Instability of charged anti-de Sitter black holes

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\textbf{A B S T R A C T}

We have studied the instability of charged anti-de Sitter black holes in four- or higher-dimensions under fragmentation. The unstable black holes under fragmentation can be broken into two black holes. Instability depends not only on the mass and charge of the black hole but also on the ratio between the fragmented black hole and its predecessor. We have found that the near extremal black holes are unstable, and Schwarzschild-AdS black holes are stable. These are qualitatively similar to black holes in four dimensions and higher. The detailed instabilities are numerically investigated.

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1. Introduction

Since the discovery of the Higgs particle at the Large Hadron Collider (LHC) [12], there have been detailed studies on the Higgs potential in the high-energy regime, which suggests that the present universe is metastable. Given this assumption, it is possible that the present false vacuum can tunnel into the more stable vacua, whose decay and lifetime is calculated with the method developed by Coleman and De Luccia [3–5]. Its lifetime turns out to be large enough to be consistent with the age of universe. However, gravitational impurities such as black holes generate inhomogeneities and act as sites for vacuum decay. These inhomogeneities reduce an energy barrier to vacuum decay, and they also can significantly reduce the lifetime of the metastable state to millions of Planck times [6,7]. In this analysis, an understanding of black holes, including AdS ones, is important.

A \(D\)-dimensional gravity theory in the anti-de Sitter spacetime (AdS\(_D\)) corresponds to a \((D-1)\)-dimensional conformal field theory (CFT\(_D-1\)) on the AdS\(_D\) boundary. This correspondence is well known as the AdS/CFT duality [8–11]. Through this duality, the thermodynamic properties of a bulk gravity theory appear as those of the CFT on the AdS boundary [12]. For example, AdS black holes can be described as finite temperature CFTs with a Hawking temperature \(T_H\), and the instabilities of AdS black holes are related to the phase transition of CFT systems. The dual CFT residing on a Reissner–Nordström-AdS (RN-AdS) black hole is a field theory with a chemical potential for the charged black hole [13,14]. An RN-AdS black hole becomes unstable in the presence of a charged scalar field [15], which is interpreted as a superconducting instability [16,17] in the dual CFT. Through AdS/CFT correspondence, a rotating holographic superconductor is the dual of a Kerr–Newman-AdS black hole [18]. The instability of the charged black hole has been used to understand the phase transition of a holographic superconductor [19,20].

The instabilities of black holes have been investigated under perturbation or thermodynamics. In the asymptotically flat spacetimes, a Schwarzschild black hole is stable under perturbation. A Reissner–Nordström (RN) black hole is stable under perturbation by a neutral charge in scalar fields [21,22] and charged [23,24]. In AdS spacetime, a Schwarzschild-AdS black hole is perturbatively stable. Determining the stability of an RN-AdS black hole depends on the theory in question. An RN-AdS black hole in Einstein–Maxwell gravity is stable under perturbation [25]; in the \(\mathcal{N} = 8\) gauged supergravity theory, the instability of an RN-AdS black hole comes from the tachyon mode of the scalar field [26,27]. Also, the horizon of rotating black holes is stable under a perturbation that is constrained by the particle energy equation in this theory [28–33]. In contrast, thermodynamic stabilities are different from those of perturbation. A Schwarzschild black hole has negative heat capacity, so it is thermodynamically unstable. An RN black hole is also thermodynamically unstable within specific parameter regions [34–36]. A Schwarzschild-AdS black hole is unstable under...
thermodynamics. An RN-AdS black hole occurs during the second order phase transition [37,38], so it is also unstable. In addition, the angular momentum of the black hole affects the instability. The Myers–Perry (MP) black hole is one of the higher-dimensional rotating black holes. The Myers–Perry (MP) black hole with large angular momentum can be unstable under fragmentation, which breaks the black hole into several pieces [39]. This provides dynamically the upper bound for the stability and angular momentum. Instability under fragmentation is a type of thermodynamic instability based on black hole entropy. The instability of fragmentation occurs in MP-AdS black holes in perturbatively stable regions [40]. According to the dilaton-Gauss–Bonnet (DGB) theory, a static black hole becomes unstable under fragmentation in specific parameter regions [41].

