Galileogenesis: A new cosmophenomenological zip code for reheating through R-parity violating coupling

Sayantan Choudhury a,*, Arnab Dasgupta b

a Physics and Applied Mathematics Unit, Indian Statistical Institute, 203 B.T. Road, Kolkata 700 108, India
b Centre for Theoretical Physics, Jamia Millia Islamia, Jamia Nagar, New Delhi 110025, India

Received 19 January 2014; received in revised form 16 February 2014; accepted 23 February 2014
Available online 6 March 2014
Editor: Tommy Ohlsson

Abstract

In this paper we introduce an idea of leptogenesis scenario in higher derivative gravity induced DBI Galileon framework aka Galileogenesis in presence of one-loop R-parity violating couplings in the background of a low energy effective supergravity setup derived from higher dimensional string theory framework. We have studied extensively the detailed feature of reheating constraints and the cosmophenomenological consequences of thermal gravitino dark matter in light of PLANCK and PDG data. Finally, we have also established a direct cosmophenomenological connection among dark matter relic abundance, reheating temperature and tensor-to-scalar ratio in the context of DBI Galileon inflation.

© 2014 The Authors. Published by Elsevier B.V. This is an open access article under the CC BY license (http://creativecommons.org/licenses/by/3.0/). Funded by SCOAP3.

The post Big Bang Universe passed through various phases in which reheating plays the crucial role in explaining production of different particle species from inflaton. Particle cosmologists have a clear picture of this hot Big Bang phase because ordinary matter and radiation were driving it and also the physical processes that characterize it involve terrestrial physics. These particles interact with each other and eventually they come to a state of thermal equilibrium. This process completes when all the energy of the inflaton transfer to the thermal energy of elementary particles. Amongst all particles degrees of freedom the production of thermal gravitinos during reheating [1–7] and its decay play a pivotal role in the context of leptogenesis [8–11] and dark

* Corresponding author.
E-mail addresses: sayanphysicsisi@gmail.com (S. Choudhury), arnab@ctp-jamia.res.in (A. Dasgupta).
matter detection [12–15]. In a most general prescription usually two types of gravitinos are produced in this epoch—stable and unstable. Both of them stimulate the light element abundances during BBN [16–18] and directly affect the expansion rate of the universe. The gravitino energy density is proportional to gravitino abundance which is obtained by considering gravitino production in the radiation dominated era following reheating [19–21].

In this paper we perform our complete phenomenological analysis with a potential driven DBI Galileon framework in the background of $\mathcal{N} = 1, \mathcal{D} = 4$ supergravity [22–30] which can be obtained from the dimensional reduction from higher dimensional string theory setup [31–33]. The total phenomenological model is made up of the following crucial components:

- Higher order correction terms in the gravity sector are introduced in the effective action as a perturbative correction to the Einstein–Hilbert counterpart coming from the computation of Conformal Field Theory disk amplitude at the two loop level [34–36].
- The matter sector encounters the effect of $\mathcal{N} = 1, \mathcal{D} = 4$ supergravity motivated DBI Galileon interaction which is embedded in the D3 brane.
- Additionally, we have considered the effect of R-parity violating interactions [37–40] in the matter sector which provide a convenient framework for quantifying quark and lepton-flavor violating effects.

The low energy UV protective effective action for the proposed cosmophenomenological model is described by [31,32]:

$$ S = \int d^4 x \sqrt{-g} \left[ K(\Phi, X) - G(\Phi, X) \Box \Phi + B_1 R 
+ (B_2 R^{\alpha \beta \gamma \delta} R_{\alpha \beta \gamma \delta} - 4 B_3 R^{\alpha \beta} R_{\alpha \beta} + B_4 R^2) + B_5 \right] \quad (1) $$

