MATCHING INTO THE HELICAL BUNCH COALESCING CHANNEL FOR A HIGH LUMINOSITY MUON COLLIDER

A. Sy#, Y. Derbenev, V.S. Morozov, Jefferson Lab, Newport News, VA 23606, U.S.A.
C. Ankenbrandt, R. P. Johnson, Muons, Inc., Batavia, IL 60510, U.S.A.
D. Neuffer, K. Yonehara, C. Yoshikawa, Fermilab, Batavia, IL 60510, U.S.A.

Abstract

For high luminosity in a muon collider, muon bunches that have been cooled in the six-dimensional helical cooling channel (HCC) must be merged into a single bunch and further cooled in preparation for acceleration and transport to the collider ring. The helical bunch coalescing channel provides the most natural match from helical upstream and downstream subsystems. This work focuses on the matching from the exit of the multiple bunch HCC into the start of the helical bunch coalescing channel. The simulated helical matching section simultaneously matches the helical spatial period $\lambda$ in addition to providing the necessary acceleration for efficient bunch coalescing. Previous studies assumed that the acceleration of muon bunches from $p=209.15$ MeV/c to 286.816 MeV/c and matching of $\lambda$ from 0.5 m to 1.0 m could be accomplished with zero particle losses and zero emittance growth in the individual bunches. This study demonstrates nonzero values for both particle loss and emittance growth, and provides considerations for reducing these adverse effects to best preserve high luminosity.

INTRODUCTION

A high luminosity muon collider requires fast cooling of muons by six orders of magnitude in phase space to obtain short, intense bunches. This six-dimensional cooling is envisioned to be achieved in two stages: initial multi-bunch cooling of trains of small bunches, and single bunch cooling of a larger density bunch in preparation for the final cooling towards collision densities. The transition between multi-bunch cooling and single bunch cooling requires an intermediate bunch recombination process with minimal beam loss.

The Helical Cooling Channel (HCC) [1-4] has been proposed as a means to achieve the six-dimensional cooling of both multiple bunches of muons and single bunches of muons. The HCC allows for continuous emittance exchange for efficient cooling, with longitudinal momentum restored through RF acceleration. In this cooling scheme, muons traverse a series of helical channels, with successive sections utilizing smaller helical spatial period $\lambda$ and higher RF frequency as the muon emittances are reduced. Bunch recombination to a large single bunch necessitates a return to a larger $\lambda$ and corresponding lower RF frequency, and the single bunch cooling then proceeds in a similar manner, through helical channels with successively smaller $\lambda$ and higher RF frequency. A helical bunch coalescing channel provides the most natural match from the upstream multi-bunch HCC to the downstream single bunch HCC and has been explored extensively [5-7] as part of the general HCC design. The helical bunch coalescing channel uses varying RF frequencies to induce an energy-time correlation in the muon bunch train. This energy-time correlation imparts relative velocities to individual bunches such that all bunches in the bunch train align in time at the end of a short drift section and can be captured into a single RF bucket. Previous studies of the helical bunch coalescing channel demonstrated particle transmission of over 90% with an RF fill factor as low as 25%, with room for optimization, but used lower RF frequencies that are no longer relevant to the system. The previous studies also excluded transition sections necessary to match the muon bunches from the exit of the upstream multi-bunch HCC to the entrance of the helical bunch coalescing channel. This work addresses the transition section required to match the muon bunches from $\lambda = 0.5$ to 1.0 m and total momentum $p=209.15$ MeV/c to 286.816 MeV/c in preparation for bunch merging in the helical bunch coalescing channel previously explored.

TRANSITIONING TO THE HELICAL BUNCH COALESCING CHANNEL

Muons exit the upstream multi-bunch HCC with $\lambda = 0.5$ m, RF frequency of 650 MHz, and total momentum on the reference orbit of 209.15 MeV/c. This bunch coalescing method has been found to be more efficient at higher muon momentum [6]: these most recent studies assumed that the acceleration of muons to 286.816 MeV/c could be accomplished with zero emittance growth and zero beam loss. The bunch coalescing channel described in [6] also utilizes $\lambda = 1.0$ m and RF frequency of 200 MHz. This parameter mismatch necessitates a transition section to evaluate the emittance growth and particle losses induced by the matching section and to allow for an end-to-end simulation of the complete six-dimensional cooling channel. The most efficient transition section will simultaneously accelerate muons while adiabatically increasing $\lambda$. We note that the previous bunch coalescing channel was simulated using RF frequency of 200 MHz, and the bunch merge performance is expected to scale with higher RF frequency.

