Cosmological selection of multi-TeV supersymmetry

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1. Introduction

The non-discovery of any supersymmetric partners of the standard model particles (sparticles) at the Large Hadron Collider (LHC) experiments so far has excluded sparticle masses in the range of a few hundred GeV [1]. Besides, the observed Higgs boson mass of a 125 GeV [2] suggests that sparticle masses most probably lie in the multi-TeV range [3] (see also [4] for a recent analysis), in case supersymmetry (SUSY) is really realized in nature. For such a high-scale SUSY scenario, serious questions, however, arise regarding the qualification of SUSY as a solution to the fine-tuning problem in the Higgs potential. Why would nature prefer low-scale SUSY, but end up with a SUSY scale that is not completely natural? What are the physical constraints preventing the SUSY-breaking scale from being lower and, hence, perfectly natural?

In this Letter, we show that these fundamental questions could be possibly answered if the scale of the last inflation is very high. As we are going to argue, the key element to a better understanding of a high SUSY scale is the domain wall problem related to the spontaneous breaking of a discrete R-symmetry in the early universe. Since the formation of domain walls after the end of inflation is disastrous for the habitability of the universe, any given inflation scale implies a lower bound on the scale of R-symmetry breaking [5]. Meanwhile, the SUSY-breaking scale (and hence the masses of sparticles) and the R-breaking scale are strictly tied to each other by virtue of the flatness condition of the universe. As a result, invoking cosmological selection for habitable universes, we find that the probable range of sparticle masses deduced from the idea of naturalness can indeed lie at the multi-TeV, if the Hubble rate is of $O(10^{14})$ GeV.

The organization of this paper is as follows. In Section 2, we discuss the constraint on the R-breaking scale resulting from the cosmological domain wall problem. In Section 3, we then discuss the probable range of sparticle masses according to the concept of naturalness. The final section is devoted to summary and conclusions.

2. Spontaneous breaking of discrete R-symmetry

For low-scale SUSY to be realized in nature, the SUSY-breaking scale, $|F|^\frac{1}{2}$, should not be too large. In addition, the vacuum expectation value (VEV) of the superpotential, $W_0$, is also required to be small, so as to achieve an almost vanishing cosmological constant, i.e.,

$$V_0 = |F|^2 - \frac{3}{M_{Pl}^2} |W_0|^2 \simeq 0,$$

(1)
where \( M_{Pl} \) denotes the reduced Planck scale. Thus, in order to realize low-scale SUSY, it is inevitable to invoke a symmetry—an \( R \)-symmetry—which prevents \( W_0 \) from being very large. A small, but non-vanishing VEV of the superpotential can then be provided by the spontaneous breaking of this \( R \)-symmetry.

Let us emphasize here that global symmetries are generically believed to be violated by quantum gravity (see e.g. [6] and references therein). Therefore, if we require that our \( R \)-symmetry is an exact symmetry, it must be a remnant of a gauge symmetry, which appears as a discrete symmetry in the low-energy effective theory. In the following argument, we will have a very strict attitude towards global symmetries, rejecting them as possible candidate symmetries suitable to protect the VEV of the superpotential. Instead, we will assume that the \( R \)-symmetry protecting the VEV of the superpotential is an exact discrete symmetry, \( Z_{HR} \).

A serious drawback when invoking an exact discrete symmetry and its spontaneous breaking is the domain wall problem [7,8]. If the Hubble rate during the last inflation, \( H \), is very high, spontaneous \( R \) breaking takes place after the end of inflation, which is accompanied by the formation of domain walls. Since we assume an exact discrete symmetry, these domain walls are stable and they immediately dominate the energy density of the universe once they are formed. Therefore, we need to require that \( R \)-symmetry breaking takes place before/during the last inflation, which leads to a constraint on the scale of \( R \)-symmetry breaking.