where the model dependent characteristic functions $K(\Phi, X)$ and $G(\Phi, X)$ are the implicit functions of Galileon and its kinetic counterpart is $X = -\frac{1}{2} g^{\mu \nu} \partial_\mu \Phi \partial_\nu \Phi$. Additionally, $B_i \forall i$ are the self-coupling constants of graviton degrees of freedom appearing via dimensional reduction from higher dimensional string theory. Specifically $B_5$ be the effective four dimensional cosmological constant. In general, $B_2 \neq B_3 \neq B_4$ which implies that the quadratic curvature terms originated from two loop correction to the CFT disk amplitudes are not topologically invariant in 4D effective theory. In Eq. (1) for potential driven DBI Galileon model once we embed DBI theory in the Galileon background we can write, $K(\Phi, X) = P(\Phi, X) - V(\Phi)$, where the kinetic term of the effective action is given by:

$$ P(\Phi, X) = -\frac{G_1}{f(\Phi)} [\sqrt{1 - 2 G_2 X f(\Phi)} - G_3] - G_4 G(\Phi, X) - 2 G_5 X \quad (2) $$

with an effective Klebanov–Strassler frame function

$$ f(\Phi) = \left( \sum_{q=0}^{2} f_{2q} \Phi^{2q} \right)^{-1} \quad (3) $$

which characterizes the throat geometry on the D3 brane. Here $G_i \forall i$ and $f_{2q} \forall q$ are originated from dimensional reduction. It is important to note that the functional $G(\phi, X)$ appearing in Eq. (1) and Eq. (2) are exactly same in the context of DBI Galileon theory. For more details on this issue see Refs. [31,32]. In the canonical limit when the contributions from the DBI Galileon sector is switched off then we get, $K(\phi, X) = X - V(\phi)$, for $G_5 = -1/2$. In such a case the
contributions from the higher derivative terms are highly suppressed by the UV cut-off scale of the effective theory and finally we get back the usual results as obtained from Einstein gravity. But once the contribution of DBI Galileon is switched on, the complete analysis deviates from canonical behaviour and contribution from the higher derivative gravity sector plays crucial role to change the dynamical behaviour during inflation as well as reheating.

Moreover, the one-loop effective Coleman–Weinberg potential is given by [31,32]:

$$V(\Phi) = 2 \sum_{m=-2,m \neq -1} \left[ \beta_{2m} + \delta_{2m} \ln \left( \frac{\Phi}{M} \right) \right] \left( \frac{\Phi}{M} \right)^{2m}$$

(4)

where $\beta_{2m} \forall m$ are the tree level constants and $\delta_{2m} \forall m$ are originated from one-loop correction. In the present setup using Eq. (1) the Modified Friedman equation can be expressed as [31,32]:

$$H^4 = \frac{\rho}{3M_p^2 \theta}$$

(5)

where the energy density can be written in terms of DBI Galileon degrees of freedom as:

$$\rho = 2K_X X - K - 2G_\phi X - 2X(1 - \theta) + V(\phi).$$

(6)

Here the subscripts represent the derivatives with respect to $X$ and $\phi$. Moreover, $\theta$ be a constant which is appearing through dimensional reduction from higher dimensional stringy setup as:

$$\theta = \frac{B_1}{M_p^2} + B_2 - 4B_3 + B_4 + \frac{B_5}{M_p^4}.$$  

(7)

In the present setup, $M_p = 2.4 \times 10^{18}$ GeV be the reduced Planck mass. It is important to note that the Friedman equation obtained in the present context is completely different from the Friedman equation as appearing in the context of Einstein’s General Relativity which will further modifies the leptogenesis framework in the present context.

