What is triggering the Higgs mechanism and inflation?

F. Jegerlehner

Humboldt-Universität zu Berlin, Institut für Physik
Newtonstrasse 15, D-12489 Berlin, Germany and
Deutsches Elektronen-Synchrotron (DESY)
Platanenallee 6, D-15738 Zeuthen, Germany

ricevuto il 4 Febbraio 2014

Summary. — We review a recent analysis presented in arXiv:1304.7813 [hep-phys] and 1305.6652 [hep-phys]. After the discovery of the Higgs the most relevant structures of the SM have been verified and for the first time we know all parameters of the SM within remarkable accuracy. Together with recent calculations of the SM renormalization group coefficients up to three loops we can safely extrapolate running couplings high up in energy. Assuming that the SM is a low energy effective theory of a cutoff theory residing at the Planck scale, we are able to calculate the effective bare parameters of the underlying cutoff system. For my specific set of \( \overline{\text{MS}} \) input parameters, it turns out that the bare mass term changes sign not far below the Planck scale, which means that in the early universe the SM was in the symmetric phase. The sign-flip, which is a result of a conspiracy between the SM couplings and their screening/antiscreening behavior, triggers the Higgs mechanism. Above the Higgs phase transition the bare mass term in the Higgs potential must have had a large positive value, enhanced by the quadratic divergence of the bare Higgs mass. The Higgs mass term thus provides the large dark energy density in the early universe, which triggers Gaussian slow-roll inflation, i.e. the SM Higgs is the inflaton scalar field. Reheating is dominated by the decay of the heavy Higgses into (in the symmetric phase) massless top/anti-top quark pairs. The Higgs mechanism stops inflation and the subsequent electroweak phase transition provides the masses to the SM particles in proportion to their coupling strength. The previously most abundantly produced particles are now the heaviest and decay into the lighter ones, by cascading down the Cabibbo-Kobayashi-Maskawa (CKM)-matrix from top and bottom to normal matter. Baryon-number \( B \) violating interactions are naturally provided by Weinberg’s set of close-by dimension 6 four-fermion effective interactions. Since matter is produced originating in the primordial heavy Higgs fields via \( C \)- and \( CP \)-violating decays we have actually a new scenario which could explain the baryon-asymmetry essentially in terms of SM physics.

PACS 12.15.-y – Electroweak interactions.
PACS 13.40.-f – Electromagnetic processes and properties.
PACS 12.15.Lk – Electroweak radiative corrections.
1. – Introduction

With the discovery of the Higgs boson by ATLAS [1] and CMS [2] at the LHC all relevant ingredients of the Standard Model (SM) have been established experimentally. In particular, for the first time we know all the basic SM parameters with remarkable accuracy [3]. The Higgs mass, found to be $M_H = 125.9 \pm 0.4 \text{ GeV}$, turned out to have a value just in the window which allows or almost allows us to extrapolate SM physics up to the Planck scale [4, 5] without the need to assume some new non-SM physics\(^{(1)}\). This together with the fact that so far no hints for a supersymmetric extension or extra dimensions etc. have been found, sheds new light on the structure of the SM and its self-consistency. The SM together with its specific values for the couplings, the gauge couplings $g', g, g_s$, the top-quark Yukawa coupling $y_t$ and newly, the Higgs self-coupling $\lambda$ are supporting the picture of the SM as a low energy effective theory of some cutoff system residing at the Planck scale. In such a framework the relation between bare and renormalized physical low energy parameters attains a physical meaning and from the knowledge of the physical parameters we can calculate actually the bare parameters relevant at the high (short distance) scale. Renormalizability of the SM as well as all known conditions which where required to get the SM as a minimal renormalizable extension of its low energy effective structure now are a consequence of the low energy expansion. As we do not see the infinite tower of non-renormalizable effective operators, the low energy effective theory actually has more symmetry than the underlying cutoff system at the Planck scale, which is largely unknown in its details. In such a scenario simplicity and symmetries are expected to be naturally generated dynamically as a consequence of our blindness for the details of the underlying cutoff system. This is our low energy effective SM (LEESM) scenario [6, 7].

The key questions asked here are 1) How does SM physics look like at much higher energies? and 2) What does the Higgs potential look like at the bare level, when going to the Planck scale? The first question can be answered, under the assumption that no substantial effect come in by possible physics beyond the SM, by studying the evolution of couplings as determined by the SM renormalization group (RG), which now is known to three loops in the $\overline{\text{MS}}$ renormalization scheme [8-10]. The initial $\overline{\text{MS}}$ values have to be obtained by appropriate matching conditions from the physical on-shell parameters. The top Yukawa coupling and the Higgs self-coupling are only known via their measured masses via the mass coupling relations, which derive from the Higgs mechanism. The appropriate renormalization scheme is the $\overline{\text{MS}}$ scheme and the $\overline{\text{MS}}$ input parameters have to be calculated by applying appropriate matching conditions as e.g. discussed in refs. [11-15]. Figure 1 in ref. [6] shows the solutions of the RG equations and the $\beta$-functions up to $\mu = M_{\text{Pl}}$.

