SIGNAL PROCESSING FOR RADIATION DETECTORS

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ABSTRACT

In this series of lectures some fundamental aspects of signal processing for various particle and photon detectors will be discussed. The principal topics will be:

- Charge collection and signal formation in detectors.
- Physical sources of noise and fundamentals of amplification, low noise devices and preamplifiers.
- Noise and filtering. Amplitude, time and waveform measurements.
- Position sensing methods.
- Signal processing for semiconductor detectors, drift and time projection chambers, and for ionization chamber calorimeters.
- Signal processing circuits, role of hybrid and monolithic technology.

Examples illustrating joint optimization of the detector and the signal processing will be presented.

OUTLINE OF LECTURES

Lecture: Time:

1. Signal formation in detectors.

2. Physical sources of noise.
Charge amplification and related noise.
Filtering; amplitude, time and waveform measurements.
Some circuit considerations.

3. Position sensing methods:
   Delay line sensing, charge division, centroid finding methods.

4. Drift chambers and proportional detectors for very high counting rates.
TPCs and "pad" detectors.
Semiconductor detectors.
Ionization chamber calorimeters.
1. Signal formation in detectors.

2. Physical sources of noise.
   Charge amplification and related noise.
   Filtering; amplitude, time and waveform measurements.
   Some circuit considerations.

3. Position sensing methods
   (Charge division, delay line sensing,
    centroid finding methods).

4. Drift chambers and proportional detectors for
   very high counting rates.
   TPC's and "pad" detectors.
   Semiconductor detectors.
   Ionization chamber calorimeters.

References
Motivation:

Detectors of interest:

1. Drift and proportional chambers.
2. TPC's and "pad" detectors.
4. Ionization chamber calorimeters.
5. Special detectors for neutrino, proton decay experiments.

Devices, circuits and technology

1. Amplifying devices.
2. Circuit configurations.
3. Technology, hybrids and monolithics.
4. Detector design (electrodes).

Basics

1. Signal formation.
2. Physical origins of noise.
3. Amplitude, time and waveform measurements, (i.e., energy, time and position), filtering.
4. Detector-amplifier charge transfer.
5. Position sensing methods.
LECTURE 1.

SIGNAL FORMATION
IN
DETECTORS
Image Charge

Electrode

Image charge

\[ q_2 = \frac{x}{d} q \]
\[ q_1 = \frac{d-x}{d} q \]

Note: only two of the infinite series of image charges are shown for simplicity.
Current induced by the motion of charge

\[ V_1 = \text{1 volt} \]

\[ i_1 = \frac{dQ_1}{dt} = \frac{d(2_wV_m)}{dt} \]

\[ 2_w = \text{const.} \]

\[ i_1 = 2_w \frac{dV_m}{dt} \frac{d\ell}{d\ell} \]

Weighting field \( \vec{E}_w \) (for 1 volt on electrode 1):

\[ \frac{dV_m}{d\ell} = -E_w \cos \theta \]

Current:

\[ i_1 = -2_w \vec{E}_w \vec{U} \]

1. Find weighting field \( \vec{E}_w(x, y, z) \)
2. \( \vec{V}(x, y, z) \) the charge velocity
3. \( x(t), y(t), z(t) \)

By reciprocity:

\[ 2_wV_m = Q_1V_1 \]

Normalize: \( V_1 = 1 \text{ volt} \)

\[ Q_1 = \frac{2_wV_m}{V_1} \]

\[ \frac{d}{dt}(x(t), y(t), z(t)) \]

\[ \text{charge coordinates} \]

In general solution with floating electrodes see NIM 193 (1982) 651
Induced signal in planar electrode geometry

\[ i = \frac{q}{d} \overrightarrow{E_w} \cdot \overrightarrow{v} \]

\[ \overrightarrow{E_w} = \frac{1}{d} \; \text{and} \; \overrightarrow{v} = -v \; \text{and} \; q = -e \]

**CURRENT**

\[ l = \frac{v}{d} \; \text{and} \; e = \frac{e}{t_d} \]

