Observational Constraints on Thermal Evolution with Decaying Cosmological Terms.

A THESIS SUBMITTED TO GRADUATE SCHOOL OF KYUSYU UNIVERSITY IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF DOCTOR OF SCIENCE IN PHYSICS

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Data analysis was in part carried out on a general common user computer system at the Astronomical Data Analysis Center of the National Astronomical Observatory of Japan.

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Contents

1 Introduction ................................................. 1
  1.1 Historical Review ........................................ 1
  1.2 The cosmological term as dynamical quantities ............ 4

2 Big-bang nucleosynthesis in the Brans-Dicke cosmology with a decaying $\Lambda$ term 9
  2.1 Historical remark ....................................... 9
  2.2 Brans-Dicke cosmology with a decaying-\(\Lambda\) term ....... 10
  2.3 Primordial nucleosynthesis in Brans-Dicke model with \(\Lambda\) term 13
  2.4 Effects of \(e^+e^-\) annihilation on BBN .................. 17
  2.5 Concluding remarks ..................................... 20

3 Thermal evolution with decaying cosmological term in the context of Friedmann universe. 22
  3.1 Cosmological Evolution .................................... 22
  3.2 Thermal evolution at photon decoupling epoch .............. 31
  3.3 SNIa constraint ........................................... 33
    3.3.1 Cosmological test from SNIa ......................... 33
    3.3.2 SNIa constraint ..................................... 34
  3.4 Effects on CMB anisotropy and constraints by Markov Chain Monte Carlo analysis ........... 35
    3.4.1 Cosmological perturbation theory – Overview – .... 36
    3.4.2 Energy momentum tensor in a Phase space .......... 40
    3.4.3 Cosmological perturbation theory with a decaying cosmological term .................. 49
    3.4.4 Markov chain Monte Carlo analysis ................. 56
CONTENTS

4  Summary and Discussion 67

A  Thermodynamics in the early Universe 77
   A.1  Thermal equilibrium epoch . . . . . . . . . . . . . . . . . . 78
   A.2  Electron-positron annihilation epoch . . . . . . . . . . . 79
   A.3  After annihilation . . . . . . . . . . . . . . . . . . . . . . . 80

B  Tight coupling approximation 83

C  Results of WMAP first and three year 87
   C.1  WMAP first year . . . . . . . . . . . . . . . . . . . . . . . . 87
   C.2  WMAP three year . . . . . . . . . . . . . . . . . . . . . . . 88
List of Figures

1.1 Illustration of effective magnitude-redshift relation of Type Ia supernovae from Supernovae Cosmology Project and the Calán/Tololo Supernova Survey. The effective magnitude $m_{\text{eff}}$ includes corrections for the apparent magnitude. The solid curves are the theoretical $m_B(z)$ for a range of cosmological models with zero cosmological constant: $(\Omega_M, \Omega_{\Lambda}) = (0, 0)$: top, $(1, 0)$: middle and $(2, 0)$: bottom. The dashed curves are for a range of flat cosmological models: $(\Omega_M, \Omega_{\Lambda}) = (0, 1)$, $(0, 0.5)$, $(1, 0)$ and $(1.5, -0.5)$ [5]. .......................................................... 2

1.2 Contours of 68.3%, 95.4% and 99.7% confidence regions on amount of matter component $\Omega_m$ and equation of state of dark energy $w$ from the Supernova Legacy Survey alone (solid lines), baryon acoustic oscillation alone (dotted lines), and the joint confidence contours (dashed line) [8]. .......................................................... 3

1.3 Full sky Mollweide projection of the 4-year DMR map, excluding the dipole [15]. .......................................................... 6

1.4 Overview of the main events after the last-scattering surface $z = 1100$ with the top axis showing the age of the universe and the bottom axis the corresponding redshift. Blue represents atomic formation regions, and red, ionized regions [45]. .... 7
# LIST OF FIGURES

1.5 Illustration of the estimation of first objects formation epoch and their masses obtained from the relation between the virial temperature and redshift in standard CDM (medium dotted line), SΛCDM (thin dotted lines) and a decaying-Λ model (thick dotted lines). This cooling diagram are characterized by the time scales of the free-fall $t_{\text{ff}}$, cooling $t_{\text{cool}}$ and Hubble expansion $t_{\text{H}}$. Three regions for the formation of the first objects are indicated by $t_{\text{H}} < t_{\text{cool}}$, no formation occurs; $t_{\text{ff}} < t_{\text{cool}} < t_{\text{H}}$, the molecule clouds become the first objects though it is not effective; and $t_{\text{H}} > t_{\text{cool}}$, the formation occurs most efficiently. We should note that the formation of the first objects should be determined by the detailed hydrodynamic simulations, where we can compare the theoretical prediction with future observation (From [36]).