To see how the \( R \)-symmetry breaking scale is constrained, let us consider a pure SUSY Yang–Mills theory, which is, in fact, the simplest model exhibiting spontaneous \( R \)-symmetry breaking. In pure SUSY Yang–Mills theories, \( R \)-symmetry is spontaneously broken by dynamical gaugino condensation [9]. For example, in an \( SU(N_c) \) Yang–Mills theory, discrete \( Z_{2N_cR} \) symmetry is spontaneously broken down to \( Z_{2R} \) symmetry and the resulting \( W_0 \) is roughly given by

\[
W_0 \simeq \Lambda^3, \tag{2}
\]

with \( \Lambda \) denoting the dynamical scale and where we have omitted all numerical coefficients. In the early universe, spontaneous \( R \)-symmetry breaking takes place when the Hubble parameter \( H \) (or the temperature of the universe \( T \)) drops below the dynamical scale, \( H \lesssim \Lambda \) (or \( T \lesssim \Lambda \)). Therefore, to avoid the formation of domain walls after inflation, the dynamical scale is required to be higher than the Hubble rate during inflation, i.e., \( \Lambda \gtrsim H \) [5], which puts a lower limit on the scale of \( R \)-symmetry breaking.

In Fig. 1, we show the corresponding lower bound on the gravitino mass, where \( m_{3/2} \equiv W_0/M_{Pl}^2 \), for a given Hubble parameter during inflation,

\[
m_{3/2} > m_{3/2}^* (H) = \mathcal{O}(1-100) \text{TeV} \times \left( \frac{H}{10^{14} \text{GeV}} \right)^3. \tag{3}
\]

Here, the range \( \mathcal{O}(1-100) \) reflects our ignorance of the precise relation between \( W_0 \) and \( \Lambda \) as well as between the critical time during the \( R \)-symmetry breaking phase transition and \( \Lambda \). To obtain a more precise constraint, further quantitative understanding of the non-perturbative aspects of SUSY Yang–Mills theory is necessary.\footnote{If we rely, for example, on so-called naive dimensional analysis [10,11] the right-hand side of Eq. (2) is suppressed by a factor of \( \mathcal{O}(4\pi r^2) \). In that case, the corresponding lower limit gets weaker by the same factor if the condition is given by \( \Lambda \gtrsim H \).}

To give a representative example, we show the constraint by taking the relation \( W_0 \simeq \Lambda^3 \) literally, although we should keep in mind the above uncertainties. In the shaded region, the corresponding dynamical scale is large enough so that \( R \)-symmetry breaking takes place before the end of inflation. Therefore, there is no domain wall problem in the shaded region.

According to the above argument, we find that \( m_{3/2} \) below the TeV range is prohibited in consequence of the domain wall problem, unless the Hubble scale is much lower than \( \mathcal{O}(10^{14}) \) GeV. Interestingly enough, such a large Hubble rate corresponds to a rather large fraction of tensor modes in the primordial fluctuations seen in Cosmic Microwave Background (CMB),

\[
r \simeq 0.16 \times \left( \frac{H}{10^{14} \text{GeV}} \right)^2. \tag{4}
\]

Future experiments such as CMBPol [13] and LiteBIRD [14] are expected to reach values of the tensor-to-scalar ratio \( r \) of \( \mathcal{O}(10^{-3}) \). This, thus, opens up the possibility to put a stringent lower limit on the gravitino mass through CMB observations, assuming that the spontaneous breaking of \( R \)-symmetry is accounted for by gaugino condensation.

Before closing this section, let us remind ourselves that there are possibilities to evade the domain wall problem, even if we assume an exact discrete \( R \)-symmetry. For example, let us consider a model of \( Z_{2nR} \)-symmetry breaking where a gauge singlet \( \phi \) with \( R \)-charge 2 possesses a superpotential of the following form,

\[
W = v^n \phi + \frac{\lambda^n}{n+1} \phi^{n+1} + \cdots. \tag{5}
\]

Here, \( v^n \) and \( \lambda \) are parameters and we take the reduced Planck mass to be unity.\footnote{The parameter \( v^n \) should be small to explain the smallness of \( W_0 \). Such a small \( v^n \) could be achieved by assuming some additional dynamics behind \( v^n \).} The ellipsis denotes higher-dimensional terms in \( \phi \) which are consistent with the \( Z_{2nR} \) symmetry. In the vacuum, \( R \)-symmetry is broken by the VEV of \( \phi \), leading to the VEV of the superpotential,

\[
W_0 \simeq v^n + \lambda^n. \tag{6}
\]

In this example, it is always possible to avoid the domain wall production if the singlet obtains a large negative Hubble mass squared. Including a negative Hubble mass squared, \( R \)-symmetry is forced to be broken during inflation. In this case, there is no constraint on the gravitino mass from the requirement of no do-
main wall formation after inflation, since $R$-symmetry has already been broken during inflation.\(^3\)