In this paper, we will investigate the instability of four- or higher-dimensional charged AdS black holes under fragmentation, divided into two black holes with arbitrary sizes. Through fragmentation, the thermal instability can be shown in terms of the size of mass and charge fluctuation. The repulsion from electric charges will play an important role in instability so that instability mainly appears in near extremal cases. These results are consistent with the instability established from the perturbation that occurs in black holes with a large charge.

This paper is organized as follows. In section 2, a higher-dimensional charged AdS black hole is introduced. In section 3, fragmentation instabilities are represented and shown in several approximations. In section 3.2, fragmentation instabilities are numerically given as phase diagrams with respect to black hole mass $M$ and charge $Q$. In section 3.3, the phase diagrams are shown in terms of fragmentation ratios of the black hole, $\epsilon_m$ and $\epsilon_q$, with given mass $M$ and charge $Q$. In section 4, we summarize our results.

2. Charged anti-de Sitter black holes

The charged anti-de Sitter (AdS) black hole is a static solution of Einstein–Maxwell gravity with a negative cosmological constant. The charged AdS black hole with a mass $M$ and electric charge $Q$ is given in the $D$-dimensional spacetime [37,42] as

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)} dr^2 + r^2 d\Omega_{D-2},$$

$$f(r) = 1 - \frac{2M}{r^D-3} + \frac{Q^2}{r^{2D-6}} + \frac{r^2}{\ell^2},$$

where $\Omega_{D-2}$ is $(D-2)$-dimensional sphere. We set the $c = G = 1$ in this paper. The radius of the outer horizon $r_h$ satisfies

$$f(r_h) = \frac{r_h^{2D-4}}{\ell^2} + \frac{r_h^{2D-6}}{\ell^2} - 2Mr_h^{D-3} + Q^2 = 0.$$ (2)

The radius of AdS curvature $\ell$ is described in terms of cosmological constant $\Lambda = -\frac{D-1}{2(2D-3)}$. The black hole solution becomes Schwarzschild-AdS black hole for $Q = 0$ and Reissner–Nordström black hole for $\ell \rightarrow \infty$. Hawking temperature $T_H$ and electric potential $\Phi_H$ are given at the outer horizon

$$T_H = \frac{(D-3)\hbar}{4\pi} \left[ \frac{r_h^{2D-6} - Q^2}{r_h^{D-5}} + \frac{(D-1)r_h}{\ell^2} \right],$$

$$\Phi_H = \frac{Q}{r_h^{D-3}}.$$ (3)

Bekenstein–Hawking entropy $S_{BH}$ is proportional to the horizon area $A_H$

$$S_{BH} = \frac{A_H}{4\hbar} = \frac{\Omega_{D-2}r_h^{D-2}}{4\hbar}.$$ (4)

The black hole entropy should be increased in the natural process along to the 2nd law of thermodynamics.

The charge $Q$ had an extremal bound for given mass $M$, because the power of mass term is smaller than charge term in $D \geq 4$. The radius of the outer horizon for an extremal black hole is a solution of

$$D - 2\frac{r_h^{D-1}}{\ell^2} + r_h^{D-3} - M = 0,$$ (5)

which comes from the combination of $f(r) = 0$ and its derivative with respect to $r$. We define the dimensionless variables rescaled by the AdS radius

$$\frac{s}{\ell} \rightarrow s, \quad \frac{t}{\ell} \rightarrow t, \quad \frac{r}{\ell} \rightarrow r, \quad \frac{M}{\ell^{D-3}} \rightarrow M, \quad \text{and} \quad \frac{Q}{\ell^{D-3}} \rightarrow Q.$$ (6)

3. The instability of fragmentation

The instability of the charged AdS black hole can grow due to the repulsion of the electric charge. This instability has been tested by a perturbation. To investigate that instability depends on the size of fluctuation, we assume that the unstable AdS black hole with an electric charge can be broken into two black holes of the same symmetries with an arbitrary ratio. To describe this fragmentation, the initial and final states will be defined, and the thermodynamic preference will be applied between them. In the initial state, there is a charged AdS black hole with mass $M$ and electric charge $Q$ at rest. After fragmentation, in the final state, one black hole has mass $\epsilon_m M$ and charge $\epsilon_q Q$, and the other has mass $(1 - \epsilon_m)M$ and charge $(1 - \epsilon_q)Q$. Under the conservations of mass and electric charge. Note that the mass and charge ratios are defined as $0 \leq \epsilon_m \leq 1$ and $0 \leq \epsilon_q \leq 1$. To clarify the final state, we treat one of the fragmented black holes as a test particle, moving in a charged AdS black hole spacetime given by the other one. Moving slowly, the charged particle with mass $m$ has energy $E$ around the black hole, having mass $M'$ and charge $Q'$.

