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THE MASS SPECTRUM OF CHARGED COSMIC RAYS AT MOUNTAIN ALTITUDE

by

Herbert Bradford Barber

A Dissertation Submitted to the Faculty of the
DEPARTMENT OF PHYSICS
In Partial Fulfillment of the Requirements For the Degree of
DOCTOR OF PHILOSOPHY
In the Graduate College
THE UNIVERSITY OF ARIZONA

1976
I hereby recommend that this dissertation prepared under my direction by Herbert Bradford Barber entitled The Mass Spectrum of Charged Cosmic Rays at Mountain Altitudes be accepted as fulfilling the dissertation requirement of the degree of Doctor of Philosophy.

Dissertation Director

Date

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SIGNED: H. Bradford Barlow
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ABSTRACT

A cosmic ray mass spectrometer using a superconducting magnet, digitized wire spark chambers, and a scintillation counter time of flight system has been operated at an altitude of 2750 meters (747 gm/cm$^2$). The apparatus is most sensitive to charge particles with momentum to mass ratios between 0.2 and 2.0. Operation of the apparatus is analyzed. Results for the momentum spectra of protons, deuterons, $^4$He and muons are presented and upper limits are obtained for anti protons, charged k-mesons, $^3$H, $^3$He, hypothetical superdense nuclei, and hypothetical massive particles in the 5 to 10 GeV/c$^2$ mass range. Results are discussed and interpreted in terms of a cosmic ray atmospheric cascade calculation developed for that purpose.
CHAPTER 1

INTRODUCTION

Historically, cosmic rays were one of the first recognized and utilized sources of particulate radiation. The study of the secondary radiation resulting from the interaction of cosmic ray primaries in the atmosphere has resulted in the discovery of a large fraction of all the known elementary particles with lifetimes longer than $10^{-10}$ seconds. Although accelerators now produce beams of much higher intensity, they are far from producing the energies available in cosmic rays, where single particles with energies as high as $10^{21}$ eV may be present. Since higher energies are required for the production of more massive particles, particle searches continue to represent a significant fraction of cosmic ray experiments. These experiments have in general not benefited from the extremely sophisticated techniques and apparatus developed for high energy physics. Ideally, a cosmic ray experiment should resemble an accelerator experiment designed for low beam intensity and high contamination. Also, the most sensitive cosmic ray particle searches have looked for anomalously charged particles using charge sensitive detectors (Fleischer et al., 1971; Cox et al., 1972; Beauchamp et al., 1972; Ashton, 1973, p. 119). Yet all known elementary particles are neutral, singly charged or at most have a few times the unit charge.

A number of mass sensitive experiments at various altitudes (Ashton, Edwards, and Kelly, 1969; Galper et al., 1971; Yock, 1974) have
used range and energy loss techniques to set limits for massive singly charged particles in the region of $10^{-8} - 10^{-9}$ cm$^{-2}$ sec$^{-1}$ sr$^{-1}$. These experiments are characterized by low mass resolution and large amounts of material in the detection path, and hence are most sensitive for particles of large mean free path.

A search for massive particles using a magnetic mass spectrometer was done by Kasha and Stefanski (1968). This experiment set a limit on massive particles of $2.4 \times 10^{-8}$ cm$^{-2}$ sec$^{-1}$ sr$^{-1}$ at a zenith angle of 75° at sea level.

As a result of the experience obtained in some of the previously mentioned charge sensitive particle searches (Cox et al., 1972; Beauchamp et al., 1972) our Arizona High Energy Physics and Cosmic Ray Group decided to attempt to determine the mass spectrum of secondary cosmic rays at mountain altitude, using sophisticated accelerator techniques (Bowen et al., 1973). For this purpose we have designed and constructed a mass spectrometer utilizing a superconducting magnet and wire spark chamber and time of flight techniques, which is sensitive to individual charged particles with momentum to mass ratios between 0.2 and 3.5.

A number of new particles have been predicted which might be found in the secondary cosmic ray spectrum using such an apparatus. Han and Nambu (1965) have developed a quark model in which the quarks are singly charged. Such quarks, if they exist in the free state, might be too massive to be produced in accelerators of the present energy range, but may exist in the secondary cosmic ray spectrum.
Weinberg (1967, 1971) and Salam (1968) have recently had remarkable success with gauge models that renormalize the weak interaction. These models require the existence of either massive intermediate bosons, heavy leptons or both (Llewellyn-Smith, 1973; Weinberg, 1974). Such models have received and added attention due to the recent confirmation of neutral currents (Hassert et al., 1973; Benvenuti et al., 1974; B. Aubert et al., 1974; Barish et al., 1974). Some evidence exists for the presence of heavy leptons in cosmic rays (DeRüJula, Georgi, and Glashow, 1975) and at accelerator energies (Perl et al., 1975). However, intermediate bosons and other heavy leptons may be massive enough to be seen only in the secondary cosmic ray spectrum at the present time.

Recently a number of groups; Augustin et al. (1974), J. Aubert et al. (1974), Abrams et al. (1974), Braunschweig et al. (1975), and Feldman et al. (1975) have reported the discovery of new particles having masses three to four times the nucleon mass. This has brought forth a flood of theories and predictions of new particles yet to be found. At least two of these newly discovered particles are surprisingly long-lived and appear to have the quantum numbers of vector mesons (Boyarski et al., 1975; Lüth et al., 1975). The most popular explanation hypothesizes a new quantum number, charm, and a corresponding charmed quark, and with it a whole new spectrum of massive charmed meson and baryon states (Appelquist et al., 1975; Eichten et al., 1975; Gaillard, Lee, and Rosner, 1975). A number of candidates for such charmed particles and their effects have recently been put forward (Benvenuti et al., 1975a,b; Cazzoli et al., 1975; Jain and Girard, 1975; Barger, Weiler, and Phillips, 1975; Deden
et al., 1975; Perl et al., 1975; Blietschau et al., 1976; Mapp et al., 1976). Whether these results are evidence of charmed particle production or if they are even related to one another remains to be seen. But they quite possibly attest to a wealth of hitherto unsuspected high mass particles.

In order to assess the probability of success of a cosmic ray search for high mass particles using a mass spectrometer, it might be instructive to calculate an upper limit for the intensity of these particles in the cosmic ray secondary spectrum.

The differential cosmic ray momentum spectrum for protons at the top of the atmosphere is given by:

\[ I_0 = \frac{2.0}{p^{2.75}} \quad p > 10 \text{GeV/c} \quad (1.1) \]

where \( I_0 \) is in units of cm\(^{-2}\) sec\(^{-1}\) sr\(^{-1}\)(GeV/c\(^{-1}\)). This spectrum is adapted from Ryan, Ormes, and Balasubrahmanyan (1972). If the cross-section, \( \sigma_A(p) \) for production of a massive particle (M) by nucleons in collisions with air nuclei (atomic number A) is given, then the vertical intensity of this particle may be approximated by using the one dimensional diffusion equation for atmospheric depth \( X \) (gm/cm\(^2\)) as:

\[ I_M = \frac{N_0(2.0)}{A \left( \frac{1}{\lambda} - \frac{1}{\lambda_0} \right)} e^{-x/\lambda_0} \left[ 1 - e^{-\left( \frac{1}{\lambda} - \frac{1}{\lambda_0} \right)x} \right] \int_{p_t}^{\infty} \frac{\sigma_A(p)dp}{p^{2.75}} \quad (1.2) \]

where \( N_0 = 6.024 \times 10^{23} \), \( p_t \) is the laboratory momentum threshold for production of M, and \( \lambda_0 \) and \( \lambda \) are the attenuation mean free paths for nucleons and M particles, respectively, in the atmosphere (units of gm/cm\(^2\)).
If we neglect shielding of nucleons by other nucleons in the nucleus and nuclear transparency to product particle \( M \), both of which would decrease the cross-section, as well as coherent production effects which would tend to increase the cross-section we may approximate

\[
\sigma_A(P) = A \sigma_n(P) \tag{1.3}
\]

where \( \sigma_n(P) \) is the cross-section for production of \( M \) particles in nucleon-nucleon collisions. Gaisser and Halzen (1975) find that the production cross-section of two hadrons in nucleon-nucleon collisions is related by:

\[
\sigma_2(q) = \frac{2J_2+1}{2J_1+1} \frac{2I_2+1}{2I_1+1} F(B_1, B_2) \left( \frac{M_1}{M_2} \right)^2 \sigma_1 \left( \frac{M_1}{M_2} q \right) \tag{1.4}
\]

where \( q = E - E_t \); \( E \) is the center of mass energy; \( E_t \) is at threshold; \( J \) is the spin, \( I \) is the isospin; \( B \) is the baryon number; \( M \) is the mass in GeV/c^2 and \( F(B_1, B_2)[F(1,1) = 1, F(1,0) = 6 = 1/F(0,1)] \) is a factor from quark counting statistics of mesons and baryons. Relating the production of hadrons \( M \) to the production of \( P-P^\prime \) pairs in nucleon-nucleon interactions for which a production cross-section curve is given in Gaisser and Halzen (1975), we find

\[
I_M = \frac{(1.20 \times 10^{24})e^{-x/\lambda_o}}{(1/\lambda - 1/\lambda_o)} \left[ 1 - e^{-\left( \frac{1}{\lambda} - \frac{1}{\lambda_o} \right)x} \right] (2J_M + 1) 2I_M + 1) F(1, B_M) \left( \frac{M_p}{M_M} \right)^2 
\int_0^\infty \frac{\sigma_F \left( \frac{M_p}{M_M} q \right)}{(q + E_t)^{4.2}} \, dq \tag{1.5}
\]

where \( M_p \) and \( M_M \) are the masses of nucleons and \( M \) particles respectively.
and \( E_t = 2M_p + 2M_M \). We have approximated the center of mass energy by \( E = \sqrt{2M_pP} \). It turns out that for any reasonable parameterization of the \( \sigma_P \) production curve \( \frac{M_P}{M_M}q \) the integral above is not easily found analytically. However if \( M_M \) is specified a Simpson's rule computation can be done on a computer yielding the result shown in Fig. 1, where \( \lambda_0 = \text{gm/cm}^2 \) and for simplicity \( M \) particles have been assumed to have the quantum numbers of nucleons. From Eq. (1.5) the generalization to other quantum numbers is obvious. Only pair production of \( M \) particles is considered here \((N + N' \rightarrow N'' + M + M')\) but for processes analogous to associated production a good approximation would be obtained by replacing \( M_M \) by \( (M_M + M')/2 \) where \( M' \) is the mass of the associated particle.

Reasonable area-solid angle-time products for mass spectrometers might be of the order of \( 10^8 \) to \( 10^9 \text{ cm}^2 \text{ sec sr} \) at the present time. It should also be noted that Fig. 1 considers \( M \) production at all momenta whereas real experiments are limited in momentum acceptance. A short lifetime for \( M \) would also reduce the number available significantly since low velocity is required for time of flight determination resulting in negligible time dilation. Particle \((v=c)\) transit times in the atmosphere are of the order of microseconds. Keeping in mind these qualifications, the results in Fig. 1 are still somewhat encouraging except in the higher mass region. On the other hand, it is possible that new kinds of interactions exist with substantially higher cross-sections than those considered here, yielding even higher intensities. The cross-sections considered here are far higher and hence more promising for massive particle production than those obtained using.
Fig. 1. Estimated Integral Intensity of Massive Particles in Secondary Cosmic Rays at Mountain Altitude as a Function of the Particle Mass for Two Different Particle Attenuation Mean Free Paths.
statistical models of particle production and assumptions of exponential
dependence of the density of particle states with rising energy (Hagedorn,
1970). The very richness in phenomena and complexity of atmospheric
cosmic ray interactions as well as the unprecedented energies involved
may be fertile ground for discovery.

Lee and Wick (1974) have hypothesized the existence of metastable
superdense nuclei with binding energies of 150 MeV to 500 MeV per nucleon.
Such states, if produced by the interaction of cosmic ray nuclei with
air nuclei, or in astrophysical sources, might be relatively stable to
disruption in nuclear collisions and hence penetrate to the lower levels
of the atmosphere. Lee (1975) has pointed out that superdense nuclei
should be most stable for \( N = Z \) (neutron number equal to proton number);
indicating that for very massive superdense nuclei charge sensitive
experiments such as plastic track detectors or emulsions might provide
the most detection sensitivity. However, if low charge types or frag­
ments of superdense nuclei exist, mass sensitive experiments would
provide the best limits.

In addition to particle searches studies of the cosmic ray
secondary spectrum have import for a number of interesting problems.
Although extensive theoretical and experimental work has been done on
both primary and secondary cosmic rays over a period of more than 60
years (Hess, 1912, and Winckler and Hofmann, 1967), their origins and
acceleration mechanisms remain obscure.

Since attempts to find cosmic ray anisotropies (Turver, 1973,
p. 185-6; Porter, 1973), as well as flux changes with time in the solar
systems geologic record (Lai, 1973) have in general not been successful, primary composition and spectra remain the main source of data about these problems. The interaction of elementary particles in the region accessible to accelerators is for the most part well understood. However the complexities of multiple interactions with decays makes understanding of the secondary cosmic ray component more difficult.

Of particular astrophysical interest is the possible survival down to mountain altitudes of part of a primary antiproton component, although this spectrum would be contaminated by antiproton production in the atmosphere. The shape and intensity relative to protons of the primary antiproton spectrum may indicate the presence of antimatter on an astrophysical scale as well as the acceleration and interaction history of primary cosmic ray nucleons. Some limits on the primary antiproton component may possibly be set with an experiment of the type considered here. (See Chapter 5.)

Due to their low intensity, high energy cosmic rays are usually detected by the showers of lower energy particles they produce in the atmosphere. Thus, understanding the lower energy cosmic ray secondary spectra is crucial to understanding that at high energies. This is particularly true of the light nuclear fragment flux which is accessible to this type of mass spectrometer and which is not as well known as other components.

As with any natural phenomena the low energy cosmic ray secondary spectrum is interesting in its own right. It is also worth noting that since the secondary cosmic rays make up the dominant part of
terrestrial background radiation surveys of all components are important and may have impact in fields far removed from those discussed here.

Scope of This Work

The work reported in the chapters to follow represent the initial phase of a long-term effort by University of Arizona experimenters to measure intensities of rare cosmic ray components, such as antiprotons, and to search for hypothetical long-lived massive charged particles. Although the data accumulation time of this initial experiment was relatively short (≈1 week), it was sufficient both to demonstrate and thoroughly understand the technique and to measure mountain-altitude spectra for: muons, protons, and deuterons as well as an intensity for \(^4\text{He}\) and intensity upper limits for \(^3\text{He}, \, ^3\text{H}\) and \(^\pi^\pm\). Comparison of these results to predictions (see Chapter 5 and Appendix III) support the conclusion that the light nuclei component at mountain altitudes is now well understood; and a correction in interpretation of the sea level results of Ashton et al. (1969) is proposed. Preliminary intensity upper limits in the range \(10^{-6}\) to \(10^{-7}\text{cm}^{-2}\text{sec}^{-1}\text{sr}^{-1}(\text{GeV/c})^{-1}\) are set for all types of massive particles including charmed hadrons and heavy leptons in the mass range 3 to 10 \(\text{GeV/c}^2\). No fundamental limitations precluding a much more sensitive mass search with this apparatus were found, but further runs did not immediately follow this phase due to funding limitations. Preliminary intensity upper limits are obtained for hypothetical superdense nuclei \(< 10^{-6}\text{cm}^{-2}\text{sec}^{-1}\text{sr}^{-1}\) and secondary antiprotons \(< 10^{-5}\text{cm}^{-2}\text{sec}^{-1}\text{sr}^{-1}\ (\text{GeV/c})^{-1}\).
CHAPTER 2

SITE

The University of Arizona Cosmic Ray Laboratory which was constructed during the summer of 1968 is near the summit of Mt. Lemmon in the Santa Catalina Mountains, about 72 km by road north of The University of Arizona. This site has year-round accessibility. The laboratory building has an unobstructed ceiling height of 10 m, a floor space of 340 m$^2$, a separately air conditioned room for electronics, and a second floor level at the superconducting magnet height. The building also has some of the living amenities such as a kitchen, toilet and shower. Sleeping and other kitchen facilities exist in a second building at the Mt. Lemmon Infrared Observatory about 1 km away. The laboratory has 208/120 V three phase power, a 10 kw propane powered generator for emergency electric power and a 30 kVA A.C. motor generator set for isolation during the frequent electrical storms. Vehicle access to the main floor of the laboratory together with a motor driven winch allowed easy loading and unloading of the heavy equipment associated with this experiment. The Arizona cosmic ray experiment mass spectrometer shares the building with the previously operated Arizona cosmic ray quark search experiment.

This site is at an altitude of 2750 m which corresponds to an atmosphere depth of 747 gm/cm$^2$ and has a geomagnetic vertical cutoff rigidity of 5.59 GV. (Shea, Smart and McCall, 1968).
CHAPTER 3
APPARATUS

The University of Arizona cosmic ray mass spectrometer, shown schematically in Fig. 2, and pictorially in Fig. 3, determines the mass of an entering charged cosmic ray secondary by first determining its momentum by its deflection, $\theta$, in the magnetic field of a superconducting magnet. Track delineation is by wire spark chambers (SC1 to SC6). The velocity ($\beta = v/c$) is determined by measuring the time of flight between a set of scintillation counters (S1, S2, S3), and the charge state ($Q =$ number of unit charges) of the particle is found by measuring the energy deposited in S1. The momentum is given by:

$$P = \beta \gamma m_o c \quad \text{where} \quad \gamma = \left(1 - \beta^2\right)^{-\frac{1}{2}} \quad (3.1)$$

and experimentally by;

$$P = Q \int_{0}^{Bd\ell} \frac{d\ell}{\theta} \quad \text{GeV/c} \quad (3.2)$$

where $\int Bd\ell$ ($\approx 0.07$ for $P$ in GeV/c and $\theta$ in radians) can be determined for each event from a knowledge of the field of the magnet as a function of position and the particle trajectory. The above information suffices to determine the particles rest mass $M_o = P/\beta \gamma$ in GeV/c$^2$ when the momentum is in GeV/c. Various parts of this apparatus will be considered separately.
Fig. 2. Diagram of Apparatus.

Chambers 1-6 are spark chambers, S1-6 are scintillation counters. The y-axis aligned approximately North-South.
Fig. 3. Picture of Apparatus.