SIMULATION GEOMETRY

The transition section to the helical bunch coalescing channel was simulated using G4Beamline/GEANT4 [8]. RF cavities of length 5 cm were used throughout the transition section, with 60 $\mu$m thick beryllium windows.

---

# Abstract

For high luminosity in a muon collider, muon bunches that have been cooled in the six-dimensional helical cooling channel (HCC) must be merged into a single bunch and further cooled in preparation for acceleration and transport to the collider ring. The helical bunch coalescing channel provides the most natural match from helical upstream and downstream subsystems. This work focuses on the matching from the exit of the multiple bunch HCC into the start of the helical bunch coalescing channel. The simulated helical matching section simultaneously matches the helical spatial period $\lambda$ in addition to providing the necessary acceleration for efficient bunch coalescing. Previous studies assumed that the acceleration of muon bunches from $p=209.15$ MeV/c to 286.816 MeV/c and matching of $\lambda$ from 0.5 m to 1.0 m could be accomplished with zero particle losses and zero emittance growth in the individual bunches. This study demonstrates nonzero values for both particle loss and emittance growth, and provides considerations for reducing these adverse effects to best preserve high luminosity.

# INTRODUCTION

A high luminosity muon collider requires fast cooling of muons by six orders of magnitude in phase space to obtain short, intense bunches. This six-dimensional cooling is envisioned to be achieved in two stages: initial multi-bunch cooling of trains of small bunches, and single bunch cooling of a larger density bunch in preparation for the final cooling towards collision densities. The transition between multi-bunch cooling and single bunch cooling requires an intermediate bunch recombination process with minimal beam loss.

The Helical Cooling Channel (HCC) [1-4] has been proposed as a means to achieve the six-dimensional cooling of both multiple bunches of muons and single bunches of muons. The HCC allows for continuous emittance exchange for efficient cooling, with longitudinal momentum restored through RF acceleration. In this cooling scheme, muons traverse a series of helical channels, with successive sections utilizing smaller helical spatial period $\lambda$ and higher RF frequency as the muon emittances are reduced. Bunch recombination to a large single bunch necessitates a return to a larger $\lambda$ and corresponding lower RF frequency, and the single bunch cooling then proceeds in a similar manner, through helical channels with successively smaller $\lambda$ and higher RF frequency. A helical bunch coalescing channel provides the most natural match from the upstream multi-bunch HCC to the downstream single bunch HCC and has been explored extensively [5-7] as part of the general HCC design. The helical bunch coalescing channel uses varying RF frequencies to induce an energy-time correlation in the muon bunch train. This energy-time correlation imparts relative velocities to individual bunches such that all bunches in the bunch train align in time at the end of a short drift section and can be captured into a single RF bucket. Previous studies of the helical bunch coalescing channel demonstrated particle transmission of over 90% with an RF fill factor as low as 25%, with room for optimization, but used lower RF frequencies that are no longer relevant to the system. The previous studies also excluded transition sections necessary to match the muon bunches from the exit of the upstream multi-bunch HCC to the entrance of the helical bunch coalescing channel. This work addresses the transition section required to match the muon bunches from $\lambda = 0.5$ to 1.0 m and total momentum $p=209.15$ MeV/c to 286.816 MeV/c in preparation for bunch merging in the helical bunch coalescing channel previously explored.

# TRANSITIONING TO THE HELICAL BUNCH COALESCING CHANNEL

Muons exit the upstream multi-bunch HCC with $\lambda = 0.5$ m, RF frequency of 650 MHz, and total momentum on the reference orbit of 209.15 MeV/c. This bunch coalescing method has been found to be more efficient at higher muon momentum [6]: these most recent studies assumed that the acceleration of muons to 286.816 MeV/c could be accomplished with zero emittance growth and zero beam loss. The bunch coalescing channel described in [6] also utilizes $\lambda = 1.0$ m and RF frequency of 200 MHz. This parameter mismatch necessitates a transition section to evaluate the emittance growth and particle losses induced by the matching section and to allow for an end-to-end simulation of the complete six-dimensional cooling channel. The most efficient transition section will simultaneously accelerate muons while adiabatically increasing $\lambda$. We note that the previous bunch coalescing channel was simulated using RF frequency of 200 MHz, and the bunch merge performance is expected to scale with higher RF frequency.