$$E^2 = m^2 \left( 1 - 2\frac{M'}{R^{D-3}} + \frac{Q'^2}{R^{2D-6}} + R^2 \right),$$ (7)

where the impact parameter of the particle is $R$. We suppose the flat limit of the impact parameter is sufficiently large to cancel the gravitational attraction and AdS potential by electric repulsion $\ell \gg R$, so the gravitational interaction of the black holes and the effect of the cosmological constant are negligibly. Under the flat limit, Eq. (7) is reduced to

$$E^2 \cong M^2.$$ (8)

Under this assumption, the analysis is valid for the charged AdS black hole with small mass. To be thermodynamically preferred, the fragmentation should increase the entropy, so that the entropy of the final state is larger than that of the initial state. The entropy of a black hole is given by Eq. (4), which is proportional to the power of the horizon of a black hole. Then, we may introduce the entropy ratio for the initial and final states

$$R = \frac{S_{BH,f}}{S_{BH,i}} = \frac{r_h[\epsilon_m M, \epsilon_q Q]^{D-2} + r_h[(1 - \epsilon_m)M, (1 - \epsilon_q)Q]^{D-2}}{r_h[M, Q]^{D-2}},$$ (9)
where $r_h[M, Q]$ is the radius of black hole horizon with given mass $M$ and charge $Q$. When the entropy ratio is larger than 1, the black hole is unstable and fragmented into two black holes, and the other is stable.

### 3.1. Analytical approximation for the instability

We can test the general behaviors of instability through analytical approximation. Instability depends on the mass and charge ratios $\epsilon_m$ and $\epsilon_q$, as shown in Eq. (9). We test the instability of the massive Schwarzschild-AdS black hole under small ratio $\epsilon_m$. We will show that the black hole becomes unstable if its mass is greater than $M_{\text{max}}$, which depends on $\epsilon_m$, as such

$$M_{\text{max}} \sim \epsilon^{-1(D-1)/(D-2)}_m,$$

(10)

where the maximum mass is sufficiently large for the small ratio $\epsilon_m$. Therefore, a Schwarzschild-AdS black hole is stable for a small ratio under fragmentation, and this is consistent with a stable property of the black hole under perturbative analysis [43,44].

Now, we assume that the fragmented black hole is divided into two identical black holes; that is, the fragmentation ratio of mass $\epsilon_m$ and charge $\epsilon_q$ satisfies $\epsilon_m = \epsilon_q = 1/2$. Then, the entropy ratio in Eq. (9) is simplified as

$$R = 2^{\frac{1}{D-1}} \left( \frac{r_h[M/2, Q/2]}{r_h[M, Q]} \right)^{D-2}.$$

(11)

For extremal black holes with small mass and charge limit, $M_{\text{ext}}$. $Q_{\text{ext}} \ll 1$, the horizon is obtained from the limit of the small mass. The entropy ratio can be rewritten as

$$R = 2^{\frac{1}{D-1}} < 1.$$

(12)

Note that $Q_{\text{ext}} \ll M_{\text{ext}}$ due to the extremal condition. Since the entropy ratio is always smaller than 1, the charged AdS black hole with the limit of small mass and charge is stable under fragmentation. For a massive extremal black hole, the entropy ratio is in the leading order

$$R = 2^{\frac{1}{D-1}} > 1,$$

(13)

where it is always larger than 1. Therefore, the extremal black hole is unstable under fragmentation, and similar instability also appears in perturbation for AdS black holes with a large charge [45-47]. For this half of the fragmentation, there is a distinguishable instability in the limit of large mass and small charge $M \gg 1$ and $Q \ll 1$. In this case, the charge term in Eq. (2) is approximately zero. In addition, $r_h^{2D-4} \gg r_h^{2D-6}$ with every $D$ for the large mass limit. It leads to the entropy ratio

$$R = 2^{\frac{1}{D-1}} > 1,$$

(14)

and it is always larger than 1. Thus, the charged AdS black hole with the limit of large mass and small charge is unstable under fragmentation. Therefore, we expect the instability to be dependent on the size of the fragmentation. Fragmentation becomes similar to perturbation within a small ratio limit, but we can expect different instabilities in large ratios. Detailed behaviors will be investigated numerically in the following section.