**CHARGE**

\[ Q_s(t) = \frac{e}{t_d} \left( \frac{t}{t_d} \right) \]

\[ Q_s(t) = N \cdot e \left[ \frac{t}{t_d} - \left( \frac{t}{t_d} \right)^2 \right] \]

\[ i = N e \frac{v}{d} \left( 1 - \frac{t}{t_d} \right) \]

If \( N \) is very large, fluctuations in pulse shape are small (e.g., liquids, solids) in calorimeters
Electron transport in LA

2mm

5.5 α-source MeV

charge vs time

Drift time: ~ 190 nsec/mm
Ionization Chamber with Frisch Grid

\[ i = -q E_w v \]

Weighting field

\[ \vec{v} = \mu \vec{E} \]

"True" (applied) field

Potential (energy) for electrons

Electron current

\[ i_a = q \frac{d_1 - x}{d_1} \]

Pos. ion current

\[ i_g = q \frac{x}{d_1} \]
Proportional Detector, Cylindrical Geometry

\[ \mathbf{v} = \text{drift velocity} = \mu \mathbf{E} \]

\( E_w = \text{weighting field} \)

\( E = \text{"real" applied field} \)

\( \mu = \text{mobility} \)

\[ E = \frac{V}{\ln \frac{r_e}{r_a}} \cdot \frac{1}{r} = E_a \frac{r_a}{r} \]

1. \[ E_w = \frac{1}{\ln \frac{r_e}{r_a}} \frac{1}{r} \]

2. \( u(r) = ? \rightarrow u = \frac{dr}{dt} = \mu E_a \frac{r_a}{r} \)

\[ i(r) = \frac{q}{2} E_w \mathbf{v} = q \frac{\mu E_a}{\ln \frac{r_e}{r_a}} \frac{r_a}{r^2} \]

3. \( r(t) = ? \rightarrow r^2 = r_a^2 + 2 \mu E_a r_a t \quad r \propto t^{1/2} \)

\[ \frac{i(t)}{i_m} = \frac{1}{1 + \frac{t}{t_o}} \]

\[ t_o = \frac{r_a}{2 \mu E_a} \]

\[ i_m = \frac{q}{2 t_o \ln \frac{r_e}{r_a}} \]

Observed (signal) charge from an avalanche \( Q_a \):

\[ \frac{Q_S}{Q_a} = \frac{1}{2 \ln \frac{r_e}{r_a}} \ln (1 + \frac{t}{t_o}) \]

\[ = \frac{\ln \frac{r_e}{r_a}}{\ln \frac{r_e}{r_a}} \]

- Diagram showing current-time relationship
- Graph indicating the charge over time
Anode wire  \( \rightarrow r \rightarrow \text{positive ions} \rightarrow r \text{[\mu m]} \)

\begin{align*}
0 & \, 50 \text{ nsec} \quad \rightarrow t \quad 60 \mu \text{sec} \\
10 & \, 60 \, 80 \, 120 \, 170 \, 2000 \\
\end{align*}

\text{Anode "current"}

(a)

With tail cancellation

(b)

\[ i = \frac{1}{1 + \frac{t}{t_0}} \]

\[ t_0 = \frac{r_a}{2 \mu + E_a} \approx 3-4 \text{ nsec in Xe} \]

\[ i_m = \frac{Q_{\text{w}}}{2 t_0 \ln \frac{r_e}{r_a}} \]

\[ Q(t) = \frac{Q_{\text{w}}}{2 \ln \frac{r_e}{r_a}} \left(1 + \frac{t}{t_0}\right) \]

\[ t_0 = 1.5 \mu \text{sec at Xe} \]

\[ r_a = 10 \mu \text{m} \]

\[
\begin{array}{c|c|c|c|c}
 t & 20 \text{ nsec} & 1 \mu \text{sec} & \frac{Q(20 \text{ nsec})}{Q(1 \mu \text{sec})} \approx 0.55 \\
\hline
\frac{Q(t)}{Q_{\text{w}}} & \approx 0.2 & \approx 0.55 \\
\end{array}
\]

\text{CHARGE COLLECTION IN GAS PROPORTIONAL DETECTORS}
Strip electrodes (semiconductor detectors, parallel plate chambers, etc.)