2.1 Time evolution of the scale factor for BD?? ($\mu = 0.7$, $t_0 = 13.7$ Gyr, and $\eta_{10} = 6.1$) and Friedmann model.

2.2 Illustration of the nuclear reaction network during the primordial nucleosynthesis.

2.3 Illustration of baryon-to-photon ratio $\eta$ for light elements abundances $^4$He, D and $^7$Li/H, in standard BBN calculation. The blue, green, and red regions show the uncertainty of nuclear reaction rates, observational abundances, and the baryon-to-photon ratio by WMAP, respectively.

2.4 Light element abundances against $\eta$ in BD?? for $\mu = 0.7$ and possible values of $B^*$. 

2.5 Same as Fig. 2.4 but for $B^* = 0$ and various values of $\mu$.

2.6 Constraints on the $\mu - B^*$ plane corresponded to 1, 2 and 3$\sigma$ confidence regions from observational abundance and baryon density obtained by WMAP.

2.7 Light-element abundances vs. $\eta$ in BD?? for $\mu = 0.7$ and $B^* = -2.5$. Dashed lines show $\pm 2\sigma$ uncertainties in nuclear reaction rates. The dark-shaded area indicates the constraint by WMAP and light-shaded areas denote regions of observational abundances.
2.8 Evolution of the scalar field as function of cosmic time during BBN epoch. The solid line refers the integration of Eq. (2.13) with $B^* = -2.43$, and the broken line is for Eq. (2.10) with $B^* = -2.50$. ................................................................. 19

2.9 Likelihood function as a function of $\eta_{10}$ for $^4\text{He}$ ($L_{4\text{He}}$), $D$ ($L_{2\text{D}}$) and $^7\text{Li}$ ($L_{7\text{Li}}$). The vertical lines indicate the upper and lower limit to $\eta$ by WMAP. ...................................................... 21

2.10 Combined likelihood function for two $^4\text{He}$ and $^7\text{Li}$ ($L_{4\text{He}}$) and three-elements D, $^4\text{He}^7\text{Li}$ ($L_{247\text{Li}}$). We note that the combined likelihood function is consistent with the WMAP data, because theoretical prediction of deuterium by BBN agrees very well with the observational data of D/H and the prediction of $\eta$ by WMAP. ......................................................... 21

3.1 Evolution of energy densities of photon, neutrino, matter and $\Lambda$ as a function of the scale factor in D$\Lambda$CDM with $(\Omega_{\Lambda 2}, m) = (10^{-4}, 1.2)$ ($\Omega_{\Lambda 1} = \Omega_{\Lambda} - \Omega_{\Lambda 2}$). ...................................................... 25

3.2 Upper panel: the evolution of the photon temperature in D$\Lambda$CDM with $\Omega_{\Lambda 2} = 10^{-4}$ after hydrogen atom recombination. Lower panel: the ratios of $T_\gamma$ to that of S$\Lambda$CDM. ......................... 26

3.3 Upper panel: the evolution of the photon temperature in D$\Lambda$CDM. Lower panel: the ratios of $T_\gamma$ to that of S$\Lambda$CDM. ................. 27

3.4 Upper panel: comparison of observational and theoretical photon temperature in D$\Lambda$CDM with $\Omega_{\Lambda 2} = 10^{-4}$. Lower panel: the ratios of $T_\gamma$ to that of S$\Lambda$CDM. We note that the observed redshift translate a scale factor by $a = 1/(1 + z)$. ......................... 28