Remarkably, as previously found for the running couplings in refs. [14, 16-18], all parameters stay in bounded ranges up to the Planck scale if one adopts our matching conditions together with the so far calculated RG coefficients. A number of analyses adopting essentially the input parameters of ref. [14] find that the Higgs self coupling $\lambda$ gets negative, which may happen at a scale as low as $10^9 \text{ GeV}$. We note that including

\(^{(1)}\) We will discuss here the scenario assuming that the Higgs vacuum is stable up to the Planck scale, which depends on the precise value of the $\overline{\text{MS}}$ top Yukawa coupling calculated with the experimental top quark mass as an input. In my opinion the issue of the precise value of $y_t^{\overline{\text{MS}}}$ is not really settled.
all known terms no transition to a metastable state in the effective Higgs potential is observed for our set of \( \overline{\text{MS}} \) input parameters, i.e. no change of sign in \( \lambda \) occurs, in agreement with refs. [14,9]. Results at various scales are collected in table 1 of ref. [6].

2. – The quadratic divergences in the SM

In the unbroken phase the only quadratic divergences show up in the renormalization of Higgs potential mass \( m \). Since the UV structure is the same in the broken phase, there are no other problems in this direction. Here we encounter the “fine tuning” relation

\[
m_0^2 = m^2 + \delta m^2, \quad \delta m^2 = \frac{\Lambda^2}{32\pi^2} C,
\]

where, at one-loop, the coefficient function \( C_1 \) has been discussed within this context by Veltman [19], and modulo small lighter fermion contributions is given by

\[
C_1 = \frac{6}{v^2} (\mu H^2 + \mu Z^2 + 2 \mu W^2 - 4 \mu t^2) = 2 \lambda + \frac{3}{2} \phi'^2 + \frac{9}{2} \phi^2 - 12 v_t^2.
\]

The key point here is that \( C_1 \) is universal and depends on dimensionless gauge, Yukawa and Higgs self-coupling only, the RGs of which are unambiguous. Similarly, for the two-loop coefficient \( C_2 \) (where however results differ by different groups (non-universal?)). The correction is numerically small, fortunately. Recently, Hamada, Kawai and Oda [18] have investigated the coefficients to two loops in terms of running couplings and found the coefficients of the quadratic divergence to have a zero not too far above the Planck scale. We find the zero to lie below the Planck scale, this, like the issue of the vacuum stability, is again a matter of the adopted input value for \( y_{\text{MS}}^t \). The SM makes a prediction for the coefficients \( C_i \) and hence for the bare mass parameter in the Higgs potential, which we displayed in fig. 3 of ref. [6]. In the broken phase given by \( m_{\text{bare}}^2 = \frac{1}{2} m_H^2 \), \( m_{\text{bare}}^2 \) is calculable and is exhibiting the following properties: i) the coefficient \( C_1(\mu) \) exhibits a zero, for \( \mu_H = 126 \text{ GeV} \) at about \( \mu_0 \sim 1.4 \times 10^{16} \text{ GeV} \), not far below \( \mu = \mu_{\text{Pl}} \), ii) at the zero of the coefficient function the counterterm \( \delta m^2 = m_{\text{bare}}^2 - m^2 = 0 \) vanishes and the bare mass changes sign, iii) this represents a first order phase transition which triggers the Higgs mechanism and seems to play an important role for cosmic inflation, iv) at the transition point \( \mu_0 \) we have \( v_{\text{bare}} = v(\mu_0^2) \), where \( v(\mu^2) \) is the \( \overline{\text{MS}} \) renormalized Higgs VEV, and v) the jump in the vacuum density, thus agrees with the renormalized one:

\[
\Delta\rho_{\text{vac}} = \frac{\lambda}{24} v^4(\mu_0^2), \quad \text{and thus is } O(v^4) \quad \text{and not } O(M_{\text{Pl}}^4). \quad \text{Note that the renormalized}
\]

\( m^2 \) in the symmetric phase is unknown, but we assume that \( m^2 \ll \delta m^2 \).

In any case, at the zero of the coefficient function there is a phase transition, which corresponds to a restoration of the symmetry.