\[ i = -g_{uw} \vec{E}_w \vec{v} = -g_{uw} \vec{E}_w \frac{dx}{dt} \quad \text{(pos. current into electrode)} \]

\[ q = \int idt = -g_{uw} \int \vec{E}_w dx \]

a) \[ \int \vec{E}_w dx = 0 \quad \rightarrow q = 0 \quad \text{for } t_w > t_d \]

b) \[ \int \vec{E}_w dx = 0 \quad \text{not important} \]

c) \[ \int \vec{E}_w dx = 0 \quad \text{waveform important} \]

\[ t_w = \text{measurement (observation time)} \]
REFERENCES FOR SIGNAL FORMATION IN DETECTORS


3. G. Cavalleri, G. Fabri, E. Gatti, and V. Svelto:
   3B Extension of Ramo's Theorem as Applied to Induced Charge in Semiconductor Detectors, Nucl. Instr. & Meth. 92(1971)137.

NOISE:

Origin and Properties
**Noise process**

\[ S(t-t_i) \]

\[ \bar{n} \text{ [sec]} \]

\[ t_i \quad t_{i+1} \rightarrow t \]

\[ h(t) \]

\[ o \rightarrow t \]

\[ u \rightarrow t \]

\[ \iota(t) \]

\[ \int_{0}^{t} \]

\[ \text{Poisson sequence} \]

\[ * \]

\[ \text{Impulse response} \]

\[ \text{Output} \]

\[ v_i = g \cdot h(t_o-t_i) \]

\[ \bar{v}^2 = g^2 \sum_{i=1}^{N} h^2(t_o-t_i) \]

\[ a(\bar{v}^2) = g^2 h^2(u) \; \bar{n} \; du \]

\[ \bar{v}^2 = \sigma^2 = \bar{n} \cdot g^2 \int_{0}^{\infty} h^2(u) \; du \]

\[ \text{NOISE PROCESS} \]

\[ \text{PHYSICAL SYSTEM} \]

\[ 2kT \]

\[ R \]

\[ \sigma^2 = \frac{1}{T} \int_{0}^{T} x^2(z) \; dt \]

\[ e^{-\frac{x^2}{2\sigma^2}} \]

\[ 2.35 \sigma \]

"Central limit theorem" → gaussian
Power spectrum, correlation, total power and bandwidth

\[ h(t) = e^{-\frac{t}{\tau_F}} \]

\[ K_h(\tau) = \frac{1}{2\tau_F} e^{-\frac{1}{2\tau_F}} \]

\[ |H(\omega)|^2 = \frac{1}{1 + \omega^2 \tau_F^2} \]

\[ \langle \theta^2 \rangle = \frac{\hbar g^2}{2\tau_F} \]

\[ \tau_F \to 0 \quad \theta^2 \to \infty \]

\[ \omega^2 \tau_F^2 \gg 1 \]

\[ W(\omega) = W_0 |H(\omega)|^2 \propto \frac{1}{\omega^2} \]

W.IENER-KHINTCHINE Theorem:

\[ W(f) = 2 \int_0^\infty K(\tau) \cos(2\pi f \tau) d\tau \]

\[ K(\tau) = 4 \int_0^\infty W(f) \cos(2\pi ft) df \]

For an introduction see the book:

A MODEL FOR GENERATION OF NOISE SPECTRA

Random sequence of impulses (Poisson process)

"White noise" with 1st order band limit (relaxation process)

"Random walk" (c)

1/f noise

Excitation:

white noise

Transforming filter (i.e. physical system response)
"white noise"

"1/f noise"

"random walk"

Samples of noise waveforms
"Self-similarity" of $\frac{1}{f}$ noise (scaling invariance)

\[ \sigma^2(f_w, f_l) = \int \frac{1}{f^1} df = f_w \frac{f_h}{f_l} \]

For $\frac{f_w}{f_l} = \text{const}$, $\sigma^2(f_w, f_l) = \text{const}$
VARIANCE, i.e. MEASUREMENT ERROR DUE TO NOISE