### 3.2. Instability in the $M$-$Q$ diagram

We will now investigate the instability of charged AdS black holes numerically using the dimensionless mass $M$ and charge $Q$. In this section, the decay with uniform ratio $\epsilon = \epsilon_m = \epsilon_q$ is considered in Eq. (9). The mass and charge ratios $Q/M$ are the same for black holes in the initial and final states. Other values of the ratios will be analyzed in the next section. The detailed phase diagram is summarized in Fig. 1. The analysis is valid for a large AdS radius, so small-mass cases are valid. Solid lines are within valid ranges in Fig. 1, but dotted lines are just an extension of these lines, without validity, because black holes outside of the dotted lines are too heavy to satisfy our assumption for the flat limit.

The bounds on extremal black holes are represented by a solid black line. The upper region of this solid black line is not allowed because the initial black hole cannot have a charge larger than $Q_{\text{ext}}$. Below the solid black line, we plot three different instability boundaries; these boundaries are represented by red, blue, and green lines that correspond to the fragmentation ratios $\epsilon = 1/2$, 1/4 and 1/8, respectively. For each curve, the stable region is inside of the curve, while the unstable region is outside of it. The plot for higher dimensions looks similar to that of $D = 4$, as seen in Fig. 1 (b) for $D = 6$. The shapes of boundaries are qualitatively similar for higher dimensions where $D \geq 4$. However, the stable region for a given $\epsilon$ shrinks as the dimension increases, which can be seen in Fig. 1 (b). Thus, the black hole becomes unstable more easily in higher dimensions.

The boundaries of instability are started at the point of $\epsilon$, $M_{\text{ext},\epsilon}$ and ended at the point of $M_{\text{crit},\epsilon}$ in Fig. 1. They correspond to the smallest and largest mass, respectively. All extremal black holes with a mass larger than $M_{\text{ext},\epsilon}$ will be unstable for a given $\epsilon$. 

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Fig. 1. Phase diagrams for the instability with respect to $M$ and $Q$. Black line represents the extremal bound. Dashed red, dotted blue, and dot-dashed green lines represent the boundaries of the instability with $\epsilon = 1/2, 1/4$ and 1/8. Markers indicate the parameter choices of Fig. 3 and 4. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)
$M_{\text{crit}, \epsilon}$ can be the end of the stable region for a given $\epsilon$, but this is invalid in our assumption for the small black hole. The values of $M_{\text{ext}, \epsilon}$ become smaller, but the values of $M_{\text{crit}, \epsilon}$ become larger for larger values of $\epsilon$. The detailed behaviors of $M_{\text{ext}, \epsilon}$ and $M_{\text{crit}, \epsilon}$ are introduced in Fig. 2. Colored dots represent $M_{\text{ext}, \epsilon}$ and $M_{\text{crit}, \epsilon}$ with respect to the selected values of $\epsilon$ in Fig. 1 (a). For larger $\epsilon$, the boundary of the instability starts at the larger value of $M_{\text{ext}, \epsilon}$ and ends at the smaller value of $M_{\text{crit}, \epsilon}$ in Fig. 1. Therefore, any two boundaries of the instability with different $\epsilon$ values must cross each other. The crossing point appears close to the boundary for the extremal black hole in Fig. 1. Note that $M_{\text{crit}, \epsilon}$ goes to infinity when $\epsilon$ goes to zero. Since the perturbation is similar to infinitesimally small fluctuations of the mass, this is consistent with the stability of the Schwarzschild-AdS black hole under perturbation. For the charged cases with $\epsilon \ll 1$, $M_{\text{ext}, \epsilon}$ goes to zero, and $M_{\text{crit}, \epsilon}$ goes to infinity. So, the charged black holes are unstable for near extremal cases under infinitesimally small fragmentation. Therefore, within the limit of infinitesimally small $\epsilon$, the fragmentation instability becomes similar to that of perturbation.