3. Naturalness and sparticle masses

Let us now discuss how the above observation enables us to answer a fundamental question brought upon us by the results of the first run of the LHC: why would nature prefer low-scale SUSY, but end up with a SUSY scale that is not completely natural? For that purpose, let us first review the conventional argument on the "natural" range of the SUSY breaking scale and, hence, sparticle masses, $m_{\text{SUSY}}$. To discuss natural ranges of parameters, it is often transparent to consider an ensemble of vacua (or theories) with various SUSY-breaking scales. Here, we call the ensemble of vacua the landscape of vacua, adopting the terminology coined in Ref. [15] and having a string theory landscape in our mind [16] (see also [17] for an earlier discussion). One way to find the range of $m_{\text{SUSY}}$ preferred by the concept of electroweak naturalness is to consider the distribution of different values of the electroweak symmetry breaking scale, $v_{\text{EW}}$, for a given $m_{\text{SUSY}}$. That is, we should collect all vacua with the same value of $m_{\text{SUSY}}$ from the landscape and count how often we respectively encounter each value of $v_{\text{EW}}$. Then, for a given value of $m_{\text{SUSY}}$, the distribution of $v_{\text{EW}}$ is expected to peak around $v_{\text{EW}} \approx m_{\text{SUSY}}$, since electroweak symmetry breaking is triggered by the sparticle masses.\(^5\) Thus, given the observed electroweak scale, $v_{\text{EW}} \approx 174$ GeV, we infer that, from the standpoint of electroweak naturalness, the sparticle masses are most likely to lie in the range of a few hundred GeV.

An alternative way to deduce the range of $m_{\text{SUSY}}$ from electroweak naturalness is to consider the distribution of $m_{\text{SUSY}}$ for a fixed value of the electroweak scale, instead. To illustrate the idea behind this alternative, let us start from the initial distribution of $m_{\text{SUSY}}$ in the landscape, without imposing any cuts on the ensemble of vacua (a) in Fig. 2). Here, we assume that the prior distribution is not severely biased towards large values of $m_{\text{SUSY}}$, although we do not need to know the exact distribution. Since we are interested in vacua with an almost vanishing cosmological constant, we restrict the landscape in the next step to vacua corresponding to an (almost) flat universe. In this restricted landscape, we expect that the distribution is now sharply biased towards low energies, since a flat universe can be achieved more easily for lower values of the SUSY-breaking scale [18] (b) in Fig. 2). Finally, we restrict the landscape further, so that all vacua in the landscape have the same electroweak scale $v_{\text{EW}}$. Then, the distribution of sparticle masses is cut off around $m_{\text{SUSY}} \approx v_{\text{EW}}$, since electroweak symmetry breaking is triggered by the sparticle masses in the MSSM (c) in Fig. 2). As a result, we end up with a distribution of $m_{\text{SUSY}}$ which peaks around $v_{\text{EW}}$.\(^5\) This means once again that sparticle masses most probably lie in the range of a few hundred GeV in view of electroweak naturalness.

In both approaches, we end up with more or less the same conclusion: $m_{\text{SUSY}}$ should be at around a few hundred GeV. (In the following, we shall call the former approach the frequentist approach and the later approach the Bayesian approach.) Here, it should be emphasized that our deductions of the range of $m_{\text{SUSY}}$ are based on the principle of mediocrity [20], i.e., in both approaches we have assumed that we are typical observers living in a typical habitable vacuum. This is the reason why we obtained similar conclusions in these two different approaches. The big problem now is that the resultant ranges of $m_{\text{SUSY}}$ in both approaches are in tension with the null discovery of sparticles at the LHC as well as with the observed Higgs boson mass. This means that, unless we find a way to depart from the above argument and alter the ranges deduced above, we almost lose ground on the postulation of low-scale SUSY from the standpoint of electroweak naturalness.

The above conclusion should, however, change if there are crucial restrictions not accounted for in the above argument. In the following, we are going to argue that the domain wall problem related to $R$-symmetry breaking corresponds exactly to such a missing selection rule. Since we are trying to constrain the distribution function of $m_{\text{SUSY}}$ by applying the missing selection rule for habitability along with other habitable conditions, the cosmological constant and the electroweak breaking scale, it is more transparent to take the Bayesian approach. One caveat pertaining to the Bayesian approach is that the final distribution of $m_{\text{SUSY}}$ depends on the prior distribution of $m_{\text{SUSY}}$ in the most generic landscape [21–23]. As we have already mentioned, we assume that the prior distribution is not strongly biased towards high energies, although we do not need to make any particular assumptions regarding the prior distribution in the following argument.