(1) Range Counter S4
(2) Time of Flight Counter S-3
(3) Spark Chambers
(4) Dewar of Superconducting Magnet
Superconducting Magnet

The magnet shown in Fig. 4 is composed of two coils of rectangular cross-section (26.67 cm I.D., 49.53 cm O.D., 82.6 cm thick) which are aligned axially and have a free space between them of 19.05 cm. This geometry which is approximately that of Helmholtz coils was chosen to make the field as uniform as possible in the central region. To reduce heat transfer and hence coolant boil off all non-coolant carrying areas inside the dewar were evacuated. The superconducting material is niobium titanium in a copper matrix with a 3:1 copper to superconductor ratio. The structural components of the magnet together with its associated dewars and inner heat shield were constructed of non-ferromagnetic materials such as stainless steel, copper, and aluminum to prevent distortion of the field. The smoke stack shaped section in Fig. 4 is a dewar containing a liquid helium reservoir sufficient for approximately 1 day of magnet operation. This dewar contains a superconducting switch which allows the magnet to be charged to a given current then switched into a persistent current mode in which it is isolated from any external power supply. In the persistent mode non-superconducting parts of the current path, such as solder contacts and copper current leads to the super switch, cause the magnet's current to decay with a time constant of 52.4 days. Our magnet was designed for a current of 200 amps and has been run successfully at 150 amps in the persistent mode. The data to be analyzed here was taken at a current of 100 amps at which the on axial field was 7.7 Gauss and the field integral was approximately 2 kGauss-meters.
Fig. 4. Super Conducting Magnet and Rack of Associated Electronics During a Cool Down at the Physics Department Helium Liquifier Facility.
Since the mass resolution depends strongly on the multiple scattering in the magnet region (see Appendix II) care was taken to allow as little material as possible in the particle path. Entry and exit windows to the vacuum region were of 0.25 mm mylar, the heat shield had windows of 0.025 mm aluminum, and there were 18 layers of 0.00825 mm aluminized mylar superinsulation throughout the vacuum region. The total amount of matter a particle encountered traversing the magnet amounted to approximately 0.2 gm/cm².

During the operation of the magnet, boil off was 0.97 ℓ/hr for liquid helium and about 2 ℓ/hr for liquid nitrogen. Since liquid helium cost was a major factor in the experimental operating expense, helium was reclaimed by a gas recovery system and recycled into new liquid helium at the Dept. of Physics liquifier facility.

A 200 amp D.C. power supply and a separate controller allowed slow (v.1 amp./sec) charging of the magnet. Control circuitry also included a carbon thermometer to measure temperature in the liquid helium region of the magnet and a liquid helium level monitor for the reservoir dewar. A protective resistor was also connected to the magnet to absorb the energy in case of an accidental quench. This system performed satisfactorily in at least one such accidental quench.

The magnet was cooled down at the Physics shop liquifier facility. First, liquid nitrogen was introduced into the magnet's liquid helium region and a vacuum pump was connected to the input pipe. In this way the boiling of the liquid nitrogen cooled the interior of the magnet to 52-57°K. The magnet was then connected directly to the helium liquifier
and cold helium gas was blown through it until the inside was cold enough to allow liquid helium to remain. After the magnet and its reservoir were filled with liquid helium it was transported while cold to the mountain site. Since some of the structural supports inside the magnet were very thin in order to lower heat transfer, the magnet had to be treated carefully during transportation and a specially designed frame and shock insulation system was built in the back of a pickup truck for this purpose.

The magnet was energized at the mountain site. This magnet has accumulated 786 hours of cooled down operation at the mountain site, 159 hours of this with the field on continuously.

Wire Spark Chambers

Track delineation was accomplished by six wire spark chambers with magnetostrictive readout. The orientation of these chambers with respect to the magnet is shown in Fig. 2, page 13. Spark chamber dimensions were: 1.5m x 1.5m for chambers 1 and 6; 1m x 1m for chambers 2 and 5; 0.27m x 0.43m for chambers 3 and 4 with the larger dimension perpendicular to the magnetic field direction. Each wire spark chamber was composed of two planes of wire mesh separated by 1cm. The mesh was aluminum wire crossed with nylon at 10 wires/cm for the two large chambers and copper crossed with nylon at 20 wires/cm for the other chambers. The wire planes were separated from the atmosphere by 0.0508mm mylar windows. A gas composed of 20% He and 80% Ne was constantly circulated through each chamber by a gas delivery and recirculation system. The gas recirculator used a cryogenic molecular sieve (McLaughlin and
Shafer, 1969) to clean the gas of impurities and allowed introduction of small amounts of ethyl alcohol for quenching purposes. Operation of the spark chambers was extremely sensitive to the gas purity.

The chamber pulsing system which was triggered by the event logic was composed of a Science Accessories Corporation Model 02A spark gap connected to a ferrite core transformer pulse inverter and fan-out which in turn triggered a separate 5C22 hydrogen thyratron pulser mounted on each spark chamber. The system is shown schematically in Fig. 5. Optimum chamber voltages were determined to be 3 to 5 kV for the four smaller chambers and 5 to 7 kV for chambers 1 and 6. The high voltage and ground planes of each chamber were connected by a terminating resistor whose value was determined experimentally. Two sets of two adjacent wires on opposite ends of each wire plane were used as fiducial wires and connected to a separate high voltage pulsing system.

Chamber readout was initiated by the arrival of a magnetostrictive pulse, corresponding to the first fiducial, at a pickup in a preamplifier at the end of the magnetostrictive wire. The magnetostrictive wire and preamplifier were part of a separate wand, placed perpendicularly to the spark chamber wires in each plane and electrically insulated from them. A tripolar pulse from the preamplifier was first converted to a bipolar pulse and then was used to start two scalers driven by a 20 MHz clock. The next two successive pulses in this system turned off each scaler in turn. From the scaler's readings and the sound velocity in the wire the positions of up to two sparks per plane could be determined according to the prescription in Table 1.
\[ \beta = 14.1, \quad 0.2 < \beta < 0.9 \]

To 5 more identical thyatrons driving remaining spark chambers.

Fig. 5. Schematic of the Electronics.
Table 1. Scaler Outputs for Various Spark Chamber Conditions.

<table>
<thead>
<tr>
<th>Condition</th>
<th>Scaler 1</th>
<th>Scaler 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>No spark</td>
<td>Fiducial 2 distance</td>
<td>Overflow</td>
</tr>
<tr>
<td>One spark</td>
<td>Spark Distance</td>
<td>Fiducial 2 Distance</td>
</tr>
<tr>
<td>Two or more sparks</td>
<td>Spark 1 Distance</td>
<td>Spark 2 Distance</td>
</tr>
</tbody>
</table>

With two planes for each of six spark chambers and two scalers for each plane there were a total of 24 scaler outputs written on the tape for each event.

During times when data were not being written on the tape data from the previous event were continually displayed on an oscilloscope in digital form (see Electronics section). The 24 scaler numbers could then be monitored during the operation of the experiment. Chamber breakdown was recognized by the continual reappearance of a given scaler number for a given chamber plane. The continual recurrence of the no spark condition (see Table 1) indicated that the magnetostrictive pulse was too small to trigger the preamplifier on the end of the wand. The size of the magnetostrictive pulse depended strongly on the magnetization of the wire. Magnetization of this wire was accomplished by running a given pole face of a small horseshoe magnet down the length of the wire in a given direction a few times, a process called stroking. Since the magnetization often decayed rather quickly, the process had to be repeated for each wand every few hours.
Chamber alignment was accomplished by using plumb lines in the following way: first, points exactly midway between the fiducials for each plane were found and at these four points for each chamber were placed aluminum blocks with channels cut in them. Two fine steel wires then hung over these blocks bisecting the chambers exactly. The wires were kept vertical by weights attached to the ends which hung in dash pots of mineral oil. Threaded bolts attached to the frames on which the spark chambers rested were used to move and fix the chambers while successive sightings of the hanging wires were taken with a surveyor's transit. The condition for chamber alignment was that each chamber was level as determined by a carpenter's level and that all wires were parallel and lay in a straight line bisecting the apparatus. In fact, this condition could only be approximated by successive observations in the two orthogonal directions and in addition, chambers 2 and 3 and chambers 3 and 4 had to be aligned separately to the center line of chambers 1 and 6, since there was little clearance and chamber 1 and 6 were fixed to the frame. This method sufficed to position the chambers to within a few millimeters of alignment and the residual errors were later determined by tracing particle trajectories using a computer program.

Although later analysis of the data showed that at least four of the chambers had efficiencies in excess of 77%, observation of the chambers during operation disclosed extensive electrical breakdown. One particularly common type of breakdown was arcing over between wires at high voltage and those in the same plane which were not connected to
the bus bar. Fixing this required painting the wires of both types in some region with a low resistance paste. Sometimes the high voltage would arc over to either the grounded frame or the wand carrying the magnetostrictive wire. This was fixed with mylar tape or epoxy. One of the most common forms of breakdown was between the main spark chamber high voltage wires and the fiducial wires in an area where the mylar window had separated from the frame. The fiducials and chambers were driven from different high voltage pulsing systems. This breakdown was repaired by reattaching the mylar window by squirting epoxy into the breakdown region with a hypodermic syringe. Since this breakdown behavior usually either produced a spurious pulse or prevented the wanted spark from occurring, operation of this experiment required almost continuous maintenance. However, by far the worst problem in this regard was warping of the chamber. This was probably due to structural strain from transportation but may have also been the result of the extreme conditions of temperature and humidity at the mountain site. In the case of chamber 2, this warping was severe enough to necessitate its replacement by a new chamber of our own construction. In the case of chambers 5 and 6, certain areas had to be deadened by insulating wires from the high voltage bus bar. Comparisons of the data indicated that chamber 1 operated intermittently, chambers 2 through 5 quite well, and chamber 6 hardly at all.

Fortunately the two-spark capability of the readout system allowed data to be taken in the presence of breakdown as long as that breakdown was not catastrophic. Also, six spark chambers overdetermined
the trajectory since the minimum required was four sparks in the x view, two above and two below the magnet. We have also required three colinear sparks in the y view.

As a result of this redundancy the combined efficiency of all the chambers acting as an array with events of the above criteria was 54.6%.

**Scintillation Counters and Time of Flight System**

The time of flight system is composed of three scintillation counters S1, S2, and S3, shown in the apparatus diagram, Fig. 2, page 13. Separate timings are made between S1 and S3 (TOF1) and between S1 and S2 (TOF2).

Counters S1 and S3 are large liquid scintillation counters of octagonal cross-section. The container, shown in Fig. 3, page 14, is constructed of aluminum 152 cm across the faces of the octagon with a vertical section containing the liquid 13 cm high. Above the liquid the container has the shape of an octagonal prism decreasing in diameter from 152 cm to 23 cm over a vertical height of 60 cm. Inside, the bottom of the container is polished to enhance reflection, whereas the prismatic region is roughened to diffuse the light in order to increase light detection efficiency.

The liquid scintillator which is about 13 cm deep in each counter is Nuclear Enterprises NE224, which is 1, 2, 4 trimethyl benzene in a toluene base. NE224 has a light output which is 80% of anthracene and a decay constant of 2.7 nanoseconds. Due to the fact that the scintillator is both volatile and poisoned by oxygen, the counter had to be
made gas tight and dry nitrogen was circulated through the region above the scintillator continuously. Whenever the scintillator came in contact with the air it was cleaned by bubbling dry nitrogen through it for a period of 12 to 24 hours in a closed container.

Since toluene is such a powerful organic solvent, the counter had to be constructed out of inert materials such as metals, teflon, and polyethylene.

The photomultiplier tube was a 5 in. diameter RCA 4522 with a bialkalai photocathode, a rise time of 2.7 ns, and an average transit time of approximately 100 nanosec with low jitter (Low and Leskovar, 1971). This phototube was located in the cylindrical region at the very top of the counter with its face 73 cm from the bottom of the scintillator. This geometry was chosen to minimize the differences in light transit time from the scintillator to the phototube. Light detection efficiency was also a factor in the design of these counters since pulse height information from them was used to determine the particle charge.

Counter S2 is a 20 cm x 33 cm plastic scintillator 0.159 cm thick and is located immediately above the magnet entrance window. S2 was chosen to be as thin as possible to reduce the multiple scattering. In order to increase light detection efficiency, S2 has two photomultiplier tubes which are placed at the ends of adiabatic light guides 1 m long to remove them from the high magnetic field region. Both S2 phototubes are 2 inch diameter RCA 8575's with bialkalai photo cathodes and rise times of 2.7 ns, and low jitter.
The shortest time of flight for TOF1 for a particle traveling at the speed of light \((v = c \text{ or } \beta = \frac{v}{c} = 1)\) is 24.0 ns. In practice this time fluctuates due to light transit time in the counters, different trajectory lengths, jitter in the photomultiplier tubes and various other factors such as scintillator fluorescence statistics and the characteristics of the electronics. The system was designed so that the standard deviation of these fluctuations did not exceed 2.5 ns. This then allows TOF1 to be used as a trigger with a cutoff of \(\beta = 0.9\). It should be noted that since the particle trajectory is eventually determined during data analysis the above errors due to light transit time in the counters and trajectory length may be calculated and corrected, allowing the velocity to be determined to much higher precision. The availability of two times of flight per event provided an easy way to distinguish many background events.

**Auxiliary Equipment**

Subsidiary information about each event was available from a range counter system beneath S3, and a shower counter.

The range counter system was composed of three 1.8m x 1.8m liquid scintillation counters, S4, S5 and S6, alternating with layers of iron absorber as shown in Fig. 2, page 13. The scintillation counters, which were described in an earlier experiment (Krider, 1969), were originally designed to detect e/3 charged quarks whose minimum ionization rate would be 1/9 that of singly charged particles. To avoid saturation with the much larger ionization rates expected in this experiment
the photomultiplier tubes in these counters were run at a lower voltage than in the previous experiment.

The top layer of iron was 13cm thick and the other two layers were each 10cm. This thickness of iron was chosen as it is approximately one nuclear collision length, i.e., one interaction mean free path for hadrons. A large fraction of incoming hadrons of momentum 1 to 2 GeV/c would be expected to trigger counter S4, but few would be expected to trigger S5 or S6. A muon of this momentum should penetrate the entire array triggering all of the counters. This serves to act as a check on the operation of the experiment as well as providing subsidiary information on any new particles which might be found. A new hadron would be expected to behave like a proton being absorbed in the first two layers or if it were massive enough, produce a secondary shower. Heavy leptons would probably behave like muons. For other types of particles the output of the range counters would depend upon such factors as lifetime, interaction type and cross section, and charge state.

There are two 1.8m x 1.8m liquid scintillation counters (S7 and S8) of the same type as the range counters which are located approximately 10 meters from the mass spectrometer and which act as shower counters to determine if the event is accompanied by a cosmic ray shower. The density of particles in the shower front and hence the output of the shower counter is an indication of the energy of the original primary cosmic ray and hence the energy available for particle production. Unfortunately, although this system was in operation during part of this data run, it was not in operation for the data that were chosen for reduction, so we will not consider the shower counter further.
Electronics

A diagram of the electronics is shown in Fig. 5, page 20. Pulse lengths were set by clipping lines or the previous logic unit.

The input pulses to coincidence units L1 and L2 are shown in Fig. 6a for the case where \( v = c \) (\( \beta = 1 \)), and in Fig. 6b for the case where \( v = c/3 \) (\( \beta = 1/3 \)). In Fig. 6a, the presence at L2 of an anti-coincidence input pulse which is the output of L1 through L3, suppresses the output of L2 for all events with \( v \) close to \( c \). How close \( V \) is to \( c \) is determined by the widths of pulses S1 and S2 into L1. These widths were chosen so that all events with \( v \geq 0.9c \) (\( \beta \geq 0.9 \)) were rejected as described in the time of flight section. This condition is referred to as the \( v < c \) trigger. Other triggers are available. If the L1 input cable to L3 is removed, the \( v = c \) anti is not available to L2 so that all events are allowed, which is called the \( v \leq c \) trigger. To obtain only the \( v = c \) events, trigger 3, the above mentioned L1 input cable to L3 is placed on the fourth coincidence input to L2 so that L2's input resembles Fig. 6c. For TOF1 the maximum time of flight (smallest \( \beta \)) is determined in the case of triggers 1 and 2 by the sum of the length of pulses S1 and S2 entering L2 (Fig. 6b). This length was chosen as 120 ns somewhat arbitrarily to allow triggering for the slowest protons not bent out of the solid angle acceptance of the spectrometer; those that would trigger S3. The event trigger (output of L2) was placed in coincidence with S1's discriminator pulse (L4) and the resultant pulse was used as the gate for quad integrator 3. Properly delayed discriminator pulses from counters S2 and S3 were then introduced into quad-integrator 3,
a.) \( V = C \) EVENT FOR \( V < C \) TRIGGER (REJECTED AT L2)

b.) \( V = C/3 \) FOR \( V < C \) TRIGGER (ACCEPTED)

c.) \( V = C \) EVENT FOR \( V = C \) TRIGGER (ACCEPTED)

Fig. 6. Pulse Timing for Various Trigger Conditions.
where the overlap of these pulses with the gate pulse was a measure of the time delay between the counters, i.e., the time of flight. TOF1 was the time delay between S1 and S3 and TOF2 was that between S1 and S2. The two other input channels of quad-integrator 3 were used for raw phototube pulses from counters S1 and S3, which, when integrated, measured the energy deposited in the scintillator and hence indicated the charge of the particle traversing the counter. Two other quad integrators, 1 and 2, were available to provide pulse height information about the range counters S4, S5, and S6, as well as a shower counter S7 or S8. Two other scalers allowed the addition of running totals such as number of events and number of triggers of a given counter up to that point. Other modules allowed preset information such as run number or trigger number (\(\nu < c\), \(\nu = c\), or \(\nu \geq c\)) to be written on the tape for each event.

As was noted in the section on the spark chambers system, the event trigger also fired a pulser which fires the spark chambers and initiates their readout cycle. The time delay from the event trigger to the firing of a given spark chamber was from 250 to 400 ns depending on the chamber. In order to prevent spark rf noise or a subsequent event from changing the data before they could be read out, the event logic (L2) was gated off for 1.5\(\mu\)s until the Lecroy controller took over the readout cycle. The controller serially interrogated the spark chamber scalers, the data scalers, and the quad-integrators, after digital conversion of their data by an ADC, and then sent these data over a data bus to the magnetic tape interface which controlled its readout onto magnetic tape by the Kennedy incremental recorder. The entire data readout cycle,
including the time to write on the tape, took approximately 0.96 sec. During this time, the controller gated off the event logic (L2), to prevent accumulation of new data. The incremental recorder writes 57, 24 bit words for each event, with 54 words containing data. From the Lecroy controller the data also went through a binary to decimal converter to a character generator and was then written in decimal form on the CRT of an oscilloscope. The controller was set to cycle continuously so that except for the time that the data were written on tape, data from the previous event were continuously displayed on the monitor. The operator of the experiment could then see what data were written on the tape as well as keep track of the operation of the counters, time of flight system, and spark chambers.

