. In each scenario, the mass eigenvalue of a fermion is given as follows:

$$m_n = \begin{cases} \sqrt{(\frac{n}{R})^2 + (f v)^2} & \text{for UED}, \\ \frac{n}{R} + f v & \text{for GHU}. \end{cases}$$

Here, for brevity, we have ignored the possible bulk mass, and $f$ is the Yukawa coupling for the zero mode fermion. Then the procedure of (4.2) gives results that are just opposite in sign for the two scenarios:

$$C \propto \begin{cases} 2 \frac{f^2}{(\frac{n}{R})^2} & \text{for UED}, \\ -2 \frac{f^2}{(\frac{n}{R})^2} & \text{for GHU}. \end{cases}$$

There is a claim that the new physics contributions of the theories with a mechanism to solve the hierarchy problem invoking some symmetry tend to reduce, e.g., the amplitude of the gluon fusion process. Intuitively, the reason might be the following. In the case of gluon fusion, the field strength in (4.1) is that of the gluon field. Since the QCD sector has no direct connection with the Higgs sector, even if we take off the gluonic field the operator is still gauge invariant and it just becomes the mass-squared operator for the Higgs field. On the other hand, any theories that aim to solve the problem of quadratic divergence of the Higgs mass should have a mechanism to reduce the quantum correction to the Higgs mass-squared by introducing, e.g. super-partners, KK modes, etc. Thus it will be natural to expect some destructive contributions from new heavy particles in this type of theory.
Let us also note that the operator (4.1) is easily generalized by replacing $\phi^\dagger \phi$ by its arbitrary function $V(\phi)$: i.e. $V(\phi) F^{\mu\nu} F_{\mu\nu}$. If we take off $F^{\mu\nu} F_{\mu\nu}$ again, we just get a Higgs potential $V(\phi)$. On the other hand, in the GHU the potential is completely finite [5–8]. Maru thus claims that the decay amplitude of $H \to \gamma\gamma$ is finite irrespective of the space-time dimension [37].

For a more recent detailed analysis concerning the di-photonic decay in GHU, refer to Ref. [35]. For a recent argument on the contribution of the non-zero KK modes of the top quark in the SO(5) x U(1) GHU on the Randall–Sundrum background, refer to Ref. [36].

Usually, the contributions of lighter generations (1st and 2nd generations) to the photonic Higgs decay are more or less negligible, since their Yukawa couplings are strongly suppressed by very small quark masses. This is why only the contribution of the top quark is seriously taken into account. In the GHU, however, the story is not so simple. Namely, the contribution of each non-zero KK mode of lighter generations of quarks is not strongly suppressed by the corresponding small quark masses. This is because only the mode function of the zero mode exhibits localization at a fixed point and the Yukawa couplings of non-zero KK modes are not exponentially suppressed, being of the order of gauge coupling. So we naturally expect possible significant contributions to the photonic decay from lighter generations as well.

Fortunately or unfortunately, it turns out that, although each KK mode makes a significant contribution to the decay amplitude, as we expected, after the summation over the contributions of all KK modes, the factor suppressed by $1/M_c^2 = 1/(1/R)^2$ (which we expect from the decoupling theorem) just goes away, and the remaining contribution is exponentially suppressed as $e^{-M_i R}$ ($M_i$: bulk mass) (K. Hasegawa et al., manuscript in preparation).

We recall that a similar exponential suppression factor due to a relatively large bulk mass was found in the process of the calculation of the quantum correction to the Higgs mass-squared [8]. This may not be so surprising, since, as we have argued above, the operator relevant to the photonic decay has some similarity to that of the Higgs potential. We may also notice that the exponential factor mimics the Boltzmann factor in the case of finite-temperature field theory. This factor goes away at the decompactification limit $R \to \infty$, thus suggesting that it comes from a non-local gauge invariant operator due to the Wilson loop. In fact, this should be the reason why we find a finite decay amplitude in GHU irrespective of the dimensionality of the space-time, as was demonstrated in Ref. [37]. We may also understand that the factor is the contribution from the sector with non-zero winding numbers at the Poisson re-summation, which clearly shows the contributions of distances larger than $R$ and is thus suppressed by the exponential factor $e^{-M_i R}$, present in the Yukawa potential.

5. Summary

In spite of the great success of the LHC experiments in discovering a particle consistent with the Higgs of the SM, it is still premature to conclude that the particle is the Higgs of the SM. It may be some SM-like Higgs particle predicted by some new physics theory. Also, the Higgs sector of the SM is still mysterious, having a few important long-standing problems, such as the hierarchy problem and the problem of too many arbitrary parameters present in the sector. Thus we should continue to ask what the origin of the Higgs is.

In this article, we have focused on higher-dimensional new physics theories. Also discussed were 4D theories, such as little Higgs (LH) and dimensional deconstruction, which possess some close relationship with the higher-dimensional theory, especially with the gauge–Higgs unification (GHU). Some important common features shared by these scenarios are the presence of the “shift symmetry”,

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which guarantees the finiteness of the Higgs mass at the quantum level, and the periodicity: physical observables are periodic in the Higgs field. In GHU the origin of the Higgs field is a gauge field and in the LH and dimensional deconstruction the origin is a (pseudo) Nambu–Goldstone boson.

In order to investigate the origin of the Higgs, precision tests of Higgs interactions will be of crucial importance. We have discussed that new physics theories are divided into two categories: theories with normal Higgs interactions, and theories with anomalous Higgs interactions.

From this point of view, we have demonstrated that, in the scenario of GHU and the related LH and dimensional deconstruction scenarios, the periodicity mentioned above (together with the breakdown of translational invariance along the extra dimension) leads to very characteristic anomalous Higgs interactions, which are never shared by the SM. This should be a very important observation in the attempt to deeply understand the origin of the Higgs particle.

The difference, i.e. normal (as in the case of UED) or anomalous Higgs interactions, also makes the new physics contribution to the main decay mode of the Higgs, \( H \rightarrow \gamma\gamma \), qualitatively different.

Such crucial precision tests of the Higgs interactions may not be easy at the LHC experiment, but we hope that they will be performed at the planned ILC experiment.

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