We should also mention that there have been several attempts in the literature to solve the question of the most probable SUSY scale by imposing further restrictions on the landscape of vacua for habitability. For example, the habitability condition has been used to restrict the landscape of vacua based on the abundance of dark matter in [24–26], which leads to a sharp lower cut-off for the distribution of $m_{\text{SUSY}}$.\(^6\) In this paper, we shall refer to this type of restriction of the landscape based on the requirement of habitability as cosmological selection.

Now, let us discuss how the domain wall problem related to $R$-symmetry breaking provides us with a means of cosmological selection. In the above argument, we have eventually restricted the vacuum landscape so that all vacua in the landscape have the same electroweak scale $v_{\text{EW}}$. Before applying this restriction, let us now hypothesize that $R$-symmetry breaking is caused by gaugino condensation in our vacuum. Then, too small values of the gravitino mass lead to uninhabitable universes for a given Hubble parameter during inflation (see Fig. 1). Correspondingly, the distribution of $m_{\text{SUSY}}$ should be cut off for $m_{\text{SUSY}} < m_{3/2}^{*}(H)$ according to cosmological selection for habitable universes (see Eq. (3) and Fig. 3).\(^7\) Finally, after applying the constraint on the electroweak scale, we obtain the distribution of $m_{\text{SUSY}}$ in the landscape for a given $v_{\text{EW}}$. The crucial difference from the previous result is that the resultant distribution of $m_{\text{SUSY}}$ in the landscape does not necessarily peak at $v_{\text{EW}}$ anymore. Instead, it now peaks at $m_{3/2}^{*}(H)$ for $m_{3/2}^{*}(H) > v_{\text{EW}}$. In this case, the most probable sparticle masses can be much higher than $v_{\text{EW}}$, which gives us an answer to the question why nature would prefer

\(^3\) One might wonder whether the domain wall might be formed once $\phi$ starts moving around its origin after inflation. However, the formation does not take place for $n \geq 3$ [12].

\(^4\) Throughout this paper, we assume the Minimal Supersymmetric Standard Model (MSSM), so that the Higgs potential is given by known couplings and soft breaking parameters of $\mathcal{O}(m_{\text{SUSY}})$.\(^5\) In "no-scale" supergravity models, the cosmological constant vanishes after supersymmetry breaking [19] at tree level. In this class of models, the distribution of $m_{\text{SUSY}}$ does not get biased towards lower energies even after imposing the condition of an almost vanishing cosmological constant. However, even in this case, it is probable that the final distribution is biased towards lower energies by imposing the observed electroweak scale, as long as the prior distribution is not severely biased towards high energies. It should also be mentioned that the cancellation of the cosmological constant is expected to be no longer exact with higher order corrections being taken account, and hence, it seems more plausible that the distribution is biased towards lower energies, after all, after imposing an almost vanishing cosmological constant.

\(^6\) See also [27] for a related discussion.

\(^7\) Here, we have assumed that SUSY breaking is mediated to the MSSM sector via gravity mediation, i.e., $m_{\text{SUSY}} \approx m_{3/2}^{*}$. 
low-scale SUSY, but end up with a SUSY scale that is not completely natural.\(^8\)

Interestingly enough, \(m_{\text{SUSY}}\) in the multi-TeV range, which is also suggested by the observed Higgs boson mass, can be reconciled with the concept of naturalness if the Hubble scale during inflation is in the range of \(10^{14}\) GeV (see Fig. 1). As mentioned earlier, such a large Hubble parameter during inflation can be tested via future measurements of the tensor fraction in the CMB fluctuations. Therefore, the fundamental (and only seemingly metaphysical) question why nature would prefer low-scale SUSY, but end up with a SUSY scale that is not completely natural can be partially tested by future measurements of \(r\). Conversely, if the tensor fraction is observed to be very small, the constraint on the gravitino mass from the domain wall argument becomes weak, and we will fail to reconcile the non-observation of sparticles with the concept of naturalness in this way.