The instability of the fragmentation is determined by competition between gravitational attraction and electric repulsion. The effect of AdS boundary have been assumed to be small. In the final state, the gravitational attraction is smaller than the electric repulsion because the power of the repulsion is higher than that of the gravitation. In a small ratio, electric repulsion can respond more sensitively than gravitational force in a short distance, allowing the fragmented black hole to be easily separated into the proper impact parameter. One can expect that the AdS effect can change instability under fragmentation for near extremal black holes. However, we have found no qualitative change in our analysis. To see this, we have worked it out numerically for $\epsilon = 1/2$. The $M_{\text{ext}, 1/2}$ occurs at $M = 0.388428$. The entropy ratio is greater than 1, about 50%, for the unstable region near $M < 1$, where the assumption of a large separation between two small black holes is reasonable. Yet, the AdS effect in this region is not expected to be big. This suggests that the AdS effect cannot rule out our analysis results on the existence of the unstable region.

### 3.3. Instability on $\epsilon_m - \epsilon_q$ diagram

Now, we numerically investigate the instability of charged AdS black holes under fragmentation, which are divided into two black holes with an arbitrary fragmentation ratio, $\epsilon_m$ and $\epsilon_q$. The detailed phase diagrams of $\epsilon_m$ and $\epsilon_q$ are shown in Figs. 3 and 4. Since Eq. (9) is symmetric under $(\epsilon_m, \epsilon_q) \rightarrow (1 - \epsilon_m, 1 - \epsilon_q)$, the figures are symmetric under the 180° rotation at $(1/2, 1/2)$. Due to the extremal bounds, black holes in the initial and final states should belong to the allowed region. This constrains on $\epsilon_m$ and $\epsilon_q$. The solid black lines in Figs. 3 and 4 represent the boundaries imposed by these constraints. The boundaries between stable and unstable regions are represented as dashed red lines. Thin, diagonal gray lines represent the uniform fragmentation ratios $\epsilon_m = \epsilon_q$, analyzed in the previous section. The red, blue, and green markers with different shapes along these lines represent the parameter choices $\epsilon_m = \epsilon_q = 1/2$, 1/4 and 1/8, respectively. The allowed regions of Figs. 3 and 4 expand as the mass of the initial black hole increases because the extremal condition is satisfied by a larger charge. This can be seen by comparing Figs. 3 (c) and 4 (b) or Figs. 3 (d) and 4 (c). It means that the area of allowed region depends on the mass and charge ratios of the initial black hole.

The boundaries of the instability, represented by a dashed red line, are shown to be complicated behaviors in Figs. 3 and 4. Most regions are unstable at $M = M_{\text{ext}}$, except the narrow stable regions close to the solid black line, as portrayed in Fig. 3 (a). As mass increases beyond $M_{\text{ext}}$, the stable region grows, and most of the allowed regions are stable, as shown in Fig. 3 (b). If the mass increases further, the stable region starts shrinking again, as demonstrated in Fig. 3 (c) and (d). This trend also can be seen in Fig. 4. As mass increases, starting with mass $M_{\text{ext}}$, the unstable region first decreases, as in Fig. 4 (b), and then increases again, as shown in Figs. 4 (c) and (d). Now, we compare Figs. 3 (c) and 4 (b), followed by Figs. 3 (d) and 4 (c), for a fixed charge comparison. Then, the unstable region expands to a diagonal direction, which implies the same results as Fig. 1; at the same time, the stable region moves to a small $\epsilon_q$ area because the increased charge creates more possibilities for small ratio of charge to become stable.

We now compare Fig. 3 with Fig. 1. Fig. 3 corresponds with the straight horizontal line with charge $Q = 0.5$ in Fig. 1. More specifically, Figs. 3 (a), (b), (c), and (d) correspond to the markers 3(a), 3(b), 3(c), and 3(d) on that line in Fig. 1 with the masses $M_{\text{ext}}$, 1.0, 1.5, and 2.0, respectively. The fragmentation ratio value $\epsilon$ determines the position along the diagonal gray lines in Fig. 3. For example, $\epsilon = 1/2$ is represented by red markers located at the center of each plot in Fig. 3. The red markers in Figs. 3 (a), (b), (c), and (d) lie in unstable, stable, unstable, and unstable regions, respectively. As Fig. 1 (a) demonstrates, the points 3(a), 3(b), 3(c), and 3(d) lie in unstable, stable, unstable, and unstable regions, respectively, and they are under fragmentation with $\epsilon = 1/2$, represented...
by a dashed red line. This is an exact match, as represented by the fixed $\epsilon = 1/2$ in Fig. 3. Similarly, one can compare other values of the fragmentation ratio in Fig. 1 and Fig. 3, shown in different colors.