To study this feature explicitly further we allow interaction of DBI Galileon scalar degrees of freedom with leptonic sector of the theory given by:

$$\mathcal{L}_{\text{int}} = \sum_k \left[ \gamma_{ijk} v_i \bar{l}_j \Phi + \gamma_{ijk}^\prime v_i \bar{l}_j \Phi + \text{h.c.} \right]$$

(8)

where the generation indices are $i, j, k = 1(e), 2(\mu), 3(\tau)$. Here after summing over all the contributions of flavor indices the corresponding charged scalar field can be written as:

$$\Phi = \sqrt{3} \left( \Phi_e \oplus \Phi_\mu \oplus \Phi_\tau \right).$$

(9)

This induces the decay of charged DBI Galileon ($\Phi\Phi^+, \Phi^-$) to the leptonic constituents through the phenomenological couplings ($\gamma_{ijk}, \gamma_{ijk}'$). In this context these couplings violate a discrete symmetry called R-parity defined as, $R_p \equiv (-1)^{3B + L + 2S}$, where $B, L$ and $S$ are the baryon, lepton and spin angular momentum respectively. Such R-parity violating interactions in the Lagrangian (8) can be identified with the lepton number violating (LNV) MSSM flat direction $LLe$ appearing in the superpotential as [39,40]:

$$\mathcal{W}_{\text{MSSM}} \supset \frac{1}{2} \epsilon_{ab} \lambda_{ijk} L^a_i L^b_j \ell_k + \text{h.c.}$$

(10)

where $a, b = 1, 2$ are weak isospin indices and flatness constraint requires $i < j$ for the lepton doublet $L$. Here $\lambda_{ijk}$ be the soft SUSY breaking trilinear coupling which violate the R-parity.
and proton-hexality $P_6$, however, they conserve baryon triality $B_3$ [41–43]. Additionally, due to the large suppression of the baryon number violating interactions via $B_3$ triality it stabilizes the proton. Now using the constraints on mass $m_{v_j} \ll m_\Phi$ the corresponding decay widths for the feasible decay channels are:

$$\Gamma(\Phi^+ \to v_1 l^+_j) = \frac{m_\Phi}{16\pi^2} \tilde{F}_{ij},$$

$$\Gamma(\Phi^- \to v_1 l^-_j) = \frac{m_\Phi}{16\pi^2} \tilde{B}_{ij}$$

(11)

where the bilinear functions $\tilde{F}_{ij}$ and $\tilde{B}_{ij}$, can be expressed as:

$$m_\Phi \tilde{F}_{ij} = \frac{1}{3} \sum_k m_{\phi_k} F_{ijk},$$

$$m_\Phi \tilde{B}_{ij} = \frac{1}{3} \sum_k m_{\phi_k} B_{ijk}.$$

(12)

Here $m_\Phi$ be the flavour independent inflaton mass and $m_{\phi_k}$ represents the $k$-th flavour dependent mass of the constituent $\Phi_k$. Additionally, the expression for trilinear functions $F_{ijk}$ and $B_{ijk}$ are explicitly mentioned in Appendix A. To understand the thermal history of the universe from our model, it is convenient to express the decay width in terms of the Hubble parameter during the epoch of reheating as:

$$m_\Phi \sum_{ij} (\tilde{F}_{ij} + \tilde{B}_{ij}) = \frac{1}{16\pi^2} \Gamma_\phi(T_r) = \frac{48\pi}{H_{rh}}$$

(13)

where $\Gamma_\phi(T_r)$ be the total decay width. In the present context the Hubble parameter during reheating is defined through the modified Friedman equation as given by:

$$H_{rh} \approx \sqrt{\frac{\rho_{rh}}{3M_P^2\theta}}.$$  

(14)

Here $\rho_{rh}$ be the energy density during reheating. Hence using Eq. (13) the reheating temperature can be expressed as:

$$T_r = \sum_{ij} \sqrt{\frac{5M_P^2\theta m_\Phi^4}{294912\pi^6 N_*}} (\tilde{F}_{ij} + \tilde{B}_{ij}) = \sqrt{\frac{5M_P^2\theta}{18432\pi^2 N_*}} \Gamma_\phi$$