The step shape of the pulse which is the S2 input to coincidence unit L1 in Fig. 6a and b has no effect on its operation and arises from the addition of the discriminator outputs of the two S2 photomultiplier tubes in the case in which one photomultiplier fires earlier than the other because the particle transited the plastic scintillator nearer to that tube. Assuming that the two discriminator outputs are equal this procedure effectively averages the time determinations of the two photomultiplier tubes since it is the overlap of this step shaped pulse with the gate pulse in quad integrator 3 which is the time of flight (T).
CHAPTER 4

DATA REDUCTION

With the exception of a few calibration runs, all the data considered here were taken during January 1974, and consisted of 24 magnetic tapes with between 18000 and 30000 events on each tape.

Preliminary Analysis

In order to facilitate utilization of the data by The University of Arizona Computer Center's C.D.C. 6400, a program called REPACK was written which read data off of a raw data tape, rejected unusable events and wrote the good events on a second tape in a denser format. REPACK rejected events with short records, unreadable records, or events with both time of flight numbers equal to zero, or events with no spark in either chamber 4 or 5. The latter condition was chosen to simplify data reduction, because chamber 6 had a very low efficiency and two chambers were needed on each side of the magnet in the x projection. This procedure resulted in the loss of less than 4% of the good data.

During the time data was being recorded on the first 10 tapes, there was an intermittent mechanical malfunction of the tape recorder which caused the interrecord gap, the gap between successive events, to vary in length. Since it was found that these tapes were more difficult and hence more costly to read, these 10 tapes were not analyzed, but held in reserve. Late in the run there were two accidental magnet quenches,
when the superconductor went normal and the magnetic field was lost. Since it was not clear how the field decayed during this period, 3 tapes, at or near magnet quenches, were not reduced. Excluding calibration data, this left 11 tapes for data reduction, totaling 239,310 triggers. The disposition of these, with respect to trigger type and whether the magnet was on or off, is shown in Table 2.

Table 2. Data Summary.

<table>
<thead>
<tr>
<th>Trigger</th>
<th>Zero field (I = 0)</th>
<th>Field on (I = 100 Amps)</th>
</tr>
</thead>
<tbody>
<tr>
<td>v = c</td>
<td>17,239 Triggers</td>
<td>16,404 Triggers</td>
</tr>
<tr>
<td>v &lt; c</td>
<td>32,181 Triggers</td>
<td>173,612 Triggers</td>
</tr>
</tbody>
</table>

In the last column are the number of events of each type surviving the application of REPACK. The v < c, I = 0 triggers were not analyzed. A more detailed tabulation of the effect of REPACK on just the v < c, I = 100 amp data is shown in Table 3.

A C.E.R.N. version of SUMX (Zoll, 1966), a data analysis and histogramming program developed for bubble chamber experiments, was used to observe chamber and fiducial performance for a number of data tapes. A sample histogram is shown in Fig. 7, for chamber 3, for both x and y views for some v=c data. The first fiducial is at zero at the extreme left of each histogram, and the second fiducial is noted as a peak at the extreme right. Scalar counts correspond to the time between pulses on the magnetostrictive wire with the fiducial separations being 47.0cm for the x view, and 31.8cm for the y view. The edges of the active regions are
Table 3. Summary of $\nu c$ Data Taken with 100 Amperes Magnet Current.

<table>
<thead>
<tr>
<th>Program</th>
<th>REPACK</th>
<th>GUTS</th>
<th>BEND</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Total Triggers Analyzed</td>
<td>Short Records</td>
<td>Unreadable Records</td>
</tr>
<tr>
<td>Event Totals</td>
<td>173,612</td>
<td>1,116</td>
<td>8,472</td>
</tr>
<tr>
<td>Rejection Rate</td>
<td>7%</td>
<td>5%</td>
<td>9%</td>
</tr>
<tr>
<td>Overall Acceptance Rate</td>
<td>77%</td>
<td>18%</td>
<td>13%</td>
</tr>
</tbody>
</table>
Fig. 7. Spark Histograms for Chamber Number 3 for Some \( v = c \) Data.
denoted by the symbols E. The three most common types of breakdown are evident in the histograms as the sharp peaks in the x view, the broader peak near the left hand edge in the y view, and the part of the distribution occupying the non-active region between the edges and fiducials in each view. These histograms are characteristic of those obtained for all the chambers for both $V=c$ and $V<c$ data. The smooth background represents spark positions for real events plus perhaps some spurious sparking.

When a chamber's behavior was monitored as a function of time, both persistent and temporary breakdown peaks were evident, but the smooth background was always present. The position of the second fiducial, Fig. 7, page 35, could also be monitored as a function of time. This indicated time variation of the sound velocity in the magnetostrictive wire, since the distance between the fiducials was fixed. Such monitoring revealed small smoothly time dependent variations, probably due to temperature effects. Infrequent small discontinuous changes that seemed to correlate with wand stroking were also noted. These variations in sound velocity were probably due to small changes in wire tension. The total variation in sound velocity for a given tape usually corresponded to a shift in the position of the second fiducial of the order of a quarter of a wire spacing. The sound velocity was found for each chamber and tape, but was considered constant for a given tape. A few cases were noted with secondary fiducial peaks which spacing indicated were probably due to dropped bits in the Lecroy scaler. Event reconstruction should simply have eliminated this chamber in these rare cases. Finally, it should be noted
that the scale in Fig. 7, page 35, is far too coarse to show chamber resolution, which is of the order of a few scaler numbers (1 mm).

A display program was developed which produced a simple diagram of spark positions in both x and y projections, for each event. This allowed the reduction by hand of a few hundred events to ascertain if the spark position determination was self-consistent and to see if meaningful data were present. This procedure indicated that enough events with sufficient information for mass determination were present to warrant further reduction of the data.

**Trajectory Determination**

A program called GUTS was developed to arrange the various spark chamber spark positions into possible particle tracks, and to determine from these the most probable trajectory. Since each spark chamber was composed of a single pair of perpendicular wire planes with each chamber aligned parallel to all other chambers, it was possible to consider the x and y projections separately. Two possible sparks per projection with two projections allowed a four-fold ambiguity in the position determination if two particles transiting the chamber were assumed. This was because it was not known which spark in the x projection was associated with which spark in the y projection. If more than two particles transiting the chamber are assumed, analysis becomes even more complicated. Fortunately, the resolution of these ambiguities was unnecessary. Since the magnetic field direction was perpendicular to the x-axis (Fig. 2, page 13), the bending of the trajectory was almost entirely in the plane of the x projection with errors in the magnet alignment.
providing at most a slight variation in the y projection. However, one should bear in mind that the y projection was necessary to reconstruct the particle trajectory, since the field strength varied with y position. This allowed consideration of the two projections separately during trajectory reconstruction. Only the broadest cuts were made in the data using y projection values for this reason. In practice, an error in the choice of which tracks were associated in x and y projections would have resulted in a small error in $\int B dl$ and hence the momentum determination, broadening the mass peak.

In the x projection, separate straight line tracks were found for the 3 chambers above the magnet (top set) and the 3 chambers below the magnet (bottom set). The bend angle was then simply the angle between these straight lines. Each set was searched first for three spark tracks, then for two spark tracks. The impact parameter, defined as the closest distance from the extended track to the center of symmetry of the magnet in the x projection, was determined for each track in both sets of chambers. Any track was rejected that had an impact parameter greater than 35.6 cm, a radius associated with internal magnet clearances. GUTS then considered up to 10 pairs of top and bottom set tracks. Since the magnetic field had rotational symmetry in the x projection and was approximately constant as a function of y in the central field region, and since also the field strength dropped off quickly as a function of distance from the magnet region, impact parameter was an approximately conserved quantity. For each track pair, the difference in impact parameter was required to be less than 2.0 cm, and
if no pair satisfied this criterion, the event was rejected. The pair of tracks with the smallest impact parameter difference ($\Delta b$) was chosen as the most probable trajectory.

In the $y$ projection, all six spark chambers were considered together, and straight line tracks were searched for by the method of stringing (Duff et al., 1967). In this method, all possible combinations of two sparks in separate chambers were considered as possible candidates for a track starting with the top chamber. These two sparks determined a straight line which was projected through the magnet and the chambers below. A spark in a lower chamber was considered to belong to this candidate if it was within a small region about the projected position in this chamber. The acceptance region was a function of the positions of the previous sparks and the slope of the track taking into account the position of the edge of the chamber. An acceptable track was a candidate that transited the magnet entirely within the fiducial region and had at least three sparks with at least one spark on either side of the magnet. GUTS stored up to 10 such tracks, each of whose chi-squared was found from a least squares fit subroutine. Only tracks with chi-squared less than 10 were deemed acceptable. The most probable $y$ trajectory was then the member of the set of tracks with the largest number of sparks that had the smallest chi-squared. If no acceptable track was found, the event was rejected.
Before further reduction of the data could proceed, the relative positions of the spark chambers had to be determined. Since zero field \((I = 0)\) events with \(v = c\) should be straight lines in both \(x\) and \(y\) projections, they could be used to determine the errors in chamber alignment. It should be noted that the position of two chambers with respect to each other must be assumed in order to define a coordinate system with respect to which the positions of the other chambers may be measured. Using the \(v = c\) zero field data, each event was used to calculate the expected position of a spark in each of the other chambers. Then \text{SUMX} was used to histogram the difference of the expected spark position as determined from the best fit track and the actual position. The displacements from zero of the peaks in these distributions determined the corrections to the chamber positions. The peaks were then centered to zero displacement by additive adjustments to the chamber location coordinates. These adjustments were used for all further data reduction. The widths of these peaks measured the spark chamber resolution (Table 4). Chambers 1 and 6 had less resolution due to the fact that their wire spacing was twice that of the other chambers as well as perhaps to the warping which caused so much of their breakdown. It was not possible to determine the \(y\) projection resolution of chamber 6 due to its low efficiency.
Table 4. Wire Spark Chamber Resolution and Efficiencies

<table>
<thead>
<tr>
<th>Chamber</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>x resolution</td>
<td>2.1 mm</td>
<td>0.8 mm</td>
<td>0.6 mm</td>
<td>1.3 mm</td>
<td>0.7 mm</td>
<td>1.1 mm</td>
</tr>
<tr>
<td>y resolution</td>
<td>3.3 mm</td>
<td>1.4 mm</td>
<td>1.6 mm</td>
<td>1.4 mm</td>
<td>1.4 mm</td>
<td>---</td>
</tr>
<tr>
<td>x efficiency</td>
<td>16%</td>
<td>91%</td>
<td>98%</td>
<td>91%</td>
<td>86%</td>
<td>42%</td>
</tr>
<tr>
<td>y efficiency</td>
<td>44%</td>
<td>82%</td>
<td>77%</td>
<td>77%</td>
<td>81%</td>
<td>2%</td>
</tr>
</tbody>
</table>

The relative number of events in each of these peaks represented a measure of the spark chamber efficiency. The efficiencies for all chambers calculated from the zero field data are shown in Table 4.

Using GUTS on the $v = c$ zero field data, the angle between the top and bottom tracks in the x projection can be found for each event and a distribution of these compiled by SUMX is shown in Fig. 8. The distribution is centered about zero as expected since no bending except that due to multiple scattering and experimental error should be present. The width of this distribution is a measure of the precision with which the bend angle can be measured (angular resolution) and hence is related to the momentum resolution by Eq. (3.2) The momentum resolution corresponding to the width of the distribution in Fig. 8 is 31.3 GeV/c, and is called the maximum detectable momentum (MDM). The angular resolution is close to what would be expected considering the chamber resolutions in Table 4 and the distances between chambers. The momentum resolution is close to the result obtained from a separate treatment of the mass resolution (Appendix II).
Fig. 8. Histogram of Deflection Angle for $v=c$
Data with the Magnetic Field Off.
GUTS was applied to the $v=c$ data with the field energized ($I = 100$ amps). The distribution of impact parameter differences, $\Delta b$, between top and bottom tracks in the $x$ projection was obtained using SUMX, and is shown in Fig. 9. The centering and width of this distribution were observed for small shifts of the magnet position allowing alignment of the magnet with the spark chambers for $x$ and $z$ coordinates. It is clear from the shape of this curve that impact parameter is conserved to a high degree and a narrower cut could be made on the data with respect to $\Delta b$ without rejecting too many events.

A histogram of the bend angle between top and bottom tracks in the $x$ projection is shown in Fig. 10. The sign of the angle was chosen consistent with the sign of the charge of a particle with that deflection direction. Note that there are fewer events with negative bend angles than positive.

From Eq. (3.2), we may approximate the momentum by:

$$ P = \frac{0.07}{\theta} \text{ GeV/c} \quad (4.1) $$

Here the field integral has been assumed independent of path and all particles are presumed to be singly charged.

Equation (4.1) was utilized to plot the ratio of events with positive bend angle to those with negative bend angle as a function of momentum, Fig. 11. The errors shown are statistical only. These results are consistent with measurements of the charge ratio of cosmic ray muons...
Fig. 9. Impact Parameter Difference for \( v=c \) Data.
Fig. 10. Bend Angle Histogram for v=c Data.
Fig. 11. Charge Ratio for ν=c Data.
as expected (Kocharian, Saakian, and Kirakosian, 1959; Hume et al. (1973). The v=c data is further treated to obtain the muon momentum spectrum in Appendix IV.

Returning to the bend angle distribution, Fig. 10, page 45, the dip near zero deflection may be seen to arise from the shape of the cosmic ray muon differential momentum spectra of Eq. (1.1):

\[ I(P) = A P^{-\gamma} \quad \text{where } \gamma = 2.75 \]  

(4.2)

The bend angle near \( \theta = 0 \) may be approximated by the first term in the Maclaurin's expansion of Eq. (1.1):

\[ f(\theta) d\theta = I(P(\theta)) \frac{dP(\theta)}{d\theta} d\theta \]  

(4.3)

\[ = -A(0.07)^{-\gamma+1} \theta^{\gamma-2} d\theta \]

thus

\[ f(\theta)d\theta \propto \theta^{0.75} d\theta \quad \text{for } \gamma = 2.75 \]  

(4.4)

The distribution is a decreasing function of \( \theta \) for decreasing \( \theta \) at small angles. The dip in the bend angle distribution does not reach zero because of finite bend angle resolution.

- **v=c Data and Field Tracing**

The 133, 155 \( v < c \), field energized (I = 100 amps), events surviving REPACK were operated on by GUTS to yield 30,782 events with acceptable trajectories in both x and y projections, (Table 3, page 34). In order to determine the mass to high precision, it was necessary to determine the momentum to high precision by finding the path dependent field integral \( \int B d\ell \) for each event individually.
The magnetic field was determined by a computer program called MAGNET, which broke up each coil into a large number of discrete current loops and summed the off axis contribution of each of these current loops to the magnetic field at any point in the vicinity of the magnet. During a magnet test at a current of 150 amps, the magnetic field was measured on axis at the outer face of the Dewar using a Hall probe Gaussmeter. This result when compared to a later calculation using MAGNET agreed to within 1% which was the expected Gaussmeter accuracy. MAGNET was used to generate a field map for a fiducial region set by the magnet clearances and this map was stored on magnetic tape.

Using this field map and a first estimation of the momentum from the bend angle in the x projection and Eq. (1.1), another program BEND traced the particle through the magnetic field region, determining its exit direction (Bowen et al., 1969). A unit charge was assumed for the particle during this procedure. BEND then compared the exit slope to that of the observed trajectory and corrected the momentum accordingly. Two iterations of this procedure yielded the measured momentum for that event. A test with constructed events of known momenta showed that this process was capable of retrieving the momentum with an accuracy greater than 1%. An event which exited the magnet fiducial region at any step of this process was rejected, which was indicated by setting the momentum output equal to zero. Table 3, page 34, shows that BEND found satisfactory momenta for 23,350 of the $v = c, I = 100$ amp, events. BEND wrote all three momenta on its output tape.
By means of extensive programming using the various features of SUMX, as much as possible of the information contained in these 23,350 events (hereafter referred to as the $v_{<c}$ data), was retrieved.

**Time of Flight Corrections**

For a particle transiting two counters A and B, the velocity is given by:

$$\beta = \frac{Z}{Z + 29.98(\tau - \tau_A + \tau_B)}$$

(4.5)

where $Z$ is the trajectory length between A and B measured in cm, $\tau$ is the measured time of flight in excess of that required by a $v_{=c}$ particle (hereafter referred to as the time of flight) $\tau_A$ and $\tau_B$ are the light transit times in counters A and B measured in nanoseconds, and 29.98 is the velocity of light in units of cm/ns. Histograms of $\beta$ for the time of flight 1 ($S_1, S_3$) and time of flight 2 ($S_1, S_2$) are shown in Fig. 12, for the $v_{<c}$ data. The histogram for TOF1 has a velocity cutoff somewhat above $\beta = 0.9$. The histogram for TOF2 has no obvious cutoff near $v_{=c}$ due to multiple particle background events which mimic a $V_{<c}$ particle for TOF1 and a $v_{=c}$ particle for TOF2. Two extra peaks can be seen in the histogram for TOF2, which are due to cases in which only one of the $S_2$ photo tubes triggers its discriminator (See the discussion of the electronics in Chapter 3). Since, as we shall see, time of flight 2 was used only to cut the data in comparison with time of flight 1, and not averaged with time of flight 1 to find a resultant velocity, these extra peaks were not expected to affect the results (see Appendix V).
Fig. 12. Histograms of the Uncorrected Velocity for v<c Data.

(a) Time of Flight 1.
(b) Time of Flight 2.
Mass Distributions

Using the velocity, $\beta$, from time of flight result and the momentum calculated in BEND, the mass may be calculated for each event from:

\[ M = \frac{P}{\beta \gamma} \quad \text{where} \quad \gamma = (1 - \beta^2)^{-\frac{1}{2}} \quad (4.6) \]

where mass is in units of GeV/c$^2$, momentum in GeV/c and all particles are assumed to be single charged. Mass distributions for $Q>0$ and $Q<0$ for all $v<c$ data using TOF1 are shown in Fig. 13. A peak near the proton mass is clearly visible, whereas the corresponding antiproton peak is absent as expected for cosmic ray secondary spectra. A smooth background is present for both $Q>0$ and $Q<0$ with a slightly larger number of events for $Q>0$. The charge ratio for background events $B+/B- = 1.76 \pm 0.23$ (2.5 GeV/c$^2 \leq M \leq 3.6$ GeV/c$^2$) is consistent with the background being predominantly muons (see Fig. 11, page 46). This background was due to false triggering of TOF1 in which an unrelated particle or spurious electronic pulse triggered Sl and was followed a time later, corresponding to a time of flight $\tau$, by a muon which transited the apparatus as a valid event. The mass was then calculated from Eq. (4.5) with the real momentum of the muon and a fictitious velocity $\beta$ corresponding to $\tau$. Since TOF2 will not in general give the same erroneous velocity $\beta$ we may discriminate against this background by requiring the two time of flight velocity results to agree within preset criteria.