\(^8\) In models where \(m_{\text{SUSY}} \ll m_{1/2}\) is achieved, e.g. [28], the constraint \(m_{1/2} \gtrsim m_{1/2}^\ast(H)\) puts a lower cut-off on \(m_{\text{SUSY}}\) at a much lower scale. By taking into account such a possibility, the sharp lower cut-off on \(m_{\text{SUSY}}\) is dulled and the distribution of \(m_{\text{SUSY}}\) has a tail extending to values smaller than \(m_{1/2}^\ast(H)\). Although we do not know the prior distribution of vacua with \(m_{\text{SUSY}} \ll m_{1/2}\), we, however, do assume that the distribution of \(m_{\text{SUSY}}\) still peaks at \(m_{1/2}^\ast(H)\) in view of the typicalness of spectra with \(m_{\text{SUSY}} \simeq m_{1/2}\) in generic supergravity setups.

Another interesting observation is an implication for the infamous gravitino problem [29]. For a very high reheating temperature of the universe, such as \(T_R \gtrsim 10^9\) GeV, which is essential for successful thermal leptogenesis [30], the late-time decay of the gravitino spoils the success of Big Bang Nucleosynthesis (BBN), as long as \(m_{1/2} \lesssim O(1)\) TeV [31]. Since this problem is not very relevant for the habitability of the universe, it does not lead to a cut-off nor any constraint on the distribution of \(m_{\text{SUSY}}\) by itself — although the low-\(m_{1/2}\) region in parameter space ends up being disfavored in consequence of the observed abundances of the light elements. On the other hand, provided that \(m_{1/2}^\ast(H) = O(100)\) TeV, the gravitino problem is automatically solved since the gravitino in this mass range decays much earlier than the beginning of BBN.\(^9\)

Several comments are in order. In the above argument, we have made the hypothesis that \(R\)-symmetry breaking is caused by gaugino condensation in our vacuum, which allows us to deduce the

\(^9\) One might also wonder how the moduli problem affects the distribution of \(m_{\text{SUSY}}\). In case moduli fields should exist. In fact, the late-time decay of moduli fields could dilute the baryon asymmetry or lead to too large a dark matter abundance after their decay. Those problems are, however, controlled by the initial amplitudes of the moduli oscillations rather than by \(m_{\text{SUSY}}\). As long as the initial amplitudes are suppressed for some reason, such as an anthropic argument (see the discussion later on) or some kind of dynamical reason [32], the moduli problem does not affect the distribution of \(m_{\text{SUSY}}\).
distribution of $m_{\text{SUSY}}$ in consequence of cosmological selection. As we have mentioned in the previous section, there are, however, $R$-symmetry breaking models which do not exhibit a domain wall problem. Thus, in order to fully answer the question whether the distribution of $m_{\text{SUSY}}$ in a global landscape (for a given $v_{\text{EW}}$) really peaks at $m_{\text{SUSY}} \gg v_{\text{EW}}$, we need to know the distribution of $R$-breaking models, which goes far beyond the scope of this paper. The only thing we can say at this point is that we anyway need to live in a vacuum where $R$-symmetry breaking is caused by gaugino condensation to reconcile the idea of naturalness with $m_{\text{SUSY}} \gg v_{\text{EW}}$ by virtue of our domain wall argument.

The same caveat applies to the distribution of the Hubble scale during inflation. That is, we need to know the distribution of $H$ in a global landscape to conclude that the peak of the distribution of $m_{\text{SUSY}}$ is really above $v_{\text{EW}}$. We only note here that chaotic inflation [33] which is free from the initial condition problem [34], predicts a Hubble scale of $O(10^{14})$ GeV during inflation.\footnote{For chaotic inflation models consistent with the constraint from the Planck satellite [35], see e.g. Refs. [36] and references therein.} The absence of the initial condition problem might explain why a large Hubble scale is chosen from the global landscape. Fortunately, our ignorance of the distribution of $H$ can be compensated by future observations. That is, if our reasoning is correct, the tensor fraction of the CMB fluctuations will be measured to be rather large.

In the above argument, we have shown that $m_{\text{SUSY}} \gg v_{\text{EW}}$ can be the most probable value when the Hubble parameter during inflation is high. However, we have not tried to explain why the electroweak scale is much smaller than $m_{\text{SUSY}}$. To answer this question, we expect that there are some anthropic reasons for the strength of the weak interaction, as in the case of the cosmological constant [37,38]. In this paper, we, however, do not pursue these issues any further. Instead, we refer to Refs. [39–42] for more discussion on the anthropic selection for $v_{\text{EW}} = O(100)$ GeV.