(15)

where $N_* (= N_B + \frac{7}{8} N_F)$ be the effective number of relativistic degrees of freedom. Usually $N_* \approx 228.75$ for all MSSM degrees of freedom. Recent observational data from PLANCK suggests an upper-bound on the reheating temperature [26,44–46]:

$$T_r \lesssim 6.654 \times 10^{15} \sqrt{\frac{r_*}{0.12}} \text{ GeV}$$

(16)

where $r_*$ be the tensor-to-scalar ratio at the pivot scale of momentum $k_*$. Consequently the upper-bound of total decay width during reheating is given by:

$$\Gamma_\phi = \sum_{ij} \frac{m_\Phi}{16\pi^2} (\tilde{F}_{ij} + \tilde{B}_{ij}) \lesssim 2.772 \times 10^{-3} \sqrt{\frac{3072\pi^2 M_P^2 N_* r_*}{\theta}}$$

(17)
Fig. 1. Variation of total decay width $\Gamma_\phi$ with respect to reheating temperature ($T_r$). The dark gray shaded area shows the theoretically allowed region which lies within the upper bound of the tensor-to-scalar ratio ($r_* \leq 0.12$) at the momentum pivot scale $k_* = 0.002$ Mpc$^{-1}$ represented by a blue vertical line in light of PLANCK data. We have also pointed the excluded parameter space for the tensor-to-scalar ratio within the range $0.12 < r_* \leq 0.36$ by imposing the constraint from PLANCK data. For the numerical estimation in the present context we have used, $G_1 = 1$, $G_2 = 0.5$, $G_3 = 2$, $G_4 = 1$, $G_5 = -0.5$ and $B_1 = 2M_P^2$, $B_2 = 2$, $B_3 = 1$, $B_4 = 3$, $B_5 = 2M_P^4$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

where the stringent constraint on the slepton masses and soft SUSY breaking trilinear coupling are $m_{\tilde{e}_k} = 300$ GeV and $\sum_{ijk} |\lambda_{ijk}|^2 = 1.7786$ at the GUT scale, which are obtained by solving the one-loop renormalization group equation in $\overline{DR}$ scheme [41]. In Fig. 1 we have shown the behaviour of the total decay width as a function of reheating temperature by imposing the observational constraints in light of PLANCK data. Additionally, we have also pointed the theoretically allowed region obtained from the model as well as the observationally excluded parameter space.

It is important to note that saturating the upper-bound on $r \sim 0.12$ would yield a large reheating temperature of the universe. In this case, the gravitino abundance is compatible with the latest observational/phenomenological bound on dark matter, provided the gravitino mass, $m_{3/2} \sim \mathcal{O}(100)$ eV, see [47]. The light gravitino is a very interesting candidate for dark matter among various other candidates, since the gravitino itself is a unique and inevitable prediction of supergravity (SUGRA) theory. This prediction is very much interesting, since we can test the gravitino dark matter hypothesis at LHC or through any other indirect probes. In fact, if we had late time entropy production after the decoupling time of the gravitino, the mass of the gravitino dark matter may be raised up to a few keV. Moreover, the gravitino dark matter with a mass in the range $m_{3/2} \sim \mathcal{O}(1–10)$ keV serves as the warm dark matter candidate which has recently been invoked as possible solutions to the seeming discrepancies between the observation and the simulated results of the galaxy formation based on the cold dark matter (CDM) scenario [48–51]. See [47] for the details of such scenario. Additionally, the gravitino mass of this order is also favored from several other phenomenological issues, the interesting parameter space for the gaugino masses at the LHC, and the solution to the well known $\mu$-problem [52].

By assuming such a phenomenological prescription perfectly holds good in our prescribed string theoretic setup let us start with a situation where the inflaton field starts oscillating when the inflationary epoch ends at a cosmic time $t = t_{osc} \simeq t_f$ and the reheating phenomenology is described by the *Boltzmann equation* [24]:
\[ \dot{\rho}_r + 4H \rho_r = \Gamma_\phi \rho_\phi, \quad (18) \]

where \( H \approx \sqrt{\frac{\rho_r + \rho_\phi}{3M_P^2}} \). Here \( \rho_r \) and \( \rho_\phi \) represent the energy density of radiation and inflaton respectively. Assuming \( \Gamma_\phi \gg H \) from we get