The center of the proton mass peak in Fig. 13 may be seen to be slightly displaced from the proton mass ($M_p = 0.938$ GeV/c$^2$). In general, when this data are broken up into various different time and velocity
Fig. 13. Mass Histograms for All ν=c Data.
intervals, different peak widths and shifts are observed. The reason is that systematic errors existed in both the time of flight and momentum determinations.

The time of flight system was calibrated using v=c events and cable delays for time increments. A sample histogram of velocity for TOF1 for v=c events is shown in Fig. 14. The shape of this peak is a complicated function of the shape of the background momentum spectrum, the experimental resolution, effects due to the electronic cutoff near \( \beta = 0.9 \) and trajectory and light transit time effects. It is not clear what point on this curve represents v=c. Thus a small zero shift systematic error in the time of flight scale is probably. Since the time of flight is a non-linear function of velocity, a small time of flight error could have a complicated non-linear effect on the mass calculation.

As pointed out in the description of the magnet in Chapter 3, the field of the magnet decays with time due to non-superconducting parts of the current path. Since this changes the field integral \( \int \beta dl \) and hence the momentum determination, it also affects the mass calculation. This is in principle a simple correction providing the field is known as a function of time. The Hall effect Gaussmeter used to monitor the magnetic field during this experiment malfunctioned so that this information was not available. Since it was difficult in principle to separate the momentum and time of flight errors, it was decided to use the events in the proton peak, a sample of known mass, to simultaneously determine the systematic errors in both time of flight and momentum.
Fig. 14. Histogram of Time of Flight 1 Velocity for v=c Data.
A mathematical expression for the relative error in the mass determination due to these systematic errors in momentum and time of flight may be obtained by differentiation of Eq. (4.6) with respect to momentum and velocity and Eq. (4.5) with respect to time of flight. After substitution of the result for Eq. (4.5) into the result for Eq. (4.6) one obtains:

$$\frac{\delta M}{M} = \frac{\delta P}{P} + L \gamma^2 \delta \tau$$

(4.7)

where \(L = \frac{29.98 \text{cm/ns}}{2} = 4.17 \times 10^{-2} \text{ ns}^{-1}\) for time of flight 1. The fractional shift in mass \(\frac{\delta M}{M}\) is given in terms of the zero shift in the time of flight, \(\delta \tau\), measured in nanoseconds and the fractional shift in momentum estimate \(\frac{\delta P}{P}\) due to the magnet field decay.

Table 5 gives the position of the proton peak in GeV/c² for various intervals of \(\beta\) and for successive data tapes. The times given in the first column of Table 5 are the mean values for each data tape. Note that the peak positions increase with increasing time. Since the decay of the magnetic field results in a smaller deflection of protons, "BEND" overestimates the momentum and thus the mass is also overestimated.

The field decay should be exponential in time, the natural logarithms of the numbers in each column in Table 5 should vary linearly with time, the slope being the same for each column, i.e., for each momentum interval. A maximum likelihood fit for this data gave a single best fit slope and four intercepts. The slope, \(7.96 \times 10^{-4} \text{ hr}^{-1}\) represented a magnetic field decay time constant of 52.4 days. This is consistent with an earlier measurement during a field on test of
Table 5. Proton Mass Peak Position for Various Velocity Intervals.

<table>
<thead>
<tr>
<th>TIME (hr.)</th>
<th>0.20&lt;\beta\leq0.55</th>
<th>0.55&lt;\beta\leq0.70</th>
<th>0.70&lt;\beta\leq0.80</th>
<th>0.80&lt;\beta\leq0.90</th>
</tr>
</thead>
<tbody>
<tr>
<td>57.5</td>
<td>0.96 GeV/c²</td>
<td>0.95 GeV/c²</td>
<td>0.95 GeV/c²</td>
<td>0.95 GeV/c²</td>
</tr>
<tr>
<td>75.9</td>
<td>0.96 GeV/c²</td>
<td>0.95 GeV/c²</td>
<td>0.89 GeV/c²</td>
<td>0.91 GeV/c²</td>
</tr>
<tr>
<td>91.1</td>
<td>0.995 GeV/c²</td>
<td>0.945 GeV/c²</td>
<td>9.94 GeV/c²</td>
<td>0.92 GeV/c²</td>
</tr>
<tr>
<td>107.9</td>
<td>1.01 GeV/c²</td>
<td>0.985 GeV/c²</td>
<td>0.98 GeV/c²</td>
<td>0.91 GeV/c²</td>
</tr>
<tr>
<td>125.6</td>
<td>1.015 GeV/c²</td>
<td>0.995 GeV/c²</td>
<td>0.99 GeV/c²</td>
<td>0.95 GeV/c²</td>
</tr>
<tr>
<td>141.5</td>
<td>1.00 GeV/c²</td>
<td>1.00 GeV/c²</td>
<td>1.04 GeV/c²</td>
<td>0.98 GeV/c²</td>
</tr>
<tr>
<td>156.9</td>
<td>1.012 GeV/c²</td>
<td>1.04 GeV/c²</td>
<td>1.01 GeV/c²</td>
<td>1.00 GeV/c²</td>
</tr>
<tr>
<td>175.1</td>
<td>1.015 GeV/c²</td>
<td>1.03 GeV/c²</td>
<td>1.02 GeV/c²</td>
<td>0.99 GeV/c²</td>
</tr>
<tr>
<td>191.0</td>
<td>1.05 GeV/c²</td>
<td>1.05 GeV/c²</td>
<td>1.04 GeV/c²</td>
<td>1.03 GeV/c²</td>
</tr>
<tr>
<td>Intercept</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T = 0. hr.</td>
<td>0.904 GeV/c²</td>
<td>0.896 GeV/c²</td>
<td>0.886 GeV/c²</td>
<td>0.862 GeV/c²</td>
</tr>
</tbody>
</table>
71.2 ± 20.4 days. Since the boil off of liquid helium from the magnet represents a combination of conduction of heat into the magnet and heat dissipated due to resistive losses for non-superconducting parts of the current path, an upper limit to the field decay can be estimated from the fact that no change in boil off was observed when the field was turned on. This upper limit to the time constant, 92 days, is consistent with the above calculation. The antilogarithms of the four t=0 intercepts are shown in the last row of Table V and are related to the time of flight error, δτ, by:

$$\frac{\delta M}{M} = L \gamma^2 \delta \tau + \left(\frac{\delta p}{p}\right)_0$$

(4.8)

where \(\left(\frac{\delta p}{p}\right)_0\) is a small constant systematic error in \(\frac{\delta M}{M}\). The above equation is the equation of a straight line with dependent variable \(L \gamma^2 \delta \tau\), slope \(\delta \tau\), and intercept \(\left(\frac{\delta p}{p}\right)_0\). A linear least squares fit this equation using the values in the last row of Table V for \(\frac{\delta M}{M}\) yields a slope \(\delta \tau = 0.33 \text{ ns}\) and an intercept \(\left(\frac{\delta p}{p}\right)_0 = 0.025\). \(\delta \tau = 0.33 \text{ ns}\) is then the time of flight correction to TOF1. This systematic error of 2.5% is probably due to either systematically misjudging the mean value position of the asymmetric mass curve, or an error in magnet calibration, or a combination of both. Since this correction procedure is involved and difficult and the error is small, further iteration of the procedure is probably not worthwhile.

The correction \((\Delta \tau_2)\) for TOF2 was found in a different way. Using events from the proton peak for which mass, corrected momentum, and trajectory are now known, the expected value of the velocity for
TOF2, $\beta_2$, suitably corrected for light transit times in the counters and trajectory, may be found. When this expected value for $\beta_2$ is subtracted from the experimental value for each proton peak event, the histogram shown in Fig. 15 is obtained. The time shift in TOF2 which was needed to center this velocity difference distribution about zero was the required additive correction, $\Delta \tau_2$, for TOF2. A distribution of velocity differences was used instead of time of flight differences, which could also have been found, because the velocity distribution was more sensitive to a small time of flight shift than was the time of flight distribution as was previously noted. A similar treatment applied to TOF1 verified the correction previously found for it.

**Time of Flight Comparison**

Consider a general time of flight system with counter separation $Z$ (cm), time of flight excess, $\tau_i$ (ns), and velocity given by:

$$\beta_i = \frac{Z_i}{Z_i + 29.98 \tau_i}$$  \hspace{1cm} (4.9)

Differentiation with respect to $\tau_i$ yields

$$d\beta_i = -\beta_i^2 \frac{29.98}{Z_i} d\tau_i$$  \hspace{1cm} (4.10)

In this experiment the two time of flight systems ($i = 1,2$) share a scintillation counter S-1, such that $\tau_1 = t_{s3} - t_{s1}$ and $\tau_2 = t_{s2} - t_{s1}$ where $t_{s1}$, $t_{s2}$, $t_{s3}$ are the discriminator triggering times for the subscripted counters. For statistically independent firing times with standard deviations $\sigma_{s1}$, $\delta_{s2}$, $\delta_{s3}$, the standard deviation $\sigma_{\beta_1-\beta_2}$ of the
Fig. 15. Histogram of Experimental Minus Calculated Velocity for Time of Flight 2, Proton Peak Data.
difference of the two time of flights is:

\[
\sigma_{\beta_1-\beta_2} = 29.98 \left\{ \left( \frac{\beta_1^2 \sigma_{s1}}{Z_1} \right)^2 + \left( \frac{\beta_2^2 \sigma_{s2}}{Z_2} \right)^2 + \right. \\
\left. \left[ \left( \frac{\beta_1}{Z_1} - \frac{\beta_2}{Z_2} \right) \sigma_{s1} \right]^2 \right\}^{1/2}
\]

After correction of the time of flight for light transit time in the counters and particle trajectory the dominant source of error is expected to be the random errors in timing by the photomultiplier tubes. The width of the distribution of corrected time of flight (TOF2) for high momentum \(v = c\) data (muons) yielded a rough estimate of photomultiplier tube error of

\[
\sigma_{\text{PMT}} \leq 0.90 \text{ ns}
\]

under the assumption that all phototubes had equivalent error distributions. We may then set \(\sigma_{s1} = \sigma_{s3} = \sigma_{\text{PMT}} \leq 0.90 \text{ ns}\) and

\[
\sigma_{s2} = \sigma_{\text{PMT}} / \sqrt{2} \approx 0.64 \text{ ns}
\]

where the timing measurement of counter \(s2\) is the average of two photomultiplier tubes with assumed independent random errors. From Eq. (4.11) with \(\beta_1 = \beta_2\), \(Z_1 = 719.2\text{ cm}\) and \(Z_2 = 346.2\text{ cm}\) the standard deviation of \(\beta_1 - \beta_2\) becomes

\[
\sigma_{\beta_1 - \beta_2} = 0.078 \beta_1^2
\]

The acceptance criterion for the cut data was chosen somewhat arbitrarily as:
The event velocity for the purpose of all calculations continued to be \( \beta_1 \), the result from time of flight 1, since a weighted averaging of the results of the two time of flights would not result in a significantly lower error for reasons discussed in detail in Appendix V.

**Charge Determination**

Background events were predominantly minimum ionizing muons, as has been noted before. A sample of these events should produce the same amount of light in a given time of flight scintillation counter since the trajectory is close to vertical. Thus for this sample the counter pulse height should be a function only of geometry. The pulse height in counter S-1 as a function of radius for background events is shown in Fig. 16, where errors shown are statistical only. The curve is linear except near the maximum radius where reflections from counter walls raise the pulse height slightly. Similar data for counter S-3 had much broader pulse height distributions, and hence greater error and did not appear suitable for charge determination purposes. After pulse height position corrections for S-1 were inserted into SUMX, pulse height distributions were found for protons for various momentum intervals. The mean pulse height for each of these distributions was then plotted versus the velocity calculated from the mean momentum of the momentum interval. The result, shown as the lower curve in Fig. 17, is fortuitously linear. When the mean pulse height was plotted versus the specific energy loss \((-dE/dx)\) calculated from published data (Trower, 1966), and the known

\[
|\beta_1 - \beta_2| \leq 0.0905 \beta_1^2 \tag{4.13}
\]
Fig. 16. Pulse Height for Counter S1 vs. Radial Distance for Minimum Ionizing Particles.
Fig. 17. Counter S1 Pulse Height Corrected for Counter Efficiency vs. Velocity for Two Particle Charged States.
Fig. 18. Corrected Pulse Height vs. Specific Energy Loss for Counter S1.
momentum, the result, Fig. 18, indicated saturation effects for large specific energy loss. The solid curve in Fig. 18 is obtained from extrapolating the linear curve in Fig. 17, for \( Q=\pm 1 \). Since \( \frac{-dE}{dx}/Q^2 \) where \( Q \) is the charge of the particle is a function of of velocity alone for a given material, we may use Fig. 18 to find the pulse height for charge \( Q=\pm 2 \) particles. This is done in the following way. For a \( Q=1 \) particle of \( \beta=0.8 \), \( a(-dE/dx \text{ of } 2.16 \text{ MeV/gm/cm}^2) \) is found from the literature (Trower, 1966). Since a \( Q=\pm 2 \) particle of \( \beta=0.8 \) would have a specific energy loss in the scintillator four times as large \( (-dE/dx)_{Q=\pm 2} = 8.64 \text{ MeV/(gm/cm}^2) \) we may find from Fig. 18 that the corrected pulse height is 32.7. In this way a plot of pulse height vs. velocity can be constructed for \( Q=\pm 2 \) particles as shown by the upper curve in Fig. 17. The dotted lines represent one standard deviation limits from the original proton distributions. Note that below a velocity of \( \beta=0.5 \) there is an overlap region where there is an ambiguity in the charge of the particle. Equations for the curves in Fig. 17 were used in SUMX to determine the charge of a given event at the one standard deviation level from the corrected S-l pulse height. In ambiguous cases the particles were assumed to be \( |Q| = 1 \). Events outside of the one standard deviation region for \( |Q| = 1 \) or \( |Q| = 2 \) were rejected.
CHAPTER 5
RESULTS AND CONCLUSIONS

Using the information on each event generated in the data analysis section and the histograming program SUMX, final mass spectra may be determined for various experimental parameters. Figs. 19-28 show samples of such mass spectra for charges Q = ±1, ±2 and for ten intervals of velocity with the time of flight cut given by Eq. (4.12). The data used to generate these histograms was corrected for magnet field decay, and had an impact parameter difference given by |Δb| < 1.0 cm. Fig. 19 which is for velocity interval 0.20 < β < 0.35 does not have the time of flight cut. Because of the low background, this would provide no significant improvement. This is fortuitous since ionization energy loss greatly affects velocities in this interval only resulting in different average velocities over the two different time of flight paths, an effect which would otherwise need correction if the time of flight cut were used. In all the histograms the interval on the abcissa is the logarithm of the mass divided by charge in units of the nucleon mass because this quantity has more nearly symmetrical errors than the mass itself. Mass divided by charge is used rather than mass alone since it is mass divided by charge that is measured by the spectrometer and thus apparatus dependent effects may appear vertically aligned in the four histograms.
Fig. 19. Mass Histogram of Corrected $\nu$ Data, $0.20 < \beta < 0.35$.

$|\Delta b| < 1.0$ cm, $\Delta \theta < 0.09058^2$. Note the logarithmic mass divided by charge and proton mass scale of the abcissa.
Fig. 20. Mass Histogram of Corrected $\nu c$ Data, $0.35 < \beta < 0.50$.

$|\Delta b| \leq 1.0$ cm, $\Delta \beta \leq 0.0905 \beta^2$. 
Fig. 21. Mass Histogram of Corrected \( v < c \) Data, \( 0.50 < \beta < 0.60 \).
\(|\Delta b| \leq 1.0\,\text{cm}, \Delta \beta \leq 0.0905\beta^2\).
Fig. 22. Mass Histogram of Corrected $\nu e$ Data, $0.60 < \beta < 0.65$.

$|\Delta b| \leq 1.0\text{cm}, \Delta\beta \leq 0.0905\beta^2$. 
Fig. 23. Mass Histogram of Corrected \(\nu c\) Data, \(0.65 < \beta \leq 0.70\).

\[|\Delta b| \leq 1.0 \text{cm}, \Delta \beta \leq 0.0905 \beta^2.\]
Fig. 24. Mass Histogram of Corrected $v < c$ Data, $0.70 < \beta \leq 0.75$.

$|\Delta b| \leq 1.0\text{cm}, \Delta \beta \leq 0.0905\beta^2$. 
Fig. 25. Mass Histogram of Corrected v<e Data, 0.75<β≤0.80.

|Δb| ≤ 1.0cm, Δβ≤0.0905β².
Fig. 26. Mass Histogram of Corrected $v < c$ Data, $0.80 < \beta < 0.85$.

$|\Delta b| \leq 1.0 \text{cm}, \Delta \beta \leq 0.0905$.
Fig. 27. Mass Histogram of Corrected $\nu c$ Data, $0.85 \leq \beta \leq 0.90$.

$|\Delta b| \leq 1.0\text{ cm, } \Delta \beta \leq 0.0905\beta^2$. 
Fig. 28. Mass Histogram of Corrected $\nu c$ Data, 0.90 < $\beta$ ≤ 0.95.

$|\Delta b| \leq 1.0 \text{cm}, \Delta \beta \leq 0.0905 \beta^2.$
The histograms of Figs. 19-28 show clean proton peaks with little background except for the higher velocity intervals. This is characteristic of the data even with many of the cuts relaxed. The Q = +1 histograms for Figs. 19-23 show clear deuteron peaks in the region near twice the nucleon mass. No tritons are evident at three times the nucleon mass. The Q = -1 histograms are very clean with no antiproton peaks and there are no indications of massive Q = -1 or Q = +1 particles. The peaks in the Q = +2 histograms which are aligned vertically with the proton peaks in the Q = +1 histograms would appear to be diprotons taking into account the mass over charge abcissa scale. These are actually spillover from the Q = +1 proton peaks due to statistical fluctuations in the pulse height from which the magnitude of the charge is determined. A Q = +2 peak also occurs which is vertically aligned with the deuteron peak in the Q = +1 histograms in Figs. 20-22. Comparison of the relative numbers of events in the peaks in both Q = +2 and Q = +1 shows that the second Q = +2 peak in each case is not totally due to spillover from the deuteron peak in Q = +1, and is thus due to real events with Q = +2 and a mass of four times the proton mass or alpha particles (\(^4\)He). \(^3\)He events would occur between the spillover proton peak and the alpha peak in the Q = +2 histograms, and there is no evidence of their presence. No peaks corresponding to massive particles are seen in either the Q = +2 or Q = -2 histograms for any velocity interval.