4. Summary and discussion

In this paper, we have discussed whether we can answer the fundamental question why nature would prefer low-scale SUSY, but end up with a SUSY scale that is not completely natural. This question is inevitable, if we postulate that low-energy SUSY is indeed realized in nature despite the null observation of sparticles at the LHC experiment below a TeV. This question becomes even more severe in view of the observed Higgs boson mass of about 125 GeV, which seems to point to sparticle masses in the multi-TeV range. As we have discussed, such an multi-TeV SUSY can be reconciled with the concept of naturalness under the assumption that the spontaneous breaking of an exact discrete $R$-symmetry is achieved by gaugino condensation in a pure SUSY Yang–Mills theory. In such theories, the dynamical scale of the Yang–Mills gauge interactions is required to be higher than the Hubble scale during inflation, in order to avoid the formation of domain walls, which puts a sharp lower cut on the distribution of the gravitino mass. With this sharp cut, we find that the distribution of $m_{\text{SUSY}}$ peaks in the multi-TeV range, if the Hubble parameter during inflation is of $O(10^{14})$ GeV. Our argument can be partially tested by future measurements of the tensor fraction in the CMB fluctuations.

We should stress that what we have proposed in this Letter is nothing less than a conceptual transition in how one should think about and address the big question of why SUSY has not yet been seen at the LHC. Conventionally, many people have attempted to construct models where the electroweak scale of $O(100)$ GeV is naturally obtained, even when $m_{\text{SUSY}}$ lies in the multi-TeV. We, on the other hand, have taken a different approach in this Letter, where we deduce the probable range of $m_{\text{SUSY}}$ for a given $v_{\text{EW}}$. We take the puzzle of the absence of sparticles at $O(100)$ GeV as an important hint for the unknown structure of high energy physics. By adopting such a philosophy, we have, in fact, managed to infer the origin of $R$-symmetry breaking as well as the scale of the Hubble parameter during inflation in this paper.

In the bulk of this paper, we have not made any assumption as to the sparticle spectrum of the MSSM. Let us comment here that our argument complies particularly well with a certain class of high-scale SUSY-breaking models where the gaugino masses are dominantly generated via anomaly mediation [43] (see also [44–46] for a discussion of the anomaly mediation mechanism in the superspace formalism of supergravity). In these models, no SUSY-breaking singlet fields are required and, hence, these models are free from the so-called Polonjy problem [47–49]. Furthermore, the models in this class feature a good candidate for dark matter: the lightest gaugino (in particular the wino) in the TeV range or below. Therefore, this class of models has advantages in cosmology, which might enhance the probability of these models of being actually selected according to cosmological selection. Having gauginos in the TeV range (or below) is also important for the testability of the scenario.

Throughout this paper, we have assumed an exact discrete $R$-symmetry. Inevitably, this symmetry should be anomaly-free [50, 51]. Related to this issue, we let us consider the paradigm of pure gravity mediation [52] as an example. There, the $R$-charge of the MSSM Higgs bilinear $H_u H_d$ vanishes, and hence, a $\mu$-term of the order of the gravitino mass is naturally generated by the coupling of $H_u H_d$ to the VEV of the superpotential via Planck-suppressed operators [53,54] (see also [55]). In this case, the difference between the MSSM contributions to the $SU(3)$ and the $SU(2)$ anomalies of the discrete $R$-symmetry is 4 and, hence, the exact $R$-symmetry is found to be a $Z_{4R}$ symmetry.\footnote{Here, we assume that the discrete $R$-symmetry commutes with the $SU(5)$ grand unified group. See Refs. [57,58] for related discussions on discrete $R$-symmetries.} It should also be noted that an odd number of extra matter fields transforming in the $5$ and $5^*$ representations of $SU(5)$ and with vanishing $R$-charge are required to make $Z_{4R}$ symmetry anomaly-free [56]. The existence of those extra matter fields, therefore, provides us with an additional possibility to test our assumption of an exact $R$-symmetry in future collider experiments.

Finally, let us comment on the relation between $R$-symmetry and supersymmetric grand unified theories (GUTs). As shown in Ref. [59–62], it is generically difficult to have an unbroken $R$-symmetry below the GUT scale in a class of GUT models where the standard model gauge groups are embedded in a single $SU(5)$ group.\footnote{The GUT gauge group itself does not necessarily need to be a simple group [62].} Thus, the existence of $R$-symmetry below the GUT scale fits well together with a class of GUT models where the standard model gauge groups are differently embedded into the subgroups of the GUT gauge group [63].

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