3.5 Illustration of constraints on $m - \Omega_{\Lambda 2}$ plane from observational temperatures. The black-solid line shows the upper limits of parameters by Eqs. (3.11), (3.12), and (3.13), and other lines indicate the upper limits obtained by same analysis as [37]. Upper panel: constraints from the temperature at $z > 1$. Lower panel: constraints from the temperature at $z < 1$. ......................... 30

3.6 Illustration of the ionization fraction as a function of $1 + z$ with and without a decaying-$\Lambda$. ....................................................... 31
LIST OF FIGURES

3.7 Illustration of visibility function $g$ as function of scale factor. Peak of $g$ show epoch of photon decoupling. When $m$ increase, photon decoupling occur at earlier epoch. 32

3.8 Magnitude-redshift relation. The solid line is the theoretical curve in DÃ€CDM with $(\Omega_{\Lambda 2}, m) = (10^{-2}, 0.03)$ and the dotted line is the model with $\Lambda = k = 0$. 35

3.9 Probability distribution of $\Omega_{\Lambda}$ from the magnitude-redshift relation of SNIa with $(\Omega_{\Lambda 2}, m) = (10^{-2}, 0.03)$. The 95.4% confidence limits are indicated by the shaded regions. 36

3.10 Angular power spectrum, $l(l + 1)C_l/2\pi$, obtained by observation released before WMAP first year (top panel) and WMAP (bottom) [101]. 37

3.11 Evolution of density perturbation in DÃ€CDM model at $k = 1.183 \times 10^{-3}$ (top panel), 0.026 (middle) and 1.0 Mpc$^{-1}$ (bottom), corresponded to $l = 1$, $l = 220$ (last scattering surface) and 1.0 Mpc$^{-1}$ (cluster scale). 51

3.12 Comparison of the angular power spectrum in the decaying $\Lambda$ model with the WMAP observation data[47] and BOOMERanG [111]. The solid line is the result of SÃ€CDM. The dashed, the dot-dashed and the dotted lines are those of DÃ€CDM with $(\Omega, \Omega_{\Lambda 2}, m) = (10^{-4}, 0.5), (10^{-4}, 1.0)$ and $(10^{-4}, 1.2)$, respectively. 52

3.13 Comparison of the angular power spectrum in the decaying $\Lambda$ model with the WMAP observation data [47] & BOOMERanG [111]. The solid line is the result of SÃ€CDM. The dashed, the dot-dashed and the dotted lines are those of DÃ€CDM with $(\Omega, \Omega_{\Lambda 2}, m) = (3.2 \times 10^{-6}, 3.0), (10^{-5}, 3.0)$ and $(1.3 \times 10^{-5}, 3.0)$, respectively. 53