Muons, pions and electrons are of course present in the cosmic ray secondary spectra but are not expected to show up in these mass spectra since their masses are too low. Bend angles for these particles
at this magnetic field intensity for velocities that would trigger the
time of flight system would be too large for them to remain within the
solid angle acceptance of the spectrometer. The spectra of muons was
found another way in Appendix IV. Charged K-mesons might be expected,
but due to their short lifetimes should only appear in the high velocity
interval histograms where background is a problem. All other known
elementary particles, both mesons and baryons, have too short a lifetime
or are neutral and neither appear nor are expected to appear in these
mass spectra. Each of the previously noted possible components will now
be discussed separately.

Protons

Although the mass spectrum histograms of Figs. 19-28 could have
been used to determine the proton intensities, the number of real
protons rejected by the various data cuts would have to be determined in
order to obtain the absolute intensities. Instead histograms analogous
to Figs. 19-28 but with no data cuts and time of flight 1 only were
used. The number of events under the proton peak for each of these 10
velocity intervals was determined. Negative charge data in the vicinity
of the proton mass was used for the background subtraction. The number
of proton events in each velocity interval are shown in the column
labeled \(N_p\) in Table 6. These numbers \(N_p\) could then be compared to
numbers obtained in a similar way for the same velocity intervals but
under various data cuts to obtain scaling factors for these data cuts,
which would then be useful in determining the absolute intensity of
other types of particles.
Table 6. Proton Results at Mountain Altitude (747 gm/cm²).

<table>
<thead>
<tr>
<th>Velocity Interval</th>
<th>Momentum Interval (GeV/c)</th>
<th>$P_{av}$ (GeV/c)</th>
<th>$N_p$</th>
<th>$I_p$ (cm⁻²sec⁻¹sr⁻¹(GeV/c)⁻¹)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.20 &lt; $\beta$ ≤ 0.35</td>
<td>0.20 - 0.35</td>
<td>0.31</td>
<td>113</td>
<td>$1.49 ± 1.12 \times 10^{-3}$ †</td>
</tr>
<tr>
<td>0.35 &lt; $\beta$ ≤ 0.50</td>
<td>0.35 - 0.54</td>
<td>0.44</td>
<td>518</td>
<td>$9.37 ± 2.25 \times 10^{-4}$ †</td>
</tr>
<tr>
<td>0.50 &lt; $\beta$ ≤ 0.60</td>
<td>0.54 - 0.70</td>
<td>0.60</td>
<td>646</td>
<td>$1.14 ± 0.18 \times 10^{-3}$ †</td>
</tr>
<tr>
<td>0.60 &lt; $\beta$ ≤ 0.65</td>
<td>0.70 - 0.80</td>
<td>0.73</td>
<td>399</td>
<td>$1.14 ± 0.09 \times 10^{-3}$</td>
</tr>
<tr>
<td>0.65 &lt; $\beta$ ≤ 0.70</td>
<td>0.80 - 0.92</td>
<td>0.84</td>
<td>547</td>
<td>$1.21 ± 0.09 \times 10^{-4}$</td>
</tr>
<tr>
<td>0.70 &lt; $\beta$ ≤ 0.75</td>
<td>0.92 - 1.06</td>
<td>0.96</td>
<td>564</td>
<td>$9.45 ± 0.66 \times 10^{-4}$</td>
</tr>
<tr>
<td>0.75 &lt; $\beta$ ≤ 0.80</td>
<td>1.06 - 1.26</td>
<td>1.12</td>
<td>541</td>
<td>$6.58 ± 0.47 \times 10^{-4}$</td>
</tr>
<tr>
<td>0.80 &lt; $\beta$ ≤ 0.85</td>
<td>1.26 - 1.51</td>
<td>1.32</td>
<td>585</td>
<td>$4.87 ± 0.36 \times 10^{-4}$</td>
</tr>
<tr>
<td>0.85 &lt; $\beta$ ≤ 0.90</td>
<td>1.51 - 1.94</td>
<td>1.63</td>
<td>537</td>
<td>$2.68 ± 0.23 \times 10^{-4}$</td>
</tr>
<tr>
<td>0.90 &lt; $\beta$ ≤ 0.95</td>
<td>1.94 - 2.86</td>
<td>2.05</td>
<td>320</td>
<td>$1.22 ± 0.21 \times 10^{-4}$ *</td>
</tr>
</tbody>
</table>

† Corrected for Magnetic Field Decay Effects as Described in Text.
* Corrected for Triggering Cut Off Effects as Described in Text.
Using the proton mass peak values in Table 6, the absolute vertical differential momentum intensity for protons could then be found from:

\[ I_p = \frac{N_p}{\Delta p \cdot A\Omega(p) \cdot \Delta t_s \cdot \eta} \]  

(5.1)

where \( N_p \) is the proton mass peak number from Table 6, \( \Delta p \) is the momentum interval in GeV/c, \( \eta \) is the efficiency for event recognition, \( \Delta t_s \) is the time for which the apparatus was sensitive, and \( A\Omega(p) \) is the area times solid angle acceptance of the spectrometer. The area times solid angle acceptance is a function of magnetic deflection, and thus changes in time as the field in the magnet decays.

The time the apparatus was sensitive was determined from \( \Delta t_s = \Delta t - N_t \cdot \Delta t_d \) where \( \Delta t \) was the total time during which the apparatus was operated, \( N_t \) was the total number of triggers during this time and \( \Delta t_d \) was the dead time; that is, the time for which the apparatus was gated off after each event during which data was written on the magnetic tape. For this set of data, \( \Delta t = 4.748 \times 10^5 \) sec., \( N_t = 173,612 \) and \( \Delta t_d = 0.96 \) sec., resulting in a sensitive time of \( \Delta t_s = 3.082 \times 10^5 \) sec.

The event recognition efficiency \( \eta \) was assumed to be entirely due to the spark chambers. \( \eta \) was determined by considering how the independent inefficiency of each chamber contributes randomly to the probability of observing an event as determined by the spark chamber criteria considered in Chapter 4. The nine separate data tapes which made up this data set were separately analyzed to determine the ratio of the number of events surviving program BEND to the sensitive time \( \Delta t_s^{-} \) for each tape. A Gaussian least squares fit applied to these nine
values yielded a standard deviation of the mean of 4.8% of the mean value which was taken as the percentage error in the product $n\Delta t_s$.

How the area solid angle acceptance as a function of momentum $[\Omega(p)]$ was determined is detailed in Appendix I. The result is shown in Fig. 29. Since for high momentum $\Omega(p)$ is a slowly varying function of momentum, a simple time averaged correction to the mean momentum for each interval should suffice to account for the effects of the decay of the magnetic field. $\Omega(p)$ together with an estimated error may be determined from the corrected momentum and Fig. 29. For the case of small momentum where $\Omega(p)$ is near threshold in Fig. 29, and hence is a strongly varying function of momentum, a time dependent method must be used. Using SUMX, the number of events in the proton peak for a given momentum interval was displayed as a function of time, and hence data tape number. The display indicated an approximately linear dependence of number of events with time when corrected for different sensitive times $\Delta t_s$ for each data tape. A linear least squares fit gave the number of events in the proton peak at zero time with respect to the magnet field decay, together with the approximate error. This proton peak value was then used with the $\Omega(p)$ determined from Fig. 29, and the mean proton momentum in the interval to find the vertical proton differential momentum intensity from Eq. (5.1). The first three values for $I_p$ in Table 6 were obtained in this way. Both average momentum interval correction to $\Omega(p)$ and the time dependent fits method described above were found to yield equivalent intensities for the higher momentum intervals, with the fit method yielding a slightly higher error. The number of protons in the proton peak for the highest
Fig. 29. Area Solid Angle Produce ($A\Omega$) of the Spectrometer as a Function of Momentum.

Data is from the Monte Carlo calculation of Appendix I, for magnet current $I = 100$ amps.
velocity interval is strongly affected by the time of flight triggering cut off, which is approximately in the middle of this interval as well as the finite time of flight resolution. \( N_p \) for this interval has been corrected for these effects from a comparison of the time of flight histogram for \( v < c \) triggers with that for \( v = c \) triggers.

The final vertical proton differential momentum intensities of Table 6 were used to plot the vertical proton differential momentum spectrum of Fig. 30. The average momentum for each momentum interval, \( p_{av} \) in Table 6, was an average of the momenta of all proton peak events in that momentum interval determined in SUMX. The solid curve in Fig. 30 is the Monte Carlo cascade calculation result for our altitude and vertical geomagnetic cut off rigidity determined in Appendix III. The agreement between the Monte Carlo calculation and the experimental result is impressive when one considers that no adjustments were made in absolute intensity or shifts in momentum to achieve this fit. This increases our confidence in the procedures used in the data analysis. The two highest momentum experimental points fall slightly below the Monte Carlo curve, and this is probably due to not including enough of the high mass tail of the proton peak in the determination of \( N_p \) (see Fig. 27 and Fig. 28). The lowest momentum point may also have a systematic error due to positioning of the counter S1 and S2 with respect to the magnet in the bend angle direction. At the lowest momentum only very small areas of counters S1 and S2 contribute to \( A\Omega(p) \) and hence a small shift of position of S1 or S3 has a large effect on the rate. This amounts to about a 50% change in the rate for a 3cm
Fig. 30. Vertical Differential Proton Momentum at Mountain Altitude (747 gm/cm²).

Data is from this experiment, solid curve is from atmospheric cascade Monte Carlo discussed in Appendix III.
shift of S1 and S2 with respect to the magnet for the lowest momentum interval. Since it is not known whether an error in position of this sort existed, this possible error is not reflected in the error bars for the lowest momentum point. The lowest point should be the only point significantly affected by this type of position error.

It is recommended that any further operation of this experiment include at least one run at a lower magnet field to better determine the proton spectrum below 0.6 GeV/c as well as to accumulate data on muons and pions.

**Deuterons**

Comparison of the proton peaks in Figs. 19-28 with proton peaks in similar histograms for the uncut data used in the determination of the proton spectrum yielded a velocity independent calibration factor, f, between the two sets of distributions. The number of deuterons in each velocity interval containing a deuteron peak for the cut data, Figs. 19-25, was found after subtraction of the background by comparison of the Q = +1 and Q = -1 histograms. This deuteron number, $N_d$, is shown in Table 7. The deuteron vertical differential momentum intensity was found from an equation similar to Eq. (5.1):

$$ I_d = \frac{f N_d}{\Delta p \Delta \Omega(p) t_s \eta} $$  \hspace{1cm} (5.2)

in a manner exactly similar to that for protons. Here the average momentum was taken at the center of the momentum interval and the time averaged correction to the area times solid angle product was used to
Table 7. Deuteron Results at Mountain Altitude (747 gm/cm²).

<table>
<thead>
<tr>
<th>Velocity Interval</th>
<th>Momentum Interval</th>
<th>( P_{av} ) (GeV/c)</th>
<th>( N_d )</th>
<th>( I_d ) (cm(^{-2})sec(^{-1})sr(^{-1})(GeV/c)(^{-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.35&lt;( \beta \leq 0.50 )</td>
<td>0.70 - 1.08</td>
<td>0.89</td>
<td>10</td>
<td>1.93 ± 0.73 x 10(^{-5})</td>
</tr>
<tr>
<td>0.50&lt;( \beta \leq 0.60 )</td>
<td>1.08 - 1.41</td>
<td>1.24</td>
<td>8</td>
<td>1.57 ± 0.64 x 10(^{-5})</td>
</tr>
<tr>
<td>0.60&lt;( \beta \leq 0.65 )</td>
<td>1.41 - 1.60</td>
<td>1.50</td>
<td>3</td>
<td>0.98 ± 0.60 x 10(^{-5})</td>
</tr>
<tr>
<td>0.65&lt;( \beta \leq 0.70 )</td>
<td>1.60 - 1.84</td>
<td>1.72</td>
<td>7</td>
<td>1.79 ± 0.75 x 10(^{-5})</td>
</tr>
<tr>
<td>0.70&lt;( \beta \leq 0.80 )</td>
<td>1.84 - 2.50</td>
<td>2.17</td>
<td>1</td>
<td>3.52 x 10(^{-6})</td>
</tr>
</tbody>
</table>

† 90% Confidence Upper Limit with Poisson Statistics.
account for the magnet field decay. The results are displayed in Table 7, and plotted as the data points in Fig. 31.

The solid curves in Fig. 31 are the predictions of an adaptation of the Monte Carlo atmospheric cascade calculation described in Appendix III. This calculation assumes production of deuterons by the secondary cosmic ray nucleon component in interactions of the form:

\[ N + A \rightarrow A' + d + \pi \]
\[ N + A \rightarrow A'' + N + d \]  

where \( N \) represents a nucleon and \( A \) is the target air nucleus.

Reaction (5.3), which will be referred to as quasi-free production, is analogous to the pion production reaction:

\[ P + P \rightarrow \pi^+ + d \]  

The cross section of reaction (5.5) peaks at an incident proton momentum near 1 GeV/c with a relatively large total cross section \( \sigma(PP \rightarrow \pi^+d) \approx 3.3 \text{mb} \) (Overseth et al., 1964; Mescherjakov, Bogacer, and Neganov, 1956). Abundant accelerator data exists for the quasi-free production process, reaction (5.3) utilizing various target nuclei (Sutter et al., 1967; Dzhelelov, 1970; Azhgirei et al., 1971; Komarov, 1974). All these authors point out that

\[ \frac{d\sigma}{d\Omega} (PA \rightarrow \pi dA') = k A^{1/3} \]  

where \( k \) was approximately constant for a large number of nuclei, \( A \), over a large range of angles. Since the nuclear radius is approximately proportional to the cube root of the number of nucleons, the cross section ratio of Eq. (5.6) is proportional to the nuclear radius, and
Fig. 31. Vertical Differential Momentum Spectrum at Mountain Altitude (747 gm/cm$^2$).

The solid curves are from the atmospheric cascade calculation described in the text; (a) Quasi free production contribution (b) Quasi elastic scattering contribution, (c) sum of a and b.
thus to the nuclear circumference of the target nucleus. This indicates a basically peripheral process in which the incoming nucleon interacts with a peripheral nucleon producing a pion and a deuteron with the rest of the target nucleus acting as a spectator, hence the designation quasi free production. Presumably nucleon shadowing and the short mean free path of the deuteron in nuclear matter prevent any sizeable contribution of the non-peripheral target nucleons to this process. The numerical results of Azhgirei et al. as communicated by Komarov (1974) are consistent with \( k = 1.89 \) in Eq. (5.6). The cross section of reaction (5.3) was then assumed to be composed of two equal components directly forward and backward in the center of mass system so that the results could be utilized in a one-dimensional cascade calculation and equal cross sections were assumed for incoming protons and neutrons. The total cross section for reaction (5.1) was then found to be

\[
\sigma(PA_1 \rightarrow \pi dA_2) = 1.89 \sigma(PP \rightarrow \pi^+d) \tag{5.7}
\]

where \( A_1 \) is an air nucleus. The Monte Carlo calculation took the Fermi momentum of the target nucleons into account by assuming that a single longitudinal component of Fermi momentum, \( P_f \), was proportional to \( \exp(-P_f^2/2M_\pi^2) \), where \( M_\pi = 140 \text{ MeV}/c \). Consideration of the Fermi momentum is particularly important in cases such as reaction (5.3) where the sharp peak in the cross section, as a function of momentum, of the analogous reaction (5.5) is substantially broadened by the intranuclear motion. In reactions where cross sections vary gradually with momentum, Fermi momentum may usually be neglected.
Reaction (5.4), which is called quasi-elastic scattering, has also been well studied for various different target nuclei at accelerators (Azhgirei et al., 1958; Sutter et al., 1967; Azhgirei et al., 1971; Komarov, 1974). Sutter et al. (1967) and Komarov (1974) point out that:

$$\frac{d\sigma}{d\Omega} |_{PA \rightarrow NA^1d} \propto A^{1/3}$$

(5.8)

with the deuteron going forward for a large range of target nuclei, A, and angles. Thus quasi-elastic scattering is also a peripheral process in which an incoming nucleon scatters backward from a pair of nucleons at the nuclear surface with the rest of the nucleus acting as a spectator. Charge exchange may also take place. A total cross section for reaction (5.4) with the deuteron recoiling forward was found from the data of Azhgirei et al. as communicated by Komarov (1974) and others (Coleman et al., 1966; Bunker et al., 1968; Dubal et al., 1973) to be:

$$\sigma(NA_1 \rightarrow NA^1_d) = 7.51 \sigma(pd \rightarrow dp)$$

(5.9)

where $A_1$ is an air nucleus.

The resultant cross sections for reactions (5.3) and (5.4) as a function of energy were inserted separately into an atmospheric cascade calculation adapted from that discussed in Appendix III. These calculations included deuteron ionization energy loss as well as deuteron removal by interaction with air nuclei.

The results for 747 gm/cm$^2$ atmospheric depth are shown as the solid curves in Fig. 31 with quasi-free production, reaction (5.3), yielding curve b, and quasi-elastic scattering, reaction (5.4), yielding curve a.
Curve c in Fig. 31 represents the sum of these two processes. The measured deuteron vertical differential momentum spectrum seems reasonably well accounted for by the combination of reactions (5.3) and (5.4). Other possible processes such as pick up reactions, nuclear fragment evaporation processes and direct nuclear reactions are not expected to contribute significantly in this momentum region.

Ashton, Edwards, and Kelly (1970) have published experimental results for the vertical intensity of deuterons in sea level secondary cosmic rays, but have found their results at variance with the predictions of a simple one-dimensional diffusion equation calculation which assumed deuteron production by reaction (5.3). The two data points of Ashton et al. (1970) for deuterons are shown in Fig. 32 together with the prediction of our atmospheric cascade calculation for the sea level deuteron vertical momentum spectrum from the sum of reactions (5.3) and (5.4) solid curve. The lower momentum point is only about $1\frac{1}{2}\sigma$ above the solid curve so that agreement between our atmospheric cascade calculation and Ashton et al.'s (1970) experimental results seem acceptable. The discrepancies between the calculation of Ashton et al. (1970) and the one considered here that account for the different interpretation are that their calculation did not include reaction (5.4) or the effect of nucleon Fermi motion in reaction (5.3).

It is felt that the origins of the deuteron cosmic ray secondary component in the 0.5 to 2.0 GeV/c range are well understood.

Beauchamp et al. (1972) have pointed out that deuterons of momentum near 1 GeV/c constitute a significant background source for
Fig. 32. Vertical Differential Deuteron Momentum Spectrum at Sea Level.