\[ \rho_\phi = \rho_{\phi 0} x^4 \exp[-\Gamma_\phi(t - t_{osc})], \quad (19) \]

where \( \rho_{\phi 0} = \beta_0 \) (the energy scale of DBI Galileon inflation as appearing in Eq. (4)) and additionally we introduce a new parameter “\( x \)” defined as:

\[ x := a_{osc}/a = \left[ 1 + H_{osc}(t - t_{osc}) \right], \quad (20) \]

with \( H_{osc} = \left( \frac{\rho_{osc}}{3M_P^2} \right)^{1/4} \). For \( t \leq \Gamma_\phi^{-1} \) the exact solution of Eq. (18) can be written as

\[ \rho_r = \frac{1}{x^4} \left[ \rho_{osc} - \rho_{\phi 0} \exp\left( -\frac{(x - 1)\Gamma_\phi}{H_{osc}} \right) \right]. \quad (21) \]

Finally, we are interested in to compute the thermal dark matter gravitino relic abundance produced by the scattering of the inflaton decay products. To serve this purpose we start with the master equation of gravitino phenomenology as obtained from Boltzmann equation is given by [24]:

\[ \left( \frac{d}{dt} + 3H \right) n_{3/2} = \left\{ \Sigma_{\text{total}} | v | n^2 - \frac{m_{3/2}^3}{2} \langle \frac{E_{3/2}}{\tau_{3/2}} \rangle \right\}, \quad (22) \]

where \( n = \frac{\zeta(3)}{\pi^2} T^3 \) is the number density of scatterers (bosons in thermal bath) with \( \zeta(3) = 1.20206 \ldots \). Here \( \Sigma_{\text{total}} \) is the total scattering cross section for thermal gravitino production, \( v \) is the relative velocity of the incoming particles with \( \langle v \rangle = 1 \) where \( \langle \cdot \cdot \cdot \rangle \) represents the thermal average. The factor \( \frac{m_{3/2}^3}{\langle E_{3/2}\tau_{3/2} \rangle} \) represents the averaged Lorentz factor which comes from the decay of gravitinos can be neglected due to weak interaction. For the gauge group \( SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \) the thermal gravitino production rate is given by,

\[ \left\{ \Sigma_{\text{total}} | v | \right\} = \frac{\tilde{\alpha}}{M_P^2}, \]

\[ = \frac{3\pi}{16\xi(3)M_P^2} \sum_{i=1}^{3} \left[ 1 + \frac{M_i^2}{3m_{3/2}^2} \right] C_i g_i^2 \ln \left( \frac{K_i}{g_i} \right), \quad (23) \]

where \( i = 1, 2, 3 \) stands for the three gauge groups \( U(1)_Y, SU(2)_L \) and \( SU(3)_C \) respectively. Here \( M_i \) represent gaugino mass parameters and \( g_i(T) \) represents gaugino coupling constant at finite temperature (from MSSM RGE) [24,25]:

\[ g_i(T) \simeq \frac{1}{\sqrt{\frac{1}{g_i^2(M_Z)} - \frac{b_i}{8\pi^2} \ln \left( \frac{T}{M_Z} \right)}} \]

with \( b_1 = 11, b_2 = 1, b_3 = -3 \). Here \( C_i \) and \( K_i \) represents the constant associated with the gauge groups \( U(1)_Y, SU(2)_L \) and \( SU(3)_C \) with \( C_1 = 11, C_2 = 27, C_3 = 72 \) and \( K_1 = 1.266, K_2 = 1.312, K_3 = 1.271 \) [24,25].
Further re-expressing Eq. (22) in terms of the parameter “x” and imposing the boundary condition \( \dot{T} = 0 \) at maximum energy density \( \rho_r(x = x_{\text{max}}) \) the thermal gravitino dark matter relic abundance is given by