FREQUENCY DOMAIN:

\[ w_d(t) = |f|^\alpha \quad \text{noise spectral density (normalized to } w_0) \]

\[ \sigma^2(f_h, f_e) = \int \frac{|f|^\alpha}{f_e} df = \frac{1}{1+\alpha} \left[ f_h^{1+\alpha} - f_e^{1+\alpha} \right] \quad \text{FOR } \alpha \neq -1 \]

\[ \sigma^2(f_h, f_e) = \ln \frac{f_h}{f_e} \quad \text{FOR } \alpha = -1 \]

\[ \frac{1}{f} \text{ NOISE} \]

\[ \text{FOR } 1/f^2 \quad \alpha = -2, \quad \sigma^2(\infty, f_e) = \frac{1}{f_e} \]

\[ \text{FOR } \alpha \geq -1 \quad \text{HIGH FREQUENCY DIVERGENT} \]

\[ \alpha \leq -1 \quad \text{LOW} \]

TIME DOMAIN:

\[ \delta^2(0, t) = \int h^2(t) dt = \ln \frac{t}{\delta} \quad \frac{|f|^\alpha}{1/f^1} \text{ NOISE} \]

Transforming filter:

\[ h(t) = -\frac{\alpha}{\delta} - 1 \]

\[ s \rightarrow 0 \]

\[ t \quad 1/f^2 \]

\[ t^2 \quad 1/f^3 \]

\[ t^3 \quad 1/f^4 \]

For more detail see:
IEEE Trans. Nucl. Sci., NS-16
NOISE VARIANCE AS A FUNCTION OF MEASUREMENT (FILTER) TIME

FOR POWER LAW SPECTRA

\( f_h/f_c = \text{const.} \quad \tau_f \propto \frac{1}{f_{h,c}} \quad \text{filter parameter, i.e., measurement time} \)

Spectrum - variance relation:

\[ \frac{1}{|f|} \rightarrow \sigma^2(\tau_f) = K_d \tau_f^{-1-\alpha} \]

Spectrum:

\[ W(f) = \cdots + W_1 |f|^1 + W_0 |f|^0 + W_{-1} |f|^{-1} + W_{-2} |f|^{-2} + \cdots \]

Variance:

\[ \sigma^2(\tau_f) = \cdots + K_1 \tau_f^{-2} + K_0 \tau_f^{-1} + K_{-1} \tau_f^{0} + K_{-2} \tau_f^{1} + \cdots \]

\[ W_N \frac{1}{|f|} \frac{1}{|f|^2} \quad \text{"white noise"} \]

\[ W_{-3/2} |f|^{-3/2} \quad \text{fractional exponents} \]

\[ K_{-3/2} \tau_f^{1/2} \]

"series" white noise in charge amplifiers (FET channel thermal noise)

"parallel" white noise in charge amplifiers (leakage current and resistor noise)

AMPLITUDE MEASUREMENTS, MEASUREMENTS WITH ARBITRARY \( \tau_c \) (NMR, EPR)

\( \tau_c = \text{noise "corner" time constant} \)
Low Frequency
Divergence of $1/f^2$ Noise and of $1/|f|$ Noise

<table>
<thead>
<tr>
<th>$\alpha = -2$</th>
<th>$\alpha = -1$</th>
<th>$1/f^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1/f^2$</td>
<td>$1/</td>
<td>f</td>
</tr>
<tr>
<td>$\sigma^2(\infty, \infty) = \frac{1}{f_0}$</td>
<td>$\sigma^2(f_h, \infty) = \ln \frac{f_h}{f_0}$</td>
<td></td>
</tr>
<tr>
<td>$\sigma^2(t, o) = t$</td>
<td>$\sigma^2(t, o) = \ln \frac{t}{s}$</td>
<td>$s = 10^{-6} \text{ sec}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$[\text{sec}]$</th>
<th>$[\log_{10}(t/s)]^{1/2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^{-6}$ (1 $\mu$ sec)</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>$10^5$ ($\sim$ 1 Day)</td>
<td>11</td>
</tr>
<tr>
<td>$10^9$ ($\sim$ 30 Years)</td>
<td>15</td>
</tr>
<tr>
<td>$10^{17}$ ($\approx$ Age of Universe)</td>
<td>23</td>
</tr>
</tbody>
</table>

Random walk, $1/f^2$ noise runs out of dynamic range for space or energy variables, except in special cases, i.e. phase of oscillators.