Data is from Ashton et al. (1970). Solid curve is the result of the atmospheric cascade calculation described in the text.
their charge sensitive cosmic ray quark search for assumed 4/3 charged quarks. This should also be true for other experiments of the same genre. A precise understanding of the atmospheric deuteron secondary spectrum may allow better upper limits to be set in these cosmic ray quark searches.

\[ ^{4}\text{He}, \quad ^{3}\text{He} \quad \text{and} \quad ^{3}\text{H} \]

The number of \(^{4}\text{He}\) (alpha particles) in each velocity interval can be obtained by adding the number of particles in the peaks near \(M/|Q M_p| = 2\) for \(Q = +2\) in Figs. 20-23 after subtraction of the background from \(Q = -2\) and spillover from the \(Q = +1\) deuteron peak due to error in the charge determination. The number of spillover events, column four of Table 8, is determined by multiplying the ratio of the number of proton spillover events to the number of events in the proton peak by the number of deuterons in the deuteron peak. The resultant number of alpha particles obtained for two momentum intervals are shown in column five of Table 8. A statistical analysis showed that the probability of obtaining the 10 candidate alpha particles in the 1.39 GeV/c to 2.15 GeV/c momentum interval from a Poisson distributed spillover distribution with a mean of 4.0 was \(8.13 \times 10^{-3}\). The analogous probability for the 2.15 GeV/c to 3.65 GeV/c momentum interval was \(4.74 \times 10^{-2}\). The assumption that alpha particles are present seems justified. The alpha particle vertical differential momentum intensity was found from Eq. (5.2) in an exactly similar manner as was done for deuterons and the results are shown in column six of Table 8.
Table 8. Predicted and Measured Intensities for Light Nuclei at Mountain Altitude (747 gm/cm²).

<table>
<thead>
<tr>
<th>Light Nucleus</th>
<th>Velocity Interval</th>
<th>Momentum Interval (GeV/c)</th>
<th>Number of Spillover Events</th>
<th>Number Observed†</th>
<th>Intensity Observed (cm⁻²sec⁻¹sr⁻¹(GeV/c)⁻¹)</th>
<th>Intensity Predicted by ATMMC</th>
</tr>
</thead>
<tbody>
<tr>
<td>³H</td>
<td>0.35&lt;β≤0.70</td>
<td>1.05 - 2.76</td>
<td>0</td>
<td>≤2</td>
<td>≤2.51x10⁻⁶*</td>
<td>4.98x10⁻⁸ (a,b)</td>
</tr>
<tr>
<td>³He</td>
<td>0.35&lt;β≤0.70</td>
<td>1.05 - 2.76</td>
<td>0</td>
<td>≤2</td>
<td>≤2.51x10⁻⁶*</td>
<td>1.42x10⁻⁸ (a,b)</td>
</tr>
<tr>
<td>⁴He</td>
<td>0.35&lt;β≤0.60</td>
<td>1.39 - 2.15</td>
<td>4.0</td>
<td>6</td>
<td>4.77±2.54x10⁻⁶</td>
<td>1.60x10⁻⁸ (b)</td>
</tr>
<tr>
<td>⁴He</td>
<td>0.60&lt;β≤0.70</td>
<td>2.15 - 3.65</td>
<td>0.8</td>
<td>2</td>
<td>7.43±6.32x10⁻⁶</td>
<td>3.40x10⁻⁹ (b)</td>
</tr>
</tbody>
</table>

* 90% Confidence Upper Limit
† After Subtraction of Number of Spillover Events

References used for cross sections:
(a) Bhasin and Duck (1973), Perdrisat et al. (1973)

All predictions assume production by reactions of Type (5.10) and (5.11) only.
No peaks were seen in Figs. 20-23 corresponding to $^3\text{He}$ near $M/|QM_p| = 1.5$ in $Q = +2$ or $^3\text{H}$ near $M/|QM_p| = 3.0$ in $Q = +1$. The upper limit in each case for $0.20 < \beta < 0.70$ is two events.

In order to find a 90% confidence upper limit in the case of a small number $N_1$ of candidates, one first assumes Poisson statistics, then finds a mean number of events $N_2$, such that there is a 10% chance of finding $N_1$ or fewer candidates. For $N_1 = 2$, one finds $N_2 = 5.3$. $N_2$ is then the number used in Eq. (5.2) to find the upper limit of the intensity. The 90% confidence limits for the $^3\text{He}$ and $^3\text{H}$ vertical momentum intensity at 747 gm/cm$^2$ so obtained are shown in column six of Table 8.

Quasi-free production and quasi-elastic scattering processes similar to those invoked to account for deuteron production should be important for the production of $^4\text{He}$, $^3\text{He}$ and $^3\text{H}$ in secondary cosmic rays in the momentum region accessible to this experiment. These reactions should have the form

$$N + A \rightarrow A' + a + \pi \quad \text{(quasi-free production)} \quad (5.10)$$
$$N + A \rightarrow A'' + a + N' \quad \text{(quasi-elastic scattering)} \quad (5.11)$$

where $N$ and $N'$ are nucleons, $A$ is the target air nucleus, $a$ is the $^4\text{He}$, $^3\text{He}$ or $^3\text{H}$ produced, and $A'$ and $A''$ are the residual spectator nuclei. The cluster of peripheral nucleons which is the target in reactions (5.10) and (5.11) is not explicitly noted [it must be $a$ in reaction (5.11)]. In the case of quasi-free production and charge exchange quasi-elastic scattering, the identity of the target cluster is easily found by noting
the charge states and baryon numbers of incoming and outgoing particles and requiring both charge and baryon number conservation.

The cross sections for the various quasi-elastic scattering reactions were found in a similar manner to the analogous deuteron quasi-elastic scattering reactions using the results of Komarov et al. (1970, 1971) and Komarov (1974). For quasi-free production of $^4\text{He}$, $^3\text{He}$, and $^3\text{H}$ the total cross section was assumed to have the same shape as reaction (5.3) as a function of $q = E - E_T$ where $E$ is the center of mass energy and $E_T$ is the center of mass energy at the pion production threshold. This is somewhat analogous to the Gaisser and Halzen (1975) type scaling discussed in Chapter 1, except here the absolute magnitude of the cross section was scaled to the available data for pion production with light nuclei as targets (Akimov et al., 1962; Bhasin and Duck, 1973; Perdrisat et al., 1973) and the data for quasi-free production (Dzhelepov, 1970; Komarov et al., 1970; Komarov, 1974). The quasi-free production cross sections for $^3\text{He}$ and $^3\text{H}$ were assumed identical. The cross sections were then inserted into the atmospheric cascade calculation ATMMC and a separate differential momentum spectrum found for production of $^3\text{He}$, $^3\text{H}$ and $^4\text{He}$ by each of reactions (5.10) and (5.11). In all cases, ionization energy loss and removal of particles by collision with air nuclei was included. The quasi-elastic scattering and quasi-free production results were then added yielding a prediction for the vertical differential momentum intensity in the appropriate momentum interval for each particle type which is shown in column seven of Table 8. Table 8 also includes the references in each case from which the cross sections
were obtained. Only one data point which represented a point far out
on the tail of the cross section curve was available to scale the cross
section for quasi-free production for $^4$He. This suggests that the pre-
diction for $^4$He in Table 8 be considered somewhat uncertain. In light
of the discrepancy between measured and predicted intensity for $^4$He in
Table 8, it seems probable that some other process or processes are
responsible for the production of $^4$He in secondary cosmic rays. Since
the predicted intensities of $^3$He, $^3$H and $^4$He are so low, they might best
be considered lower limits. Alternative processes which might be impor-
tant at these momenta and intensity levels are direct nuclear reactions,
evaporation of fragments from excited nuclei, and possibly survival of
spallation produced fragments of heavier nuclei in the primary flux.
If they are considered as upper limits, the predictions in Table 8 for
$^3$He and $^3$H intensity at mountain altitude indicate that improvements to
the sensitivity of this spectrometer and a longer running time should
result in the observation of these components. More data of $^4$He would
also be useful in determining its spectrum.

**K+ Mesons**

Charged K-mesons are massive enough ($M = 0.494 \text{ GeV}/c^2 = M_p/2$) to
be detected in this experiment. However, their mean lives are short
enough that most charged K-mesons entering the spectrometer would decay
before completing their transit of it. If $\tau$ is the rest mean life of a
charged K-meson and $\beta$, its velocity in the laboratory system, the
probability of its surviving transit of the detector is
\[ \mathcal{P} = e^{-Z_1/\beta \gamma CT} \] (5.12)

where \( \gamma = (1 - \beta^2)^{-\frac{1}{2}} \) and \( Z_1 \) is the length of the spectrometer.

Since the highest probability of survival is for the largest \( \beta \gamma \), the most probable place to find charged K-mesons is in the \( Q = +1 \) and \( Q = -1 \) distributions of Fig. 28 with velocity interval \( 0.90 < \beta < 0.95 \) near \( M/|QM_p| = 0.5 \). In this region there are \( N_+ = 35 \) events with an estimated background of \( B_+ = 27 \) events in \( Q = +1 \) and \( N_- = 38 \) events, with an estimated background of \( B_- = 30 \) events in \( Q = -1 \). Although in each case, \( B_\pm \) is approximately one standard deviation below \( N_\pm \), this is not enough to strongly indicate the presence of charged K-mesons. If the Gaussian approximation to Poisson statistics is assumed, the 90% confidence upper limit can be found from \( N_{K^\pm} = X_\pm - B_\pm \) where \( X_\pm \) is the mean of a Gaussian distribution for which \( N_\pm \) or less is obtained with only 10% probability. The values of \( N_{K^\pm} \) obtained for Fig. 28 are shown in column three of Table 9. These values of \( N_{K^\pm} \) may then be used to find the 90% confidence upper limit for the \( K^\pm \)-meson vertical differential momentum intensity from an adaptation of Eq. (5.2):

\[ I_{K^\pm} = \frac{f N_{K^\pm}}{\eta \Delta t \Delta p \Delta \Omega(p)} \frac{1}{\mathcal{P}} \] (5.13)

where \( P \) can be found from Eq. (5.12).

If the length of the spectrometer is assumed to be the length of TOF1, \( z_1 = 720 \text{cm} \) and the mean life of a charged K-meson is \( \tau = 1.24 \times 10^{-8} \text{ sec} \) (Barash-Schmidt et al., 1974) and the average velocity is \( \beta = 0.925 \) for Fig. 32, then the probability of survival becomes
Table 9. Intensity Upper Limits for Various Particles at Mountain Altitude (747 gm/cm²).

<table>
<thead>
<tr>
<th>Intensity Particle</th>
<th>Velocity Interval</th>
<th>Momentum Interval (GeV/c)</th>
<th>Number of Candidates</th>
<th>Intensity Upper Limit (cm⁻²sec⁻¹sr⁻¹(GeV/c)⁻¹)</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>0.20&lt;β≤0.70</td>
<td>0.19 - 0.92</td>
<td>≤ 2</td>
<td>≤ 1.08 x 10⁻⁵</td>
</tr>
<tr>
<td>K⁺</td>
<td>0.90&lt;β≤0.95</td>
<td>1.02 - 1.50</td>
<td>≤ 15</td>
<td>≤ 4.59 x 10⁻⁵</td>
</tr>
<tr>
<td>K⁻</td>
<td>0.90&lt;β≤0.95</td>
<td>1.02 - 1.50</td>
<td>≤ 17</td>
<td>≤ 5.01 x 10⁻⁵</td>
</tr>
<tr>
<td>4M_p ≤ M ≤ 10M_p</td>
<td>0.20&lt;β≤0.70</td>
<td>0.77 - 3.68*</td>
<td>≤ 3</td>
<td>≤ 1.42 x 10⁻⁶ *</td>
</tr>
<tr>
<td>4M_p ≤ M ≤ 10M_p</td>
<td>0.20&lt;β≤0.70</td>
<td>0.77 - 3.68*</td>
<td>≤ 2</td>
<td>≤ 1.13 x 10⁻⁶ *</td>
</tr>
<tr>
<td>3M_p ≤ M ≤ 10M_p</td>
<td>0.20&lt;β≤0.70</td>
<td>0.57 - 2.76*</td>
<td>≤ 1</td>
<td>≤ 1.06 x 10⁻⁶ *</td>
</tr>
<tr>
<td>3M_p ≤ M ≤ 10M_p</td>
<td>0.20&lt;β≤0.70</td>
<td>0.57 - 2.76*</td>
<td>0</td>
<td>≤ 6.28 x 10⁻⁷ *</td>
</tr>
</tbody>
</table>

* Lower mass limit used in these calculations.
p = 0.452. The 90% confidence upper limits obtained for charged K-mesons from Eq. (5.13) is shown in column four of Table 9.

From Ashton et al., (1970) the ratio of charged π-mesons to protons at sea level is approximately 3% near 1 GeV/c. If the ratio of charged K-mesons to π-mesons is assumed to be 4% (Feinberg, 1972, p. 362) then the ratio of charged K-mesons to protons at mountain altitude might be expected to be near \(1.2 \times 10^{-3}\) near 1 GeV/c or for

\[ I_p \approx 1.3 \times 10^{-3} \text{cm}^{-2} \text{sec}^{-1} \text{sr}^{-1} (\text{GeV/c})^{-1} \]  (Fig. 30), \( I_K \approx 10^{-6} \text{cm}^{-2} \text{sec}^{-1} \text{sr}^{-1} (\text{GeV/c})^{-1}. \)

Intensities of this magnitude should be accessible to this experiment with better background suppression and a longer running time.

However, since charged K-mesons have so short a mean life, most of them should originate in the air only a few meters above the detector. Also, since the amount of production is dependent upon the amount of mass in the production region, the matter composing the upper parts of the spectrometer and the roof of the building above it are expected to dominate the production of charged K-mesons by perhaps an order of magnitude over those originating in the air, for low momentum. However, other charged particles produced in these interactions should also pass through the detector, decreasing the chance that these events will satisfy the criteria for an acceptable event. These considerations also limit the observability of charged π-mesons which have a rest frame mean life of \(\tau = 2.60 \times 10^{-8} \text{ sec} \) (Barash-Schmidt, 1974), only about twice that of the charged K-meson.
Massive Particles

No obvious peaks or clusters of events which might be due to new particles appear in Figs. 18-23. Since the region of mass below twice the proton mass has been studied extensively, the most physically interesting region in this experiment is \(4.0 \leq M/|QM| \leq 10.0\) for \(Q = \pm 1\), and perhaps \(3.0 \leq M/|QM| \leq 10.0\) for \(Q = \pm 2\) for \(0.20 \leq \beta \leq 0.70\) (Figs. 18-23) where the background is low. In the regions specified, there are two candidate events for \(Q = -1\), three for \(Q = +1\), one for \(Q = +2\) and none for \(Q = -2\). Upper limits may then be calculated under the assumption that each of the above sets of events is due to a single type of particle whose mass peak is smeared over the entire interval, a case which is somewhat worse than expected, (see Appendix II).

Using the Poisson statistics method described in the section on \(^4\text{He}, ^3\text{He}\) and \(^3\text{H}\), the 90% confidence upper limit to the vertical differential momentum intensity for each of the above cases may be found from Eq. (5.2) where the momentum is found from assuming the lowest mass in the interval. The results are shown in Table 9. Considerably better limits might be expected with an improved experiment of this type and a much longer data accumulation time.

A particularly interesting case to consider is that of super-dense nuclei discussed in Chapter 1. As already noted, \(Q/M = 1/2\) should be true in this case so that these particles should appear at the deuteron mass since the momentum determination is proportional to \(Q\) [see Eq. (3.2)].
However, since $Q$ should be large, a superdense nucleus should produce large pulses in all the counters.

SUMX was changed so that all events with mass $M \geq 1.25$ GeV/c$^2$ were printed out together with subsidiary information. This data was then scanned visually for events with large $S1$ corrected pulse heights and large pulses in the range counters ($S4$, $S5$, and $S6$). No candidate events were found. Since presumably any superdense nuclei present would have been seen, a 90% confidence upper limit to the integral intensity (subject to the time of flight triggering limitation; $0.20 < \beta < 0.95$) may be found utilizing the Poisson statistics method and an equation similar to Eq. (5.2).

$$I_{SD} = \frac{f N_{SD}}{n \Delta t \Delta \Omega(\infty)}$$

(5.14)

The result is found to be $I_{SD} < 1.16 \times 10^{-6} \text{cm}^{-2} \text{sec}^{-1} \text{sr}^{-1}$ with $0.20 < \beta < 0.95$.

**Antiprotons**

Inspection of Figs. 18 to 23 for $Q = -1$ reveals two low momentum candidates for antiproton events. Comparison with adjacent mass regions discloses that these are probably not significant. Using the Poisson statistics method and Eq. (5.2), a 90% confidence upper limit to the vertical differential momentum intensity for antiprotons can be found. The result is shown in Table 9, and as the upper limit in Fig. 33.

Since the possible presence of a primary antiproton flux is a matter of great astrophysical importance (Wayland and Bowen, 1968;
Fig. 33. Upper Limit and Predicted Vertical Differential Momentum Intensity for Anti Protons at Mountain Altitude (747 gm/cm$^2$).
Solid curves are from: (a) atmospheric secondary production, (b) hypothetical primary flux ($I_{\bar{p}}/I_p = 5 \times 10^{-3}$).
Shen and Berkey, 1968; Badhwar and Golden, 1974; Gaisser and Levy, 1974; Ganguli and Sreekantan, 1975) it is worthwhile to consider predictions of the atmospheric cascade calculation for antiprotons of both primary and secondary origin.

The atmospheric cascade calculation considers production of secondary antiprotons in collisions of secondary cosmic ray nucleons with air nuclei. The production cross section for production of either an antiproton or an antineutron was taken as $A^{2/3}$ times the cross section for production of antiprotons in proton-proton collisions, where $A$ is the mass of an air nucleus in A.M.U. and the $A^{2/3}$ factor corresponds to production with shielding by nucleons. The cross section used for antiproton production in proton-proton collisions was the same as that used by Badhwar and Golden (1974). Antinucleon inelastic collisions with nuclei have two possible consequences; inelastic nucleon scattering except that a forward scattered antinucleon survives the interaction and annihilation in which the final state has no antibaryons. Both of these scattering processes were considered for the propagation of antinucleons in the atmosphere using the data compiled by Bracci et al. (1973) and Enstrom et al. (1972). The resulting predicted antiproton vertical differential momentum spectrum at mountain altitude (747 gm/cm$^2$) due to atmospheric production is shown in Fig. 33 as curve a.

The best upper limit for primary antiproton intensity as a fraction of primary proton intensity is $I_{\bar{p}}/I_p \leq 8 \times 10^{-3}$ due to Bogomolov,
Lubyanaya, and Romanov (1971). Curve b in Fig. 33 represents the predicted intensity of antiprotons at mountain altitude \((747\text{g/m/cm}^2)\) which would result from a primary antiproton spectrum having the same spectral shape as the primary proton spectrum but reduced in intensity by a factor of \(5\times10^{-3}\). Curve b represents the result of a primary antiproton component only so that in this case the total spectrum would be the sum of curves a and b in Fig. 33. Proton and antiproton primary spectra might be expected to be the same shape if the galaxy contained significant amounts of antimatter assuming a galactic origin for primary cosmic rays in the 1 to 10 GeV/c region.