If we take the flat space limit $\epsilon \to \infty$, which is the same in section 3.2, the variables are dimensionless, and the flat space limit is equal to the small-mass and charge limit as shown in Fig. 5. However, since the mass and charge of the initial black hole are already fixed, as shown in Figs. 3 and 4, the flat limit is described with the AdS radius turned on. The parameter values of Fig. 3 (c) are chosen for the flat limit. This limit makes the unstable region contract to $(1/2, 1/2)$. This shrinking behavior of the unstable region implies that the black hole fragmentation limited to two identical black holes is highly preferable, as we discussed in a previous section. It is expected that the flat space limit will eliminate the unstable region for all parameter choices. Finally, the charged AdS black hole will be stable under the flat space limit even in non-uniform fragmentation. In other words, an RN black hole should be stable under fragmentation with an arbitrary ratio of mass and charge. This is consistent with known results that the RN black hole is stable under perturbation.

4. Conclusion

We have investigated the instability of charged AdS black holes with dimensions $D \geq 4$ due to fragmentation. During fragmentation, the charged AdS black hole of the initial state is at rest. Subsequently, the black holes have been assumed to be broken in two by the instability. In the final state, two slowly moving black holes emerge, and their total mass and charge are conserved in the process. The AdS radius and impact parameters are assumed to be properly large enough to neglect the interactions in the final state. To be a thermally natural process, the fragmentation should increase the entropy from the initial to the final state, so we have found the ratio of the entropy between states.

In the analytical approximation for the ratio of the entropy, the instability of the charged AdS black hole has been expected at extremal black holes, but the small Schwarzschild-AdS black hole has been stable under fragmentation. Thus, there exists a boundary of instability between them. We have found the detailed boundary using a numerical calculation.

In the fragmentation with $\epsilon = \epsilon_m = \epsilon_p$, the instability is shown in Fig. 1 with respect to the mass and charge of the black hole. Since our analysis is valid at a small mass, the black holes are
Fig. 4. Phase diagrams for the instability in $D = 4$ and $Q = 1.0$. (For interpretation of the references to color in this figure, the reader is referred to the web version of this article.)

Fig. 5. The phase diagrams for Fig. 3 (c) with different values of $\ell$. 
unstable at a large charge. Instability is also dependent on the ratio \( \epsilon \). When the ratio \( \epsilon \) is a small value compared with 1/2 (the largest value), the extremal black holes become unstable at smaller masses than \( M_{\text{ext}} \). This instability is compatible with the results from perturbation, because the fluctuation of the infinitesimally small \( \epsilon \) is the perturbation of the black hole. In higher dimensions, the phase structure is qualitatively similar to a four-dimensional case, except that the stable regions are smaller.

For general fragmentation cases, the instability of the black hole is dependent on the values of the ratios \( \epsilon_m \) and \( \epsilon_q \) for a given mass in Figs. 3 and 4. The instability of charged AdS black holes, with respect to the mass and charge ratios \( \epsilon_m \) and \( \epsilon_q \), is shown in Figs. 3 and 4. The allowed region increases as mass increases, but it decreases as charge increases. The stable region first increases as mass increases, and it decreases again as mass further increases for a fixed charge. In addition, it always decreases as charge increases for fixed mass.

We have studied the instability of charged AdS black holes by comparing the entropy ratio of initial and final black hole states. However, we have not studied the dynamical or tunneling processes that lead to the instability. The entropy ratio smaller than 1 is sufficient to ensure stability. On the contrary, the entropy ratio larger than 1 is not sufficient, but it is a necessary condition for the black hole to be stable under fragmentation. This is because the lifetime of the initial state can be either short or long, depending on the fragmentation process. The process leading to fragmentation may occur dynamically or through tunneling due to a thermal or quantum fluctuation. Investigation on this aspect is left for future work.

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