\[
\Omega_{3/2}(x) = \frac{n_{3/2}(x)}{s(x)} = \frac{45}{2\pi^2 x^3 N_\star T^3(x)} \left[ \frac{C_3}{4} \frac{\sqrt{C_2 + C_1(x - 1)}}{x^4} \left\{ C_1(2 - 5x) - 2C_2 \right\} + \frac{3C_1^2 x^2}{\sqrt{(C_1 - C_2)(C_2 + C_1(x - 1))}} \tan^{-1} \left( \frac{\sqrt{C_2 + C_1(x - 1)}}{C_1 - C_2} \right) \right] + \frac{1}{3x_{\text{max}}^2} \{ C_3(C_2 + C_1(x_{\text{max}} - 1)) \}^2 - \frac{C_3}{4} \frac{\sqrt{C_2 + C_1(x_{\text{max}} - 1)}}{x_{\text{max}}^4} \left\{ C_1(2 - 5x_{\text{max}}) - 2C_2 \right\} + \frac{3C_1^2 x_{\text{max}}^2}{\sqrt{(C_1 - C_2)(C_2 + C_1(x_{\text{max}} - 1))}} \tan^{-1} \left( \frac{\sqrt{C_2 + C_1(x_{\text{max}} - 1)}}{C_1 - C_2} \right) \right],
\]

where the entropy density is given by \( s(x) = \frac{2\pi^2}{45} N_\star T^3(x) \). Here the temperature can be expressed in terms of the tensor-to-scalar ratio and the parameter “x” as:

\[
T(x) = \frac{1}{\sqrt{\pi}} \left[ \frac{30}{N_\star x^4} \left( \rho_{\text{osc}} - \rho_{\phi_0} \left\{ 1 - \frac{0.48(x - 1)}{\rho_{\text{osc}}} \right\} \right) \right]^{1/4}.
\]

and we also introduce new sets of parameters defined as:

\[
C_1 = \frac{\Gamma_\phi \rho_{\phi_0}}{H_{\text{osc}}}, \quad C_2 = \rho_{\text{osc}} - \rho_{\phi_0}, \quad C_3 = \frac{30\sqrt{10\bar{\alpha}\xi^2(3)}}{H_{\text{osc}}\pi^7 M_P^2 N_\star^{3/2}}, \quad x_{\text{max}} = \frac{4}{3} - \frac{4C_2}{\left(9C_1 H_{\text{osc}}^2\right)}.
\]

In this paper we introduce the leptogenesis scenario in presence of DBI Galileon which has the following remarkable phenomenological features:

- In Fig. 1, the theoretically allowed region shows that the reheating temperature for DBI Galileon is high enough and lies around the GUT scale \((10^{16} \text{ GeV})\). This is the first observation we have made from our analysis in the context of DBI Galileon, which is remarkably different from the GR prescribed setup as using GR we can probe up to \(10^{10} \text{ GeV}\). Such high values of the reheating temperature implies that the obtained value of the tensor-to-scalar ratio from the DBI Galileon inflationary set up lies within a wide range: \(2.4 \times 10^{-3} < r_\star < 0.12\), at the pivot scale of momentum \(k_\star \sim 0.002 \text{ Mpc}^{-1}\), which confronts well the Planck data. If the signatures of the primordial gravity waves will be detected at present or in near future then the consistency between the high reheating temperature and gravity waves can be directly verified from our prescribed model using Eq. (16).
Fig. 2. Variation of gravitino dark matter relic density parameter ($\Omega_{3/2}$) with respect to reheating temperature ($T_r$). The dark gray shaded region shows the theoretically allowed region which lies within the upper bound of the tensor-to-scalar ratio ($r_\star \leq 0.12$) at the momentum pivot scale $k_\star = 0.002$ Mpc$^{-1}$ represented by a blue vertical line in light of PLANCK data. We have explicitly shown the observationally allowed region of $\Omega_{3/2}$ by imposing the constraints from the PLANCK data. Further we have also pointed the constraint parameter space obtained from the PDG catalog. Most importantly, the overlapping region within the range $0.245 < \Omega_{3/2} < 0.250$ shown by the black strip satisfies both the constraints obtained from PLANCK and PDG data. For the numerical estimation in the present context we have used, $G_1 = 1$, $G_2 = 0.5$, $G_3 = 2$, $G_4 = 1$, $G_5 = -0.5$ and $B_1 = 2M_P^2$, $B_2 = 2$, $B_3 = 1$, $B_4 = 3$, $B_5 = 2M_P^4$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