More than three orders of magnitude separate the present experimental upper limit, Fig. 33, from the lowest expected intensity of antiprotons at mountain altitude, Fig. 33b. To bridge this gap will require great effort but will be rewarded by the ability to set more precise limits on the presence of primary antiprotons when mountain altitude antiproton intensity upper limits approach \(10^{-8} \text{ cm}^{-2} \text{ sec}^{-1} \text{ sr}^{-1} \text{(GeV/c)}^{-1}\). The background in this experiment which is mainly due to false triggers from multiparticle events with one particle decayed can be greatly suppressed by simple changes in the triggering requirements. Since the data considered here represents only about one week of operation, much longer operation times would be possible. A considerable increase in sensitivity of the spectrometer is expected from improvements to it already completed in anticipation of a second experiment (Jones et al., 1975) and it is planned to increase the range of velocity sensitivity using a Cerenkov counter of novel design (Bowen et al., 1976). An attempt to measure
the antiproton intensity at mountain altitude should be a major part of a second experiment using this spectrometer.

Epilogue

The author feels that this technique shows great promise for future cosmic ray investigations, both ground based and above the atmosphere. Although many techniques achieve higher sensitivity, they do so only by the sacrifice of much information about individual events or by greatly limiting the regions of energy or particle properties to which they are sensitive. The very richness and diversity of phenomena evident in the primary and secondary cosmic radiation makes any assumptions about it in the absence of direct information dangerous. For this reason, cosmic ray physics, although it has been a gold mine of information for both astrophysics and elementary particle physics, has often suffered from conflicting results and a plethora of retracted claims. There can be no substitute for specifying the charge mass and momentum, of each charged particle passing through the detector, such as the technique of mass spectrometry provides.
APPENDIX I

AREA TIMES SOLID ANGLE ACCEPTANCE

In order to determine the absolute intensity of cosmic rays seen by a given detector its acceptance for incoming particles in terms of the area of the detector and the solid angle viewed by it must be determined. Since this mass spectrometer employs a magnet which may bend a particle trajectory away from a detector it might otherwise trigger or cause a particle to trigger a detector it might otherwise have missed, the acceptance will depend upon the bending of the trajectory and hence will in general be a function of the momentum. From Eq. (3.2), the bending for a singly charged particle is approximately:

\[ \theta = \frac{\int B \, dl}{p} \]  

(A1.1)

where the field integral, \( \int B \, dl \), depends on the trajectory. Although the magnetic field, \( B \), is a complicated function of position it is linearly dependent on \( I \), the magnet current, and therefore the bend angle, \( \theta \), scales as the ratio of the magnet current to momentum (\( I/p \)). Since the spectrometer acceptance depends on \( \theta \) only it also should scale as \( I/p \). Thus the acceptance need only be determined for one magnet current \( I \) and all \( p \) for maximum generality. It should also be noted that for given \( I \), \( \theta \) decreases for increasing momentum so that in the limit of high momentum the spectrometer appears as simply an array of
detectors, the same as the zero field case. The zero field case is considered first.

Although exact calculations with analytic solutions exist for the area solid angle acceptance of an array of parallel counters of rectangular cross section (Heristchi, 1967), we are unfortunate in having two counters, S1 and S3 of octagonal cross section. Exact calculations of this sort involve complicated integrations and are extremely difficult for anything but the simplest geometries. Fortunately, Monte Carlo calculations are well suited for this purpose. The Monte Carlo program used was adapted from one written by Crannell and Ormes (1971), also Hemmer and Crannell (1973), which was designed for an arbitrary number of parallel detectors of rectangular cross section. In this program a random number generator was used to find the \( x \) and \( y \) position of a particle in the topmost detector and two polar angles \( \theta \) and \( \phi \) to specify its direction of motion. This particle was then projected downward in a straight trajectory through a set of prisms whose bottom face was in each case the next lower counter and whose sides were the trapezoids formed by connecting the vertices of the upper counter with the vertices of the counter below it. By determining if the trajectory fell inside or outside of one or more of the sides of the lower detector, the face of the prism through which the particle exited could be determined. For each detector histograms of the positions and angles of particles hitting the detector were accumulated as well as the number that missed the detector for each of the other four faces of the prism. This program, AOMEGA, was adapted to the University of Arizona Computer Center's
CDC-6400 and tested for a simple three parallel rectangular detector array whose area times solid angle acceptance could be calculated analytically. After an acceptable comparison was obtained, AOMEGA was adapted to our spectrometer's detector configuration.

Consider two superimposed squares of equal size. If one square is rotated by an angle of 45° about an axis through the common centers of the squares the area they now share in common, the intersection, is a regular octagon. In AOMEGA the top detector of our spectrometer, counter S1, which has octagonal cross section was replaced by two parallel square detectors of appropriate size separated by 1 cm. After a particle's position and direction were chosen its coordinate system was rotated 45° and the program then required it to hit the acceptable region of the second square 1 cm below. If the particle survived the coordinate system was rotated 45° back to the original orientation and the Monte Carlo procedure continued as before. All other detectors and clearances such as the magnet fiducial region were representable by parallel rectangular detectors of various sizes, except S3, which was octagonal and was treated in the same manner as S1. In this way, the area times solid angle acceptance for zero field was found to be

\[ A\Omega_{p\rightarrow\infty} = 32.32 \pm 0.44 \text{ cm}^2 \text{ sr}, \]

where the error is statistical. The histograms for each detector were carefully checked to verify the behavior of the program.

In order to check the results of AOMEGA, another program AOM was used which divided two of the spectrometer's detectors, counter S2 and the bottom magnet window, into 400 small equal rectangular elements
each and then required the line between the centers of any two elements, one in the top detector and one in the bottom detector to pass through all the detectors. A clearance was replaced by a fictitious detector of the same dimensions. The area times solid angle acceptance determined in this way is given by

\[ A\Omega' = \frac{N \Delta A_1 \Delta A_2}{R^2 \cos^2 \theta} \]

where \( \Delta A_1 \) and \( \Delta A_2 \) are the areas of the small rectangular elements of the detectors used, \( N \) is the number of pairs that passed the criteria and \( R \cos \theta \) is the distance between the two elements of the two detectors used. The result obtained from AOM for our detector was 33.7 cm\(^2\) sr which compares well with the result of AOMEGA.

A field tracing routine similar to BEND was then incorporated into AOMEGA. Each particle was traced through the field region while being required to remain within the magnet fiducial region. As before an acceptable particle trajectory was required to penetrate all detectors. It should be noted that particles with trajectories bending outside of the magnet fiducial region but penetrating all detectors were rejected by AOMEGA as were real events of the same characteristics by BEND.

AOMEGA was then run for 10 momenta from .3 GeV/c to 200 GeV/c with an assumed magnet current \( I = 100 \) amps. The result at the lowest momentum 0.3 GeV/c was nearly zero whereas at the highest momentum 200 GeV/c it was 31.9\( \pm \)1.3 cm\(^2\) sr, agreeing within statistics with the zero field or infinite momentum result. The plotted result for area times solid angle acceptance as a function of momentum, \( A\Omega(p) \), is shown
in Fig. 29, page 82, where the errors are statistical and a reasonable curve is drawn in. With $10^5$ particles per momentum, computation costs limited both the number of momenta considered and the statistical error at each momentum. In Fig. 29, the values for AOM and AOMEGA are shown on the ordinate with the asymptote at the AOMEGA value.
APPENDIX II

MASS RESOLUTION DETERMINATION

In order to be sure that the operation of the experiment is well understood, it is worthwhile to calculate the dependence of the mass resolution on velocity, momentum and mass, and compare this to the observed widths of the proton peaks.

From Eqs. (4.5) and (4.6), the mass is given by:

\[ M = \frac{p}{\beta Y}, \quad Y = (1 - \beta^2)^{-\frac{1}{2}} \]  

(A2.1)

where the mass, \( M \), is in GeV/c\(^2\), the momentum, \( p \), is in GeV/c, and the velocity, \( \beta \), is determined from the time of flight, \( \tau \), by:

\[ \beta = \frac{Z}{Z + 29.98(\tau - \tau_A + \tau_B)} \]  

(A2.2)

\( Z \) is the distance in cm between the time of flight counters A and B, corrected for trajectory, and \( \tau_A \) and \( \tau_B \) are the light transit times in counters A and B respectively, with all times in counters given in nanoseconds. The errors in determination of the light transit time and trajectory corrections are small compared to the measurement error in \( \tau \), the time of flight.

The momentum is determined from the bend angle in the particle trajectory and may be approximated by Eq. (4.1)
\[ p = \frac{0.07}{\theta} \text{ GeV/c} \]  

where \( \theta \) is in radians, for a magnet current of 100 amps.

The standard deviation of the mass measurement, the mass resolution, due to independent random errors in the determination of bend angle and time of flight may be found from:

\[ \sigma_m^2 = (\frac{\partial M}{\partial \theta})^2 \sigma_\theta^2 + (\frac{\partial M}{\partial \tau})^2 \sigma_\tau^2 \]  

Using Eqs. (A2.1), (A2.2), and (A2.3), we find

\[ \frac{\sigma_m^2}{M^2} = T^2 p^2 \sigma_\theta^2 + L^2 \beta^2 \gamma^4 \sigma_\tau^2 \]  

where \( T = 14.3 \text{ (GeV/c)}^{-1} \), \( L = \frac{29.98}{2} \text{ ns}^{-1} \) (\( L = 4.17 \times 10^{-2} \text{ ns}^{-1} \) for time of flight 1), and \( \sigma_\theta \) and \( \sigma_\tau \) are the standard deviations of measurements of the bend angle and the time of flight respectively.

Since as explained in Chapter 4 and Appendix V, the velocity is determined from time of flight 1 alone, \( \tau \) is the difference between the triggering times of two counters S1 and S3 respectively:

\[ \tau = \tau_3 - \tau_1 \]  

For independent random errors in \( \tau_1 \) and \( \tau_3 \), the standard deviation of the time of flight measurement becomes:

\[ \sigma_\tau^2 = \sigma_{\tau_3}^2 \sigma_{\tau_1}^2 \]
The bend angle error arises from two sources, the errors in trajectory position measurement due mostly to the finite spark chamber resolution and small deflections of the trajectory due to multiple Coulomb scattering in the material within the trajectory analyzing region of the spectrometer. Since these two types of errors are random and independent of each other, the standard deviation of the bend angle determination is given by:

\[ \sigma_\theta^2 = \sigma_{\theta_{MS}}^2 + \delta_{\theta_R}^2 \]  \hspace{1cm} (A2.8)

where \( \sigma_{\theta_R} \) and \( \sigma_{\theta_{MS}} \) are the standard deviations of measurement of bend angle due to the spark chamber resolution and multiple scattering respectively.

The spark chamber resolution has been considered in Chapter 4 where the bend angle distribution for \( v=c \) magnetic field off data of Fig. 8, page 42, was used to find the maximum detectable momentum, MDM, from the condition

\[ \sigma_{\theta_R} = \frac{0.07}{\text{MDM}} \]  \hspace{1cm} [see Eq. (A2.3)] \hspace{1cm} (A2.9)

The maximum detectable momentum obtained in Chapter 4 was MDM = 31.3 GeV/c.

Multiple Coulomb scattering in a layer of material of thickness \( L \) and radiation length \( L_R \) for a particle of velocity \( \beta \), charge \( Q \), and momentum \( p \) GeV/c has an rms projected scattering angle given by (Perkins, 1972, p. 33).
\[ \theta_{\text{RMS}} = \frac{Q(0.015)}{p} \beta \sqrt{\frac{L}{L_R}} \]  \hspace{1cm} (A2.10)

From the known characteristics of the spectrometer, the rms projected scattering angle may be found for each piece of matter in the beam path. The scattering angles then add in quadrature after being weighted by a factor \( \alpha_i = \frac{Z - Z_i}{Z} \) where \( Z_i \) is the distance from the center of the magnet to the piece of matter being considered and \( Z \) is the distance from the center of the magnet to either of the largest spark chambers. The result of this calculation gives the standard deviation of the bend angle due to multiple scattering as

\[ \sigma_{\theta, \text{MS}} = \frac{Q}{p \beta} \frac{(1.62 \times 10^{-3})}{(\text{GeV}/c)^{-1}} \]  \hspace{1cm} (A2.11)

From Equations (A2.1), (A2.5), (A2.8), (A2.9) and (A2.11) we can now represent the mass resolution by:

\[ \frac{\sigma M^2}{M^2} = \frac{a(1)}{\beta^2} Q^2 + a(2) \beta^2 \gamma^2 M^2 + a(3) \beta^2 \gamma^4 \]  \hspace{1cm} (A2.12)

where \( a(1) \), \( a(2) \) and \( a(3) \), the coefficients of the multiple scattering, chamber resolution and time of flight resolution terms, respectively, calculated from preceding treatment are shown in column three of Table 10.

If the mass dependent second term on the right-hand side of Eq. (A2.12) is ignored, it may be seen that for constant velocity, \( \beta \), and charge, \( Q \), the right-hand side of Eq. (A2.12) is constant. Therefore to first order the mass resolution or width of the mass peak is proportional to the mass. This is the reason that the mass histograms of Figs. 19-28 pages 71-78, were plotted on a logarithmic mass scale.
Table 10. Comparison of Mass Resolution Coefficients Determined by Calculation and by Fitting the Proton Mass Peak Widths to Equations (4.6) and (4.7).

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Term Origin</th>
<th>Calculation</th>
<th>Fit to Eq. (4.6)</th>
<th>Fit to Eq. (4.7)</th>
</tr>
</thead>
<tbody>
<tr>
<td>a(1)</td>
<td>Multiple Scattering</td>
<td>0.54x10^{-3}</td>
<td>1.06±0.14x10^{-3}</td>
<td>0.99±0.14x10^{-3}</td>
</tr>
<tr>
<td>a(2)</td>
<td>Maximum Detectable Momentum</td>
<td>1.02x10^{-3} (GeV/c^2)</td>
<td>3.07±1.89x10^{-3} (GeV/c^2)</td>
<td>2.44±1.89x10^{-3} (GeV/c^2)</td>
</tr>
<tr>
<td>a(3)</td>
<td>Time of Flight</td>
<td>2.81x10^{-3}</td>
<td>2.33±.59x10^{-3}</td>
<td>2.38±0.59x10^{-3}</td>
</tr>
<tr>
<td>$\chi^2$ of Fit</td>
<td>-------</td>
<td>-------</td>
<td>3.275</td>
<td>3.382</td>
</tr>
</tbody>
</table>
Using the logarithmic mass scale and the uncut data which was used to obtain the proton momentum spectrum in Chapter 5, a logarithmic standard deviation for the proton mass peak for each of ten intervals of velocity was found by a maximum likelihood fit of each peak to a Gaussian distribution. A standard deviation was then obtained for each peak and the results are the points shown in Fig. 34. The errors are statistical and the average velocity for each point was obtained from an average of proton velocities for each velocity interval within the histogramming program SUMX.

The solid curve in Fig. 34 is the result of a maximum likelihood fit of the mass resolution and velocity values above to Eq. (A2.12) to determine the values of $a(1)$, $a(2)$ and $a(3)$. The plotted values are $\sigma_M$, not $\sigma_{M/M}$ in Fig. 34. The fitted values of $a(1)$, $a(2)$ and $a(3)$ are shown in the fourth column of Table 10 with the error values determined from those of the experimental mass resolution by the fit.

Column five of Table 10 shows the result of adding a constant 2% error to the mass resolution corresponding to a mass resolution formula with an extra term in Eq. (5.13):

$$\frac{\sigma_M^2}{M^2} = \frac{a(1)Q^2}{b^2} + a(2)\beta^2\gamma^2M^2 + a(3)\beta^2\gamma^4 + 0.0004 \quad (A2.13)$$

The agreement between fitted (column five of Table 10) and calculated (column three of Table 10) values of the parameters for Eq. (A2.13) is somewhat better than a similar comparison for Eq. (A2.12)
Fig. 34. Mass Resolution.

Data is determined from widths of the proton mass peaks. The solid line is a maximum likelihood fit to Eq. AA2.13).
(columns four and three of Table 10). Notice that the values of $\chi^2$ for the fits of Eqs. 13 and 14, shown in the last row of Table 10 do not differ by very much. This result is corroborated by the fact that a plot of the mass resolution vs. velocity from Eq. (A2.13) is essentially indistinguishable from the plot for Eq. 13, shown in Fig. 34. This indicates that the fit is not very sensitive as a test of the parameters $a(1)$, $a(2)$, $a(3)$.

However, the agreement between the calculated and fitted parameters of Table 10 indicates that although a few percent residual error may remain unaccounted for, both the magnitude and functional dependence of the mass resolution seem well understood. Introduction of larger residual error up to 4.33% in a procedure similar to that above significantly worsened the fit. A residual error of 2% could easily result from the approximations used in finding the mass resolution itself in this analysis, as well as the many approximations made during data reduction, particularly in determining the magnet field decay and time of flight shift corrections. Since counters S1 and S3 are of identical construction, we have

$$\sigma_{t_3} = \sigma_{t_1} = \sigma_t$$

and thus

$$\sigma_t = \sqrt{2} \sigma_t$$

(A2.14)

where $\sigma_t$ is the standard deviation of the measurement of the time of passage of a particle as determined by either counter. The error indicated by $\sigma_t$ is mostly due to the photomultiplier tube as corrections for the light transit time in the counters have already been made.
APPENDIX III
ATMOSPHERIC NUCLEON CASCADE SIMULATION

In order to compare the results found by this experiment both to the results of previous experiments and to theoretical predictions it was necessary to develop a computer simulation, ATMMC, using Monte Carlo methods, of the atmospheric nucleon cascade which could model the nucleon flux at various altitudes and for various geomagnetic cutoff rigidities.

In agreement with Eq. (1.1), the integral primary proton momentum spectrum was taken as:

\[ I = \frac{1.14}{p^{1.75}} \text{ (cm}^{-2} \text{ sec}^{-1} \text{ sr}^{-1}) \]  \hspace{1cm} (A3.1)

for \( p \geq 10 \text{ GeV/c} \)

where \( p \) is in GeV/c. At lower momenta the spectral index was gradually lowered to 1.25 for \( P = 2 \text{ GeV/c} \). ATMMC considers a primary flux of protons only, and the calculations are one dimensional in that only the vertical flux of both primaries and secondaries is considered. Provision is also made for truncating the incoming flux at a given momentum corresponding to the vertical cut of rigidity for protons and rounding off the distribution near cut off.

