Vanishing Higgs one-loop quadratic divergence in the scotogenic model and beyond

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A B S T R A C T

It is shown that the inherent one-loop quadratic divergence of the Higgs mass renormalization of the standard model may be avoided in the well-studied scotogenic model of radiative neutrino mass as well as other analogous extensions.

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In quantum field theory, the additive renormalization of \( m^2 \) for a scalar field of mass \( m \) is a quadratic function of the cut-off scale \( \Lambda \). The elegant removal of this quadratic divergence is a powerful theoretical argument for the existence of supersymmetric particles. However, given the recent discovery of the 126 GeV particle [1,2] at the Large Hadron Collider (LHC), presumably the long sought Higgs boson of the standard model, and the nonobservation of any hint of supersymmetry, it may be a good time to reexamine the model and beyond.

It was suggested a long time ago [3] that in the standard model of quarks and leptons, the condition

\[
\frac{3}{2} M_W^2 + \frac{3}{4} m_Z^2 + \frac{3}{4} m_H^2 = \sum_N m_N^2, \tag{1}
\]

where \( N_f = 3 \) for quarks and \( N_f = 1 \) for leptons, would make the coefficient of the \( \Lambda^2 \) contribution to \( m^2_\nu \) vanish. This would predict \( m_H = 316 \text{ GeV} \), which we now know to be incorrect.

The same idea may be extended to the case of two Higgs doublets [4–7] where \( \langle \phi^0_{1,2} \rangle = v_{1,2} \), with \( v = \sqrt{v_1^2 + v_2^2} = 174 \text{ GeV} \). In that case, the vanishing of quadratic divergences would also depend on how \( \Phi_{1,2} \) couple to the quarks and leptons. In the scotogenic model of radiative neutrino mass [8], there are two scalar doublets \( \phi^+, \phi^0 \) and \( (\eta^+, \eta^0) \), distinguished from each other by a discrete \( Z_2 \) symmetry, under which \( \Phi \) is even and \( \eta \) odd. Thus only \( \phi^0 \) acquires a nonzero vacuum expectation value \( v \). This same discrete symmetry also prevents \( \eta \) from coupling to the usual quarks and leptons, except for the Yukawa terms

\[
\mathcal{L}_Y = h_{ij} (v_1 \eta^0_i - l_i \eta^+ j) N_j + H.c., \tag{2}
\]

where \( N_i \) are three neutral singlet Majorana fermions odd under \( Z_2 \). As a result, neutrinos obtain one-loop finite radiative Majorana masses as shown in Fig. 1. This is a well-studied model which also offers \( \sqrt{2} \text{Re}(\eta^0) \) as a good dark-matter candidate [9]. The lightest \( N \) may also be a dark-matter candidate [10] but is more suitable if the dark-matter discrete symmetry \( Z_2 \) is extended to \( U(1)_D \) as proposed recently [11].

The scalar potential of the scotogenic \( Z_2 \) model is given by [8]

\[
V = m_\phi^2 \phi^+ \phi + m_\eta^2 \eta^+ \eta + \frac{1}{2} \lambda_1 (\phi^+ \phi)^2 + \frac{1}{2} \lambda_2 (\eta^+ \eta)^2 + \lambda_3 (\phi^+ \eta)(\eta^+ \phi) + \frac{1}{2} \lambda_5 [(\phi^+ \eta)^2 + (\eta^+ \phi)^2]. \tag{3}
\]

Let \( \phi^0 = v + H/\sqrt{2} \) and \( \eta^0 = (\eta_R + i \eta_I)/\sqrt{2} \), then

\[
m^2(\phi) = 2 \lambda_1 v^2, \tag{4}
\]

\[
m^2(\eta^+) = m_\phi^2 + \lambda_3 v^2, \tag{5}
\]

\[
m^2(\eta_R) = m_\phi^2 + (\lambda_3 + \lambda_4 + \lambda_5) v^2, \tag{6}
\]

\[
m^2(\eta_I) = m_\phi^2 + (\lambda_3 + \lambda_4 - \lambda_5) v^2. \tag{7}
\]

The corresponding two conditions for the vanishing of quadratic divergences are

\[
\frac{3}{2} M_W^2 + \frac{3}{4} M_Z^2 + \frac{3}{4} m_H^2 + \left( \lambda_3 + \frac{1}{2} \lambda_4 \right) v^2 = 3 m^2_\nu, \tag{8}
\]
tron may acquire a radiative mass by assigning terms

The conditions for vanishing quadratic divergence in this model are

As for the Yukawa couplings $f$, vanishing quadratic divergence is necessary for this scenario. In a model with simply a second “inert” scalar doublet[12,13], vanishing quadratic divergence will not be possible. To test Eq. (10), Eqs. (5) to (7) may be used, i.e.

As for $\lambda_3$, it may be extracted[14,15] from $H \rightarrow \gamma \gamma$ using also $m_\tau$. However Eq. (11) is very difficult to test, because $h^2$ and $\lambda_2$ are not easily measurable.

Analogous extensions of the scotogenic model may also accommodate vanishing quadratic divergences. As an example, consider the addition of a charged scalar $X^+$ odd under $Z_2$, then the electron may acquire a radiative mass by assigning $\epsilon_3$ to be odd with the Yukawa couplings $\epsilon_3 N_L X^+$ as shown in Fig. 2, where $N_L$ is even under $Z_2$, but the soft Dirac mass term $N_L N_R$ breaks $Z_2$ explicitly. With the addition of $X^+$, the scalar potential has the extra terms

The conditions for vanishing quadratic divergence in this model are then:

\[ \frac{3}{2} m_W^2 + \frac{3}{4} M_Z^2 + \left( \frac{3}{2} \lambda_2 + \lambda_3 + \frac{1}{2} \lambda_4 \right) v^2 = \sum_{i,j} h_{ij}^2 v^2. \]  

\[ 3 \left( M_Z^2 - M_W^2 \right) + \left( \lambda_6 + \lambda_7 + \lambda_8 \right) v^2 = f^2 v^2. \]  

Again, verification is possible, at least in principle. Other more involved scenarios such as the scotogenic $U(1)_D$ model[11] or that of a recent proposal[16], where all quark and lepton masses are radiative with either $Z_2$ or $U(1)_D$ dark matter, may also have similar viable solutions.

It is of course well-known that the one-loop vanishing of the Higgs quadratic divergence is not invariant under the renormalization-group running of the gauge, Yukawa, and quartic scalar couplings. Thus the two-loop SM contribution has also been studied[17,18]. Whereas it is impossible to have both set equal to zero, if the latter is viewed as a perturbation to the first for a physical cutoff[19], then the approximate validity of the Veltman condition remains a plausible solution. Other ideas regarding the inherent quadratic divergence of any scalar mass have also been discussed in the recent literature[20–22].

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References