There is no divergence problem with $1/f$ noise. It has been observed (in MOS transistors, for example) down to $10^{-5}$ Hz.
ZERO-CROSSING STATISTICS OF NOISE

The noise is described by:

\[ W(\omega) \rightarrow K(\tau) \]

\[ w(\omega) \rightarrow k(\tau) \]

Then:
The frequency of positive zero crossings:

\[ n_{zc} = \frac{1}{2} \left[ - \frac{K''(0)}{K(0)} \right]^{1/2} \left[ \int_0^\infty \omega^2 W(\omega) d\omega \right]^{1/2} \]

The number of level crossings:

\[ n(u_d) = n_{zc} \cdot \exp \left( - \frac{u_d^2}{2K(0)} \right) \quad K(0) = \sigma^2 \]

\[ \sigma = \text{rms noise} \]

Example: 2-nd order high frequency cutoff
(2 RC integrations)

\[ n_{zc} = \frac{1}{2\pi (\tau_1 \tau_2)^{1/2}} \]

for \( \tau_1 \) or \( \tau_2 \) → 0 \( n_{zc} \rightarrow \infty \) !

for \( \tau_1 = \tau_2 = 5 \text{ nsec} \)

\[ n_{zc} = 30 \text{ MHz} \]

Level crossing rate:

<table>
<thead>
<tr>
<th>( \frac{U_d}{\sigma} )</th>
<th>( \frac{nLc}{nzc} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>( 1.4 \times 10^{-1} )</td>
</tr>
<tr>
<td>3</td>
<td>( 1.1 \times 10^{-2} )</td>
</tr>
<tr>
<td>4</td>
<td>( 3.3 \times 10^{-4} )</td>
</tr>
<tr>
<td>5</td>
<td>( 4 \times 10^{-6} )</td>
</tr>
</tbody>
</table>

1st order cutoff
(\( \tau_1 = 0 \) not sufficient. Higher order cutoff necessary.)
HIGH FREQUENCY LIMIT OF NOISE

1. **Thermal (Johnson) noise in resistors**

\[
\overline{V^2} = 4kT R \Delta f \cdot \frac{hf}{kT} \exp\left(\frac{hf}{kT}\right) - 1
\]

\[h = 6.63 \times 10^{-34} \text{ Joule} \cdot \text{sec}\]
\[k = 1.38 \times 10^{-23} \text{ Joules}\]

For \( \frac{hf}{kT} \ll 1 \) \[\overline{V^2} = 4kT R \Delta f\]

For \( \frac{hf}{kT} = 1 \) \[\overline{V^2} \approx \frac{kT}{h}\] at \( T = 300^\circ K\)

\[f_h \approx 6 \times 10^{12} \text{ Hz}\]

At \( T = 0.39^\circ K \) \[f_h \approx 6 \text{ GHz}\]

\( T = 3 \times 10^{-3}^\circ K \) \[f_h \approx 60 \text{ MHz}\]

2. **Shot noise** (detector leakage current, transistor collector and base current, FET gate leakage current, etc.)

\[\overline{i^2} = 2 \overline{n} \overline{g^2} \Delta f\]

\[\overline{n} = \text{average rate of impulses}\]
\[\overline{g^2} = \text{mean charge/impulse}\]

\[\overline{i^2} = 2eI_0 \Delta f\] for electrons, \( I_0 = \text{mean current}\)

High frequency limit is determined by the electron transit time:

- **Induced current:**

\[f_h \approx \frac{1}{\tau_e}\] this is usually lower than for resistors (thermal noise)
REFERENCES FOR MAXIMUM LIKELIHOOD METHODS AND
STATISTICAL TREATMENT OF SIGNALS AND NOISE

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5. P. Rehak, Detection and Signal Processing in High Energy
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