The interaction cross section for nucleons with air nuclei was assumed independent of nucleon type, and was determined from the known data for nucleon-nucleon cross sections at various energies (Hayakawa,
1969, p. 177-8; Barash-Schmidt et al., 1974). For low incoming nucleon momentum, 0.1 GeV/c to 1.0 GeV/c, and average of the neutron-proton and proton-proton cross section was assumed, whereas above 2.0 GeV/c, the proton-proton inelastic cross section (Bracci et al., 1973) was used, with the two cross sections smoothly joined in the region 1.0 GeV/c to 2.0 GeV/c. This entire cross section was then scaled to an assumed nucleon air nucleus cross section of $\sigma_{NA} = 280 \text{ mb}$ at 10 GeV/c (Feinberg, 1972, p. 257; Sens, 1974, p. 160). ATMMC used a mean free path obtained from the cross section $\sigma_{NA}$ above by

$$\lambda = \frac{A}{N_0 \sigma_{NA}} \left( \text{g} \text{m}/\text{cm}^2 \right)$$

where $A = 14.5$ is the mean molecular weight of air and $N_0$ is Avogadro's number. A graph of the mean free path, $\lambda$, as a function of momentum is shown in Fig. 35a.

The nucleon-nucleus interaction is in general a complex event with a large number of possible final states. Fortunately, at high momenta the atmospheric nucleon cascade is dominated by the effects of the leading nucleon, the highest momentum nucleon produced in the collision, due to the steeply falling momentum spectra of the nucleons producing the collisons. To preserve as much generality as possible, the following simple model was adopted for nucleon-nucleus collisions. A forward scattered nucleon was assumed with momentum:

$$p = (1 - k) p_0 \quad 0 \leq k \leq 1$$

where $p_0$ is the momentum of the nucleon producing the collision and...
Fig. 35. Momentum Dependence of Various Parameters Used in ATMMC.

(a) Nucleon interaction mean free path,
(b) Probability of change of type,
(c) Inelasticity factor.
k is called the inelasticity. The inelasticity was then given by:

$$k = f_k (p_0) k_r$$  \hspace{1cm} (A3.4)

where $k_r$ is chosen randomly from the distribution in Table 11. (Feinberg, 1972, p. 257), and $f_k(p_0)$ was a momentum dependent factor fitted from published experimental data on cosmic ray nucleon intensities at various altitudes (see Table 12).

The leading particle after a collision is usually a nucleon, a nuclear excited state (resonance), or another baryon, but for ATMMC it was assumed to have already decayed into a final state proton or neutron.

The relative probability of finding a given type of nucleon in the final state depends upon the incoming nucleon type and the momentum. The probability of a change of nucleon type (charge change) in each interaction was taken as the same for both protons and neutrons and is shown as a function of momentum in Fig. 35b. The probability of a change of type remains high at high momentum due to the high probability of producing an excited state which then decays into several charged particles. Charge exchange dominates the probability of change of type at low momenta.

A recoil nucleon is also considered whose type is chosen randomly, and whose momentum is found from the kinematics assuming backward scattering in the center of mass and the availability of a random fraction of $k p_0$ the laboratory momentum lost by the incoming nucleon. Momentum loss due to ionization was also included for charged particles.
Table 11. Probability of Obtaining a Discrete Inelasticity Factor in Eq. (3.4)

<table>
<thead>
<tr>
<th>$K_T^*$</th>
<th>$P_T^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.15</td>
<td>0.033</td>
</tr>
<tr>
<td>0.25</td>
<td>0.067</td>
</tr>
<tr>
<td>0.35</td>
<td>0.167</td>
</tr>
<tr>
<td>0.45</td>
<td>0.233</td>
</tr>
<tr>
<td>0.55</td>
<td>0.233</td>
</tr>
<tr>
<td>0.65</td>
<td>0.167</td>
</tr>
<tr>
<td>0.75</td>
<td>0.067</td>
</tr>
<tr>
<td>0.85</td>
<td>0.033</td>
</tr>
</tbody>
</table>

$\sum P_T = 1.0$

$^* K_T = \text{Inelasticity Factor}$

$^* P_T = \text{Probability}$
Table 12. Experimental Data for Proton Differential Spectra Used to Adjust ATMMC.

<table>
<thead>
<tr>
<th>Altitude (m)</th>
<th>Atmospheric Depth (gm/cm²)</th>
<th>Vertical Cutoff Rigidity (G.V.)</th>
<th>Figure Number</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>9000</td>
<td>310</td>
<td>2.4 - 6.5 (?)</td>
<td>36</td>
<td>Baradzei et al. (1958)</td>
</tr>
<tr>
<td>5200</td>
<td>538</td>
<td>13.6</td>
<td>37</td>
<td>Allkofer and Kraft (1965)</td>
</tr>
<tr>
<td>3200</td>
<td>710</td>
<td>6.5</td>
<td>38</td>
<td>Kocharian et al. (1959)</td>
</tr>
<tr>
<td>2750</td>
<td>747</td>
<td>5.59</td>
<td>30</td>
<td>This experiment</td>
</tr>
<tr>
<td>Sea Level</td>
<td>1030</td>
<td>2.43</td>
<td>39</td>
<td>(a) Meshkovskii and Sokolov (1957)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(b) Diggory et al. (1974)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(c) Brooke and Wolfendale (1964)</td>
</tr>
</tbody>
</table>
The number of nucleons of each type for each of 30 momentum intervals were accumulated for each of five atmospheric depths: 310 gm/cm$^2$, 538 gm/cm$^2$, 710 gm/cm$^2$, 747 gm/cm$^2$ and 1030 gm/cm$^2$ (sea level). The atmospheric depth of this experiment was 747 gm/cm$^2$ and the other depths were chosen because published experimental data at these depths for the vertical proton momentum spectrum were available as listed in Table 12. The inelasticity parameter $f_k(p_o)$ was adjusted to obtain the best fit by eye of the nucleon cascade calculation results to the published experimental results. The resulting fit to the published results for the various altitudes shown in Figs. 36 to 39 is excellent except perhaps for Fig. 37. The data for Fig. 37 was taken by Allkofer and Kraft (1965) at a relatively large geomagnetic cutoff (13.56 gev).

Alpha particles of momentum 6.78 GeV/c per nucleon may enter the atmosphere vertically at this location where as protons need 13.56 GeV/c. Assuming an alpha to proton ratio of 0.043 at any given momentum per nucleon and the proton integral spectrum of Eq. (A3.1), a correction to the proton differential spectrum for these alpha particles, assuming that they break up entirely in the atmosphere, can be found. The proton differential momentum spectrum corrected for this effect is shown as the dashed curve in Fig. 37, which is a great improvement. This effect should be negligible for the lower geomagnetic cutoff's of the other results, Figs. 36, 38, and 39. The result obtained for $f_k(p_o)$ as a function of momentum is shown in Fig. 35c. The result obtained for $f_k(p_o)$ as a function of momentum is shown in Fig. 35c. This result which represents a constant average
Fig. 36. ATMMC Prediction for the Proton Differential Momentum Spectrum at 310 gm/cm$^2$.

Data from Baradzei et al. (1958).
Fig. 37. ATMMC Prediction for the Proton Differential Momentum Spectrum at 538 gm/cm².

Data from Allkofer and Kraft (1965). Dashed curve includes primary ⁴He.
Fig. 38. ATMMC Prediction for the Proton Differential Momentum Spectrum at 710 gm/cm².

Data from Kocharian et al. (1959).
Fig. 39. ATMMC Prediction for the Proton Differential Momentum Spectrum at 1030 gm/cm$^2$ (Sea Level).

(a) Meshkovskii and Sokolov (1957),
(b) Diggory et al. (1974),
(c) Brooke and Wolfendale (1964).
inelasticity of 0.5 above 2.0 GeV/c is entirely consistent with previous nucleon cascade simulations (Jabs, 1968; Pinkau, 1969; Hook and Turver, 1974) and accelerator data (Feinberg, 1972). The dip in the curve in Fig. 35c may be justified on theoretical grounds as due to the domination of nucleon-nucleon elastic scattering in the nucleon-nucleus interaction in this momentum region, with single scatterings in the nucleus predominating near 1 GeV/c and multiple scattering in a single nucleus predominating at very low momentum.

Not surprisingly, the resulting proton vertical momentum spectra at high momentum determined by ATMMC are relatively insensitive to modest variations of the parameters enumerated above. This is agreement with the fact that a relatively large number of atmospheric cascade simulations and calculations with differing parameters seem to achieve reasonable fits to the high momentum proton spectrum (Jabs, 1968 and 1972; Pinkau, 1969; O'Brien, 1971; Hook and Turver, 1974).

The complexity of ATMMC arose from the attempt to fit the turnover and low momentum region of the vertical proton intensity using what is known about nucleon-nucleus interactions at low momenta. The dominant feature of these low momentum interactions was the steeply rising nucleon-nucleon cross sections.
APPENDIX IV

MUON MOMENTUM SPECTRUM

In Chapter 4, a bend angle distribution and plot of charge ratio versus momentum was obtained using $v=c$ data and it was compared to the known charge ratio for muons at mountain altitudes. For this experiment mass peaks cannot be found for muons using the $v<c$ data since the muon bend angle at low velocity is larger than the maximum acceptable bend angle of the apparatus for the magnet field intensity chosen. Mass peaks cannot be found for the $v=c$ data, since of course the velocity is not precisely known. The $v=c$ data were expected to consist mostly of muons since a majority of this sample corresponded to minimum ionizing particles which had traversed 13 cm of iron. If the $v=c$ data that survive track fitting are assumed to be mostly muons, the vertical differential momentum spectrum of muons may be obtained. The $v=c$ data available consisted of 1505 triggers from run 22, a short run prior to the main $v<c$ data and a longer data set from runs 34 and 35, 14899 triggers, for which some timing data was not available. After BEND had been run on both sets of this data, the number of events in twelve momentum intervals for both positive and negative momenta were determined.

The muon vertical differential momentum intensity was then determined in a manner exactly similar to that of protons. From Eq. (5.1) we have:
\[
I_\mu = \frac{N_\mu \Delta p \Omega(p) \Delta t_s}{\eta} \quad (A4.1)
\]

\((\text{cm}^{-2} \text{ sec}^{-1} \text{ sr}^{-1} (\text{GeV/c})^{-1})\)

where \(N_\mu\) is the number of events in momentum interval \(\Delta p\) in GeV/c, \(\Omega(p)\) is the area solid angle acceptance for that momentum corrected for magnet field decay in cm\(^2\) sr, \(\eta\) is the spark chamber efficiency and \(\Delta t_s\) is the time interval in seconds corrected for readout dead time. The large data set consisting of runs 34 and 35 was used to determine the muon intensities with the shorter run 22 being used to scale the larger runs and provide \(\Delta t_s\). The statistical errors introduced by this scaling were considered in obtaining the final intensities and their errors. Shown in Figs. 40 and 41 are the resulting differential momentum spectra for positive and negative muons respectively. Since protons with velocity above the \(v=c\) cutoff should be a contaminant in the positive muon momentum spectrum of Fig. 40, the estimated proton differential momentum intensities above 2 GeV/c were subtracted from the muon differential momentum intensities to obtain the results in Fig. 40. The proton intensities were obtained by extrapolation from Fig. 30, page 84, and were corrected for cutoff effects due to the \(v=c\) triggering determination. The subtracted proton intensity never exceeded 8.6\% of the muon intensity, so this was a small correction. The solid curves in Figs. 40 and 41 are from Kocharian et al. (1956) and Kocharian et al. (1959) at an atmospheric depth of 710 gm/cm\(^2\) which is comparable to our 747 gm/cm\(^2\).
Fig. 40. $\mu^+$ Vertical Differential Momentum Spectrum at Mountain Altitude (747 gm/cm$^2$).

Solid Curve is from Kocharian et al. (1959).
Fig. 41. $\mu^-$ Vertical Differential Momentum Spectrum at Mountain Altitude (747 gm/cm$^2$).
Solid curve is from Kocharian et al. (1959).
The agreement is remarkable considering that no adjustment of our data was made to improve the fit. The discrepancy between the solid curve and our experimental points at low momentum is at least partially due to the cosmic ray secondary electron spectrum (Beuermann and Wibberrenz, 1968).

The further operation of this experiment at a lower magnetic field intensity as suggested in Chapter 5 would allow a muon mass peak determination and hence differential momentum intensity determination in the very low momentum region around 0.2 GeV/c free from contamination by electrons as well as to determine if the excess events below 1 GeV/c in Figs. 40 and 41 were due to experimental limitations such as the magnetic field decay effect on area solid angle product.
APPENDIX V

WEIGHTED ADDITION OF VELOCITIES

The two time of flight systems TOF1 and TOF2 produce two measurements of the particle velocity for each event, \( \beta_1 \) and \( \beta_2 \) respectively. The best estimate of the velocity is, in general, a weighted sum:

\[
\beta = \omega \beta_1 + (1-\omega) \beta_2
\]  
(A5.1)

where the weight, \( \omega \), is chosen so as to minimize the error in \( \beta \). The two measurements of the velocity are given by:

\[
\beta_1 = \frac{Z_1}{t_1 - t_{III}} \quad \text{and} \quad \beta_2 = \frac{Z_2}{t_1 - t_{II}}
\]  
(A5.2)

where \( Z_1 \) and \( Z_2 \) are the path lengths of TOF1 and TOF2 respectively, and \( t_1, t_{II}, \) and \( t_{III} \) are the times measured by the three scintillation counters, S1, S2 and S3 respectively. Note that both time of flights share counter S1 (\( t_1 \)) and hence their errors for velocity determination may be correlated.

The standard deviation of measurement of velocity \( \beta \) may be found from

\[
\sigma_\beta^2 = \left( \frac{\partial \beta}{\partial t_1} \right)^2 \sigma_1^2 + \left( \frac{\partial \beta}{\partial t_{II}} \right)^2 \sigma_{II}^2 + \left( \frac{\partial \beta}{\partial t_{III}} \right)^2 \sigma_{III}^2
\]  
(A5.3)

where \( \sigma_1, \sigma_{II}, \) and \( \sigma_{III} \) are the standard deviations of the measurements of times \( t_1, t_{II}, \) and \( t_{III} \) respectively. The partial derivatives
in Eq. (A5.3) may be calculated from Eqs. (A5.1) and (A5.2) yielding:

\[ \frac{\partial \beta}{\partial t_I} = \omega \frac{\beta_1^2}{Z_1} - (1 - \omega) \frac{\beta_2^2}{Z_2} \]

\[ \frac{\partial \beta}{\partial t_{II}} = (1 - \omega) \frac{\beta_2^2}{Z_2} \]  \hspace{1cm} \text{(A5.4)}

\[ \frac{\partial \beta}{\partial t_{III}} = \omega \frac{\beta_1^2}{Z_1} \]

Eq. (A5.3) then becomes:

\[ \sigma_\beta^2 = \beta^4 \left[ \left( \frac{\omega}{Z_1} + \frac{1 - \omega}{Z_2} \right)^2 \sigma_I^2 + \frac{1 - \omega}{Z_2} \sigma_{II}^2 + \frac{\omega}{Z_1} \sigma_{III}^2 \right] \]  \hspace{1cm} \text{(A5.5)}

where the velocities have been set equal (\( \beta = \beta_1 = \beta_2 \)), since they are measurements of the same particle velocity and thus should have the same mean value.

The weight, \( \omega \), for minimum error in velocity is found by solving the equation:

\[ \frac{\partial \sigma_\beta^2}{\partial \omega} = 0 \]  \hspace{1cm} \text{(A5.6)}

using \( \sigma_\beta^2 \) from Eq. (A5.5). The result is found to be:

\[ \omega = \frac{\left( \frac{1}{Z_2} - \frac{1}{Z_1} \right) \sigma_I^2 + \frac{\sigma_{II}^2}{Z_2} + \frac{\sigma_{III}^2}{Z_{12}}}{\left( \frac{1}{Z_1} - \frac{1}{Z_2} \right)^2 \sigma_I^2 + \frac{\sigma_{II}^2}{Z_2} + \frac{\sigma_{III}^2}{Z_{12}}} \]  \hspace{1cm} \text{(A5.7)}

Let us simplify the above result by setting \( Z_2 = f Z_1 \). Since counters S1 and S2 are of the same type, we may also set \( \sigma_I = \sigma_{III} = \sigma \), and
also $\sigma_{II} = \alpha \omega$. In this experimental arrangement, S2 has two photomultiplier tubes of a type similar to S1 and S3 so that if the photomultiplier tubes are the major source of error, the independent random errors from the two phototubes will add in quadrature to give $\sigma_{II} = \frac{\sigma_2}{\sqrt{2}}$ and $\alpha = \frac{1}{\sqrt{2}}$. The simplified result for the weight $\omega$ becomes:

$$\omega = \frac{\alpha^2 + 1 - f}{\alpha^2 + 1 - 2f + 2f^2}$$

(A5.8)

and

$$\omega = \frac{3 - 2f}{3 - 4f + 4f^2} \quad \text{for} \quad \alpha = \frac{1}{\sqrt{2}}$$

Notice that for $f = 1/2$, which is approximately the condition for our apparatus, one obtains $\omega = 1$ regardless of the value of $\alpha$ and $\beta = \beta_1$. That is, minimum error in velocity is obtained by considering TOF 1 alone ($\beta_1$) and neglecting TOF 2 ($\beta_2$) entirely. This seems surprising until one considers the timing of the three counters as shown in Fig. 42. Here, each of the three counter timings are shown as a pulse for the sake of simplicity, although a similar argument can be made for the pulse shapes of Fig. 6. If the ratio of time of flight paths is one half, the pulse shapes would be evenly spaced as shown in Fig. 42a. If the timing of S2 is slightly delayed, as shown in Fig. 42b, then the weighting calculation assumes the origin from the two possible cases shown in Fig. 42c and 42d. The effect of averaging these yields the case of Fig. 42a. This ambiguity may be resolved if the two time of flights do not share a counter, or if the ratio of the paths is substantially different from 1/2.
The latter case is substantiated by Fig. 43, which is a graph of the weighting factor, \( \omega \), versus \( f \) for \( \alpha = 1/\sqrt{2} \). In fact, the ratio of time of flight lengths is actually \( f = 0.481 \) yielding from Eq. (A5.9) or Fig. 43, \( \omega = 1.018 \), so TOF 2 should actually be very slightly anti-correlated with TOF 1. It is sufficient to consider the velocity to be given by TOF 1 (\( \beta = \beta_1 \)), and these results are the reason for the latter choice in Chapter 4. It is important to note that TOF 2 still serves its main purpose of rejecting background events that have substantially different values for \( \beta_1 \) and \( \beta_2 \) [see Eq. (4.11)].

Fig. 42. Time of Flight Pulse Timing Anomaly.
Fig. 45. Time of Flight Weighting Factor as a Function of Time of Flight Path Length Ratio, with $a = 1/\sqrt{2}$.

See. Eq. (A5.9).


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