There is a close relation between the Higgs mechanism and the electroweak (EW) phase transition. To this end we have to consider the relevant finite temperature effects, which are dominating especially in the very early thermal evolution of the universe at the hot big bang. Including the leading effect only, the finite temperature effective potential reads

\[
V(\phi, T) = \frac{1}{2} (g_T T^2 - \mu^2) \phi^2 + \frac{\lambda}{24} \phi^4 + \ldots .
\]
Fig. 1. – The role of the Higgs in the finite-temperature SM. Left: for \( \mu_0 \sim 1.4 \times 10^{16} \) GeV (\( M_H \sim 126 \) GeV, \( M_t \sim 173.5 \) GeV). Right: finite-temperature delayed transition for \( \mu_0 \sim 6 \times 10^{17} \) GeV (\( M_H \sim 124 \) GeV, \( M_t \sim 175 \) GeV), the \( m^2_{\text{bare}} \) term alone is flipping at about \( \mu_0 \sim 3.5 \times 10^{18} \) GeV.

The usual assumption is that the Higgs is in the broken phase \( \mu^2 > 0 \) from the beginning at the big bang. The EW phase transition is then taking place when the universe is cooling down below the critical temperature \( T_c = \sqrt{\mu^2/g_T} \), meaning \( g_T T^2 - \mu^2 < 0 \) when \( T < T_c \). My analysis, in contrast, shows that above the phase transition point \( \mu_0 \), the SM is in the symmetric phase with \( -\mu^2 \rightarrow m^2 = (m^2_{H} + \delta m^2_{H})/2 > 0 \), and the EW phase transition is essentially triggered by the Higgs mechanism, at least it can happen only after the Higgs mechanism has taken place, thus \( \mu_{\text{EW}} < \mu_{\text{HM}} = \mu_0 \). The relevant question here is which of the terms \( \delta m^2 \) or \( g_T T^2 \) is leading in the relevant epoch of early universe? I find \( m^2(\mu = M_{\text{Pl}}) \approx 0.87 \times 10^{-3} M_{\text{Pl}}^2 \) and a coefficient \( g_T = \frac{1}{4\pi} (2m^2_W + m^2_Z + \frac{1}{2} m^2_H) = \frac{1}{16} [3 g^2 + g'^2 + 4 y_t^2 + \frac{3}{2} \lambda] \approx 0.0983 \) using the couplings at scale \( M_{\text{Pl}} \). The dramatic jump in \( m^2_{\text{bare}} \) at \( \mu_0 \) in any case drags the Higgs into the broken phase not far below \( \mu_0 \) as illustrated in fig. 1.

3. – The Higgs hierarchy and its impact on inflation

Cosmological inflation [20] requires an exponential growth of the Friedman-Robertson-Walker radius of the universe \( a(t) \), i.e. \( a(t) \propto e^H t \) with \( H(t) = \dot{a}/a(t) \) the Hubble constant at cosmic time \( t \) (dotted quantities denote time derivatives). Inflation is able to solve the flatness problem and the horizon problem. The inflation term comes in via the SM energy-momentum tensor, which is the source of the Einstein tensor of gravity, and thus affects the cosmological Friedman equations. The second Friedman equation reads

\[
\ddot{a}/a = -\frac{\ell^2}{2} (\rho + 3p),
\]

where \( \ell^2 = 8\pi G/3 \) and \( G = 1/M_{\text{Pl}}^2 \) is Newton’s gravitational constant. From (4) we learn that the condition for growth \( \ddot{a} > 0 \) requires \( p < -\rho/3 \) and hence \( \frac{1}{2} \dot{\phi}^2 < V(\phi) \).

Cosmic Microwave Background (CMB) observations strongly favor the slow-roll inflation \( \frac{1}{2} \dot{\phi}^2 \ll V(\phi) \) condition, implying the dark energy equation of state \( w = p/\rho = -1 \).
Indeed the Planck mission [21] measured $w = -1.13_{-0.10}^{+0.13}$. The first Friedman equation reads $\ddot{a}/a^2 + k/a^2 = \ell^2 \rho$ and may be written as $H^2 = \ell^2 [V(\phi) + \frac{1}{2} \dot{\phi}^2] = \ell^2 \rho$, while the field equation reads $\ddot{\phi} + 3H\dot{\phi} = -V'(\phi) = -dV(\phi)/d\phi$.

What the SM phase transition to the symmetric phase suggests is a Higgs potential where $\lambda$ remains small and positive and a bare mass square very large and positive, before it flipped to negative values at later times, this at least naively supports the Gaussian slow-roll inflation condition. The leading behavior would then be characterized by a free massive scalar field with potential $V = \frac{\lambda}{2} \phi^2$ such that $H^2 = (\dot{\phi}/a)^2 = \frac{\omega}{m} \phi^2$ and $\ddot{\phi} + 3H\dot{\phi} = m^2 \phi$ which is a harmonic oscillator with friction. It tells us that the Higgs field is decaying more or less rapidly, except during inflation where it should vary slowly such that $V(\phi)$ remains more or less constant. This is a crucial point: as the Higgs field depends on the renormalization scale logarithmically only it is not expected to be particularly large at the time of the EW phase transition. However, extrapolating the solution of the dynamical equations reveals that $\phi$ grows when we go back to the big bang. A large field is actually required in order to get a dominating dark energy which triggers inflation.