3.14 Top panel: Acoustic oscillations. Spring and balls schematically represent fluid pressure and effective mass, respectively (From Ref.[112]). Bottom: Effects of baryon density on the temperature fluctuation. When $R(\propto \rho_b/\rho_{\gamma})$ increase, the relative height of odd peaks and even peaks boosts. 54
### LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.15</td>
<td>Constraint on $m - \Omega_{A2}$ plane from WMAP first year. Drawn lines correspond to 1, 2 and 3$\sigma$ confidence limit. The labeled no Big-Bang region indicate that $T_{\gamma}$ is negative at $a &lt; 1$.</td>
</tr>
<tr>
<td>3.16</td>
<td>An example of parameter search by Markov Chain Monte Carlo. The vertical line indicates the parameter $\theta$ and the horizontal axis is the iteration numbers. Calculations began the iteration number $\sim 100$ converge to a certain value which can be determined by statistical analysis such as the $\chi$ fitting [102]. In this thesis, all parameters finally determined in this chapter are determined with the same procedure which convergence levels are checked statistically.</td>
</tr>
<tr>
<td>3.17</td>
<td>Probability distribution function of $m$ (top panel) and $\Omega_{A2}$ (top panel). The vertical lines mean the 95.4% confidence levels of individual parameters.</td>
</tr>
<tr>
<td>3.18</td>
<td>Constraints on $\log \Omega_{A2} - \Omega_b h^2$ (left top), $\log \Omega_{A2} - \Omega_{CDM} h^2$ (right top), $\log \Omega_{A2} - n_s$ (left middle), $\log \Omega_{A2} - z_{re}$ (right middle), $\log \Omega_{A2} - \Omega_A$ (left bottom), and $m - \log \Omega_{A2}$ (right bottom) from WMAP first year results. The curves correspond to the 68.3% and 95.4% confidence levels.</td>
</tr>
<tr>
<td>3.19</td>
<td>Constraints on $m - \Omega_b h^2$ (left top), $m - \Omega_{CDM} h^2$ (right top), $m - n_s$ (left middle), $m - z_{re}$ (right middle), and $m - \Omega_A$ (bottom) from WMAP first year results. The curves correspond to the 68.3% and 95.4% confidence levels.</td>
</tr>
<tr>
<td>3.20</td>
<td>Constraints on $\log \Omega_{A2} - \Omega_b h^2$ (left top), $\log \Omega_{A2} - \Omega_{CDM} h^2$ (right top), $\log \Omega_{A2} - n_s$ (left middle), $\log \Omega_{A2} - z_{re}$ (right middle), $\log \Omega_{A2} - \Omega_A$ (left bottom), and $m - \log \Omega_{A2}$ (right bottom) from WMAP three year results. The curves correspond to the 68.3% and 95.4% confidence levels.</td>
</tr>
<tr>
<td>3.21</td>
<td>Constraints on $m - \Omega_b h^2$ (left top), $m - \Omega_{CDM} h^2$ (right top), $m - n_s$ (left middle), $m - z_{re}$ (right middle), and $m - \Omega_A$ (bottom) from WMAP first three results. The curves correspond to the 68.3% and 95.4% confidence levels.</td>
</tr>
</tbody>
</table>
LIST OF FIGURES

3.22 Illustration of likelihood functions from WMAP first and three year. ................................................................. 66

A.1 Illustrations of evolution of the energy density of photons, neutrinos and electrons before the primordial nucleosynthesis era. The decoupling of electrons around $5 \times 10^9$ K is due to the pair-annihilation, where the small increase in the temperature changes the results of BBN in some degree. ................................. 80

B.1 (a): Acoustic oscillation: Photon pressure resists gravitational compression of the fluid setting up acoustic oscillations. Spring and balls schematically represent fluid pressure and effective mass, respectively. (b): Bryon drag decreases the sound horizon the increase the gravitational mass, causing more infall and a net zero-point displacement. $\Psi$ indicate the metric perturbation in the conformal Newtonian gauge and $\Theta = \delta/4$ [99]. ................. 85

C.1 Comparison of WMAP first and three year data and angular power spectrum obtained from these. Since newest WMAP results can determine the acoustic peaks up to third peak, we can obtain the cosmological parameters more precisely. ........ 90
List of Tables

1.1 Example of parameterization of cosmological term adopted by recent studies. $A, B, C, D$ and $\epsilon$ are constants. $a$ and $H$ is the scale factor and Hubble parameter ($\dot{a}/a$). Other models for $\Lambda$ term are seen in Refs. [32, 31]. .............................. 5

3.1 Effect of a decaying-$\Lambda$ on the neutrino temperature, redshift and temperature at photon decoupling, and ionization fraction at $z = 0$. ......................................................... 31

3.2 Comparison of parameters in $\Lambda$CDM and $\Delta$CDM. parameters in $\Delta$CDM model are obtained by MCMC calculation and these in $\Lambda$CDM model is by LAMBDA site [113] .............. 58

A.1 Evolution of pressure and density of electrons. ................. 82
Abstract

We investigate the evolution of the universe with a decaying cosmological term $\Lambda$. Among many functional forms of $\Lambda$, we consider two cases. First we take into account as the $\Lambda$ term the another gravitational law of the Brans-Dicke theory. This theory is based on a more fundamental approach, where the covariant formulation of the gravitational theory is satisfied. We constrain critical parameters included in the theory with use of both the observational abundances of the light elements and the WMAP data. We find that the theory is consistent with the present observations, though the differences from the standard Friedmann cosmology becomes smaller compared to the previous studies.