# SIMULATION GEOMETRY

The transition section to the helical bunch coalescing channel was simulated using G4Beamline/GEANT4 [8]. RF cavities of length 5 cm were used throughout the transition section, with 60 $\mu$m thick beryllium windows.
between cavities. The RF cavities were filled with hydrogen gas; the nominal cavities are identical to ones used for ionization cooling in the HCC, but we note that the cooling performance is less crucial in this transition section, and the hydrogen gas pressure was allowed to vary. The RF accelerating gradient was a constant 20 MV/m. A sample 3D render illustrating the helical structure of the transition section is shown in Fig. 1. The adiabatic increase of \( \lambda \) is clearly demonstrated by the increasing pitch in the helical channel.

![Figure 1: 3D render of helical transition section to helical bunch coalescing channel.](image)

**CHANNEL LENGTH EFFECTS**

The short muon lifetime (\( \tau = 2.2 \mu s \)) necessitates efficient manipulation of the muon phase space to deliver high-intensity muon beams for collision. Additional transport sections in the 6D cooling channel increase the muon path length and total transit time; thus it is important to minimize additions to the length of the helical bunch coalescing channel while balancing particle losses that can occur with aggressive acceleration. Figure 2 illustrates the muon fractional emittance growth and transmission for positive muons at the exit of the transition section as a function of transition section length with RF frequency of 650 MHz, \( H_2 \) pressure of 160 atm, \( \lambda_i \rightarrow \lambda_f = 0.5 \rightarrow 1.0 \) m, and \( p_i \rightarrow p_f = 209.15 \rightarrow 286.816 \) MeV/c. The RF frequency in the transition section was chosen to match that of the upstream HCC section feeding directly into the transition. The simulations clearly demonstrate nonzero particle losses in this transition section, with higher particle losses observed for shorter channel lengths. This is likely due to the smaller RF bucket present for the shorter channel lengths. The RF bucket area at the shortest channel length of 12 m is nearly half that of the longest channel length simulated, 30 m [9].

While the muon transmission decreases with decreasing transition section length, the overall emittance growth is minimized. Scattering processes contribute to this emittance growth even in short channels, and both the transverse and longitudinal emittances grow by as much as 20% in the transition section.

![Figure 2: Emittance growth and particle transmission as a function of transition channel length. Emittance growth is reduced in shorter channels; particle loss increases in shorter channels.](image)

**PRESSURE EFFECTS**

Because the transition section is not a dedicated ionization cooling channel, the ionization cooling process is less crucial and the hydrogen gas pressure in the transition section becomes knob that can be tuned for optimization. Figure 3 shows the fractional emittance growth and transmission for positive muons in the 12 m long channel previously simulated, with varying \( H_2 \) gas pressure.

![Figure 3: Emittance growth and transmission as a function of hydrogen gas pressure in the 12 m long transition channel. Emittance growth increases with reduced pressure; particle losses decrease with reduced pressure.](image)

For a fixed channel length, reducing the hydrogen gas pressure in the RF cavities requires adjustment of the RF phase to achieve the proper acceleration. The required energy gain per cavity is reduced due to reduced energy losses from ionization cooling, allowing for larger RF phase and larger RF bucket areas that increase the overall transmission. The emittance growth seen at lower gas pressures suggests that the ionization cooling effect...
dominates over heating processes present in the channel. Though the longitudinal emittance growth decreases with increasing gas pressure, we note that this reduction is due to the combined effects of ionization cooling as well as the omission of particles from the emittance calculation that have fallen out of the RF bucket. Figure 4 illustrates the effect of reduced gas pressure on particle transmission in the 12 m long channel.

Figure 4: Particle momentum as a function of distance in the channel for H$_2$ gas pressure of (a) 80 atm and (b) 160 atm. Particles falling out of the RF bucket are not properly accelerated and are excluded from the longitudinal emittance calculation.

**SUMMARY**

G4Beamline/GEANT4 simulations of the transition section from the upstream helical cooling channel to the helical bunch coalescing channel indicate both nonzero emittance growth and nonzero particle losses, details that must be accounted for in the overall design of the helical bunch coalescing channel. Emittance growth is reduced with reduced transition channel lengths, while transmission is improved with reduced hydrogen gas pressure in the accelerating RF cavities.

**ACKNOWLEDGMENTS**

Authored by Jefferson Science Associates, LLC under U.S. DOE Contract No. DE-AC05-06OR23177 and supported by U.S. DOE STTR Grant DE-SC0007634. The U.S. Government retains a non-exclusive, paid-up, irrevocable, world-wide license to publish or reproduce this manuscript for U.S. Government purposes.

**REFERENCES**