The SM inflation pattern seems well supported by observation, most recently by the Planck 2013 results [21]. The cosmological constant is characterized by the equation of state $w = p/\rho = -1$, and in our scenario is a prediction of the SM for times before the phase transition when $\mu > \mu_0$. Likely, the dark energy term $\rho_{\text{vac}} = \langle V(\Phi) \rangle$ is dominated by the Higgs mass term $\frac{\lambda}{2} m^2 \phi^2$ what would support Gaussian inflation, consistent with Planck mission data. Inflation in any case is stopped by the phase transition when $\mu = \mu_0$. In the symmetric phase, the effective number of relativistic degrees of freedom is $g_*(T) = g_0(T) + \frac{3}{4} g_f(T) = 102.75$ such that the Hubble constant is $H_{\text{Pl}} \sim 16.83 M_{\text{Pl}}$ at Planck time, at the beginning of the very early radiation dominated era. As the temperature as well as $\phi$ are decreasing the mass term will be dominating for some time before the phase transition, for appropriate values of $\mathcal{C}(\mu)$ and $\lambda(\mu)$ at these times.

The four Higgses near the Planck scale have an effective mass about $m_{Hb} \simeq 3.6 \times 10^{17} \text{GeV}$ and thus can be produces in processes like $WW \rightarrow HH$ or $tt \rightarrow H$ at times at and after the big bang. The big difference to standard big bang scenarios is that the Higgses are primordial, i.e. they exist as modes in the Planck medium in advance of being produced by high energy radiation processes. A Higgs in this phase has a width dominated by $H \rightarrow t\bar{t}$ decay, since direct couplings $HWW$ and $HZZ$ are absent in the symmetric phase. The SM predicts that the Higgses produce top/anti-top quark radiation most abundantly. This means that reheating is mainly provided by $H \rightarrow t\bar{t}$ decays, with known rate.

Concerning the possibility of baryogenesis, baryon-number violating interactions in the LEESM scenario naturally are the close-by dimension 6 effective four-fermion interactions discussed first by Weinberg [22]. Usually, it is assumed that some unknown very heavy particle $X$ is responsible for baryogenesis. Decays proportional to the $CP$-violating CKM matrix elements $V_{ud}$ and $V_{ub}$ $H^+ \rightarrow t\bar{d}$ and $H^- \rightarrow b\bar{u}$ are important as a condition for baryogenesis. These determine the primary densities for the lighter states including baryonic matter, which get augmented afterwards, after all particles have acquired their masses, by heavier particle decays. After the EW phase transition other $CP$-violating SM processes come into play. We note that matter production is preferably into fermion pairs with the biggest Yukawa couplings. Thus, predominantly into yet massless “would-be heavy” top, bottom, $\tau$, and so on fermions. After the EW phase transition the now heavy states decay into the lighter ones, with the smaller Yukawa couplings. The major
part of normal matter is produced via the heavy states which are cascading down the CKM matrix. Apparently in such a scenario the system likely would intermittently be far from equilibrium while approaching the EW phase transition, and the dynamics behind could be important for the explanation of the baryon-asymmetry.

4. – Conclusion

The main conclusions have been given in the abstract already. Here we would like to point out the importance of an extended analysis of the possible consequences of the SM physics. One of our main assumptions has been the one that physics beyond the SM is not needed to understand the early universe. The point is that in the LEESM scenario unseen physics can naturally be expected, however, it must be natural in the sense of a low energy expansion. Grand unified theories as well as a supersymmetrized SM are not natural, because they require an improbably high amount of conspiracy of very many modes, while the emergence of an extra $U(1)$ or a $SU(4)$ look much more natural. A hidden $SU(4)$ could provide dark matter by forming stable bosonic quartet bound states.

As we have seen a big issue is the very delicate conspiracy between SM couplings. Therefore precision determinations of parameters are more important than ever and a real challenge for experiments at the LHC and at a future ILC, which may improve substantially $\lambda$, $y_t$ and $\alpha_s$. But also low energy hadron facilities have to play an important role as needed for a better control of the non-perturbative hadronic effects in $\alpha(M_Z)$ and $\alpha_2(M_Z)$.

Our analysis shows that the role of the Higgs is not just to provide masses to SM particles, it likely plays a key role in early cosmology providing the necessary dark energy which triggers inflation. Also reheating after inflation, baryogenesis and the EW phase transition appear in a new light.

I thank the organizers of the LC 2013 Workshop at the ETC* Trento for the kind invitation, the kind hospitality and for the support.

REFERENCES