Next we consider in the context of the Friedmann cosmology a decaying-$\Lambda$ which is a function of the scale factor. The cosmological term in this model is larger in the early universe, but the radiation energy density for the redshift $z < 10^4$ is lower compared to the model with the cosmological constant. We find that the effects of the decaying-$\Lambda$ term on the cosmic expansion rate is negligible at the redshift $z < 2$. On the other hand, the decrease in the radiation density affects the cosmic thermal evolution after the photon decoupling. We estimate the photon decoupling epoch by the visibility function and find that the photon decoupling can be occurred at $z \sim 2000$ with upper limit of parameter set, which indicates that the ionization occurs faster by $\Delta z \sim 1000$ compared to the standard $\Lambda$CDM model.

Furthermore we investigate the effects of the decaying-$\Lambda$ term on the temperature fluctuation of the cosmic microwave background. We construct the Bolzmann equation for photons based on the cosmological perturbation theory, where effects of decaying-$\Lambda$ is included. We calculate the CMB anisotropy with decaying-$\Lambda$ and find that decaying-$\Lambda$ affects CMB significantly. Comparing the angular power spectrum with the Wilkinson Microwave Anisotropy Probe data, we derive the best fit values of the cosmological parameters. Using the statistical analysis of the Markov chain Monte-Carlo, we have determined six standard cosmological parameters and two parameters in our
model. As a consequence, we find that cosmological parameters can be varied by ten percents within the standard deviation of 95.4% confidence regions.

We conclude that at present observational levels, we cannot exclude possibilities of variable-$\Lambda$ such as investigated in this thesis of special forms of Brans-Dicke $\Lambda$ cosmology and decaying-$\Lambda$ models. We propose that if the reionization parameters are constrained severely by the observation, the possibility of varying cosmological terms should be clarified. This also limits the properties of the first object which is the most preferential subjects in astrophysics.
Chapter 1

Introduction

1.1 Historical Review

Big-bang cosmology has succeeded in many observations from the early universe to produce light elements, H, He, D and Li, to present cosmic microwave background in the context of the expanding universe. On the other hand, some non-standard cosmologies have been proposed as another possibility instead of general relativity. Non-standard cosmologies correspond to the model that include scalar field such as scalar-tensor gravity, or contain usually unknowns matter and/or energy [1, 2, 3, 4].

Albert Einstein introduced constant cosmological term \( \Lambda \) for the universe to be static. However he discarded it due to the discovery of the expanding universe by Edwin Hubble. It is contradictory that analytical solutions of the Einstein equation with a constant-\( \Lambda \) designate the accelerating universe, which had been found by Gerge Lemaitre, where a static solution of the above equation are unstable. After that, the \( \Lambda \) term has not been paid attention, since many observations seem to be explained by the simple solution of the Einstein equation, where only homogeneous and isotropic metric has been assumed.

On the other hand, during the progress of the elementary particle physics, the inflation theory has been proposed as a consequence of the grand unified theory. While the inflation theory explains the long-standing problems of horizon and flatness, the theory assumes the vast release of the vacuum energy [2, 3]. The vacuum energy is now considered to be \( \Lambda \)-term (the historical review of the cosmological term the inflation theory are introduced in
Fig. 1.1: Illustration of effective magnitude-redshift relation of Type Ia supernovae from Supernovae Cosmology Project and the Calán/Tololo Supernova Survey. The effective magnitude $m_{\text{eff}}$ includes corrections for the apparent magnitude. The solid curves are the theoretical $m_B(z)$ for a range of cosmological models with zero cosmological constant: $(\Omega_M, \Omega_{\Lambda}) = (0, 0)$: top, $(1, 0)$: middle and $(2, 0)$: bottom. The dashed curves are for a range of flat cosmological models: $(\Omega_M, \Omega_{\Lambda}) = (0, 1), (0.5, 0.5), (1, 0)$ and $(1.5, -0.5)$ [5].

Recent observations of high redshift type Ia supernova (SNIa) suggest strongly that the expansion of the Universe is accelerating [5, 6, 7, 8]. Figure 1.1 shows the results of luminosity-redshift relation of SNIa by Supernova Cosmology Project [5]. These observational results suggest the accelerating expansion of the universe because the distant supernovae (large effective magnitude $m_B$ in Fig. 1.1) have smaller velocity where $z$ is related to the velocity of SNIa from observers, compared to the cold dark matter model.