In Fig. 2 we have explicitly shown the behaviour of gravitino relic abundance with respect to reheating temperature in light of PLANCK and PDG data. The overlapping region within the range $0.245 < \Omega_{3/2} < 0.250$ satisfies both the dark matter constraints obtained from PLANCK and PDG data as given by [53,54]:

\[
\begin{align*}
\Omega_{\text{PLANCK}}^{\text{DM}} & = 0.26 \pm 0.01 \\
\Omega_{\text{PDG}}^{\text{DM}} & = 0.22 \pm 0.03.
\end{align*}
\]  

(28)

In the present article we have studied cosmological consequences of reheating and dark matter phenomenology in the context of DBI Galileon on the background of low energy effective supergravity framework. We have engaged ourselves in investigating for the effect of perturbative reheating by imposing the constraints from primordial gravitational waves in light of the PLANCK data. Further we have established a cosmological connection between thermal gravitino dark matter relic abundance, reheating temperature and tensor-to-scalar ratio in the present context. To this end we have explored the model dependent features of thermal relic gravitino abundance by imposing the dark matter constraint from PLANCK + PDG data, which is also consistent with the additional constraint associated with the upper bound of tensor-to-scalar ratio $r_{0.002} \leq 0.12$ obtained from PLANCK data.

Acknowledgements

S.C. thanks Council of Scientific and Industrial Research, India, for financial support through Senior Research Fellowship (Grant No. 09/093(0132)/2010). S.C. also thanks Centre for Theoretical Physics, Jamia Millia Islamia, for extending hospitality. A.D. thanks Council of Scientific and Industrial Research, India, for financial support through Senior Research Fellowship (Grant No. 09/466(0125)/2010).
Appendix A

In Eqs. (11,13,15,17) the trilinear functions are given by:

\[ F_{ijk} = |\lambda_{ijk}|^2 \left[ 2 + \frac{9g^4}{4c_w^4} \left( \frac{1}{4} + 2s_w^4 - s_w^2 \right) I_1^2(m_{\phi_k}^2, 0, 0) \right] \]

\[ B_{ijk} = |\lambda_{ijk}|^2 \left[ 2 + \frac{9g^4}{4c_w^4} \left( \frac{1}{4} + 2s_w^4 + s_w^2 \right) I_1^2(m_{\phi_k}^2, 0, 0) \right] \]

where the integral \( I_1(m_k^2, m_i^2, m_j^2) \) is defined as

\[ I_1(m_k^2, m_i^2, m_j^2) = -i \int_0^1 \int_0^1 dx \, dy \left[ \gamma_E + \frac{1}{2} + \frac{N(x, y)}{2Q^2(x, y)} + \ln \left( \frac{Q^2(x, y)}{4\pi\mu^2} \right) \right] \]

with

\[ N(x, y) = x(1-x)m_i^2 + y(1-y)m_i^2 + \frac{1}{2}[(1-x-y)(m_k^2 - m_i^2 - m_j^2)] \]

\[ Q^2(x, y) = x^2m_i^2 + y^2m_i^2 - xy(m_k^2 - m_i^2 - m_j^2) + (1-x-y)m_z^2. \]

In Eq. (30) \( \gamma_E = 0.5772 \) is the Euler–Mascheroni constant originating in the expansion of the gamma function. Here \( c_w = \cos \theta_w, s_w = \sin \theta_w \) (where \( \theta_w = \) Weinberg angle) and \( m_z \) be the mass of the Z boson.

References