Since the standard Friedmann cosmology with the matters and radiations cannot explain the recent SNIa observation, we should need other theoreti-
Fig. 1.2: Contours of 68.3%, 95.4% and 99.7% confidence regions on amount of matter component $\Omega_m$ and equation of state of dark energy $w$ from the Supernova Legacy Survey alone (solid lines), baryon acoustic oscillation alone (dotted lines), and the joint confidence contours (dashed line) [8].

To explain this cosmic acceleration, there are two theoretical models to be employed. First, we can assume the existence of dark energy which is a cosmic fluid with a negative pressure; for instance cosmic quintessence [9] or phantom energy [10]. Second, we could adopt modified gravitational laws having a scalar field of the Brans-Dicke cosmology [11] or dark matter inflowing from the extra dimension in the brane world cosmology [12, 13] (More details of theoretical approaches for cosmic acceleration are reviewed in Refs.[4, 14]).

Recently many astronomical observations, e.g. temperature fluctuation of cosmic microwave background (CMB) [16, 17], baryon acoustic oscillation (BAO) [18] and high-redshift gamma-ray burst [19], are able to put constraint on property of dark energy, especially equation of state (EOS); the ratio of the
CHAPTER 1. INTRODUCTION

pressure relative to the energy density, which is expressed as \( w \). Figure 1.2 shows the constraint on between the matter component \( \Omega_m \) and constant EOS of dark energy, where the constraint is obtained from luminosity-redshift relation of SNIa and BAO [8]. This constraint indicate that EOS is close to \(-1\), namely present dark energy is similar to the cosmological term.

If we consider that the dark energy is equivalent to \( \Lambda \), there remains a *cosmological constant problem* [20]. Observationally we know that the present energy density of \( \Lambda \) is estimated from Hubble constant \( H_0 \):

\[
\rho_\Lambda \sim \rho_{cr} = \frac{3 H_0^2}{8 \pi G} \sim 10^{-47} \text{GeV}^4.
\]

Meanwhile, the vacuum density at Planck scale is evaluated as follows [14, 21]

\[
\rho_{\text{vac}} \sim 10^{74} \text{GeV}^4
\]

which is about \(10^{121}\) orders of magnitude larger than the observed value. To solve this problem, it is natural to consider that the cosmological term decreases from the large value at the early epoch to the present value. Many functional forms of the cosmological term have been suggested; for example the function of the cosmic scale factor [22] and the scalar field [23]. The model of a \( \Lambda \) term is seen in Tab. 1.1. The cosmology with variable-\( \Lambda \) term have been investigated related to the evolution of the cosmic scale factor [32], the matter power spectrum [26, 24], Galaxy number count and apparent magnitude relations [30].

### 1.2 The cosmological term as dynamical quantities

The possibility of \( \Lambda \) (or dark energy) interacted with other energy has been discussing. The vacuum energy coupled with baryon is ruled out because baryon-antibaryon created by vacuum decay cause pair-annihilation and produce gamma ray flux [33].

The vacuum energy decayed into photon could affect the cosmological evolution significantly. Assuming the ratio of the vacuum energy to the radiation is constant at radiation dominated era \((z > 10^5)\), Freese et al. investigated effects on primordial nucleosynthesis and obtain limit of vacuum to photon ratio is \(\lesssim 0.07\) [33]. In this model, constraints from CMB intensity on
Table 1.1: Example of parameterization of cosmological term adopted by recent studies. $A, B, C, D$ and $\epsilon$ are constants. $a$ and $H$ is the scale factor and Hubble parameter $(\dot{a}/a)$. Other models for $\Lambda$ term are seen in Refs. [32, 31].

<table>
<thead>
<tr>
<th>Decay Law</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_\Lambda = A + B a^{-m}$</td>
<td>[22, 35]</td>
</tr>
<tr>
<td>$d\rho_\Lambda/d \ln H \propto H^2$</td>
<td>[24]</td>
</tr>
<tr>
<td>$\rho_\Lambda \propto H$</td>
<td>[25]</td>
</tr>
<tr>
<td>$\rho_\Lambda \propto a^{-n}$</td>
<td>[26, 30]</td>
</tr>
<tr>
<td>$\rho_\Lambda = C + D a^{-3+\epsilon}$</td>
<td>[27]</td>
</tr>
<tr>
<td>$\rho_\Lambda \propto a^{-2}$</td>
<td>[29]</td>
</tr>
<tr>
<td>$\rho_\Lambda \propto \dot{a}/a$</td>
<td>[28]</td>
</tr>
<tr>
<td>$\rho_\Lambda \propto H^n$</td>
<td>[30]</td>
</tr>
</tbody>
</table>

Vacuum energy and radiation ratio give the ratio to be $\sim 10^{-3}$ [34]. On the other hand, the thermal evolution with a phenomenological decaying-$\Lambda$ term has been found to affect the cosmological evolution after recombination. In a $\Lambda$ term as a function of the scale factor, the radiation and matter temperature are lower compared to the standard cold dark matter model with constant $\Lambda \Lambda$CDM) [35] and the molecular formation is occurred at earlier epoch by $\Delta z < 10^3$ [36]. Furthermore, the radiation temperature in this model is able to be higher compared with SACDM model depending on parameters and is found to be consistent with the observational result at $z < 4$ [37].

The existence of CMB was predicted by the hot Big-Bang model invented by Gerge Gamow [38]; the relic of the hot state in the early universe will be observed by a few kelvin radiation at present [39]. Their prediction was discovered accidentally by Penzias and Wilson [40]. Although the sky map of CMB shows almost isotropic temperature distribution, small anisotropy is predicted from the inflation theory. Temperature anisotropy was observed by Cosmic Background Explorer (COBE) satellite in 1992 [41] 1. Figure 1.3 illustrates the observational results of the spatial properties of the cosmic microwave background radiation based on the full 4 years of COBE Differential Microwave Radiometer (DMR) experiment [15]. On the other hand, CMB

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1Dr. John C. Mather and George F. Smoot, Main staff of COBE team, have won the 2006 Nobel Prize for Physics for their discovery of the blackbody form and anisotropy of the cosmic microwave background radiation.
temperature fluctuation is able to put constraint on cosmological parameters. The CMB anisotropy observed by Wilkinson Microwave Anisotropy Probe (WMAP) [42] put tightly constraints on cosmological parameters. Paying attention to CMB observed by WMAP, we examine the limits on the thermal evolution with a decaying-$\Lambda$ cosmology.

As one of the recent interesting topic, the first star formation has been investigated. Figure 1.4 shows cosmic thermal evolution after photon last scattering at $z \sim 1100$. The first star is formed between the last scattering surface and the most distant object known at present $z = 6.96$ [43]; the period between $z \sim 1100$ and time of first star formed is called as *dark age* as shown in Fig.1.4. From the estimate using colling diagram, the first star could in the decaying $\Lambda$ model be formed at earlier epoch by $\Delta z \sim 20$ and its mass is $\sim 10^6 M_\odot$ as shown in Fig.1.5 [36]. In the meantime, from an observational approach, the CMB polarization observed by WMAP satellite predicts the first object formed around $z = 10$ via measured reionization redshift [17, 44] (the relation of the first star and reionization is explained in Refs.[45, 46]). We can also estimate the first star era using the CMB anisotropy in the decaying-$\Lambda$ cosmology.

In the present paper, we first study the Brans-Dicke cosmology with a
Fig. 1.4: Overview of the main events after the last-scattering surface $z = 1100$ with the top axis showing the age of the universe and the bottom axis the corresponding redshift. Blue represents atomic formation regions, and red, ionized regions [45].

decaying-$\Lambda$ term. The Big-Bang nucleosynthesis limits the parameters. Next we investigate the thermal history of the universe with a decaying $\Lambda$ into photon (hereafter we call DACDM). We derive the best fit values of the cosmological parameters, by comparing the CMB angular power spectrum with the WMAP data. We also discuss the epoch of the first object formation using the reionization redshift. In chapter 2, we explain the evolution of the universe in Brans-Dicke cosmology with a $\Lambda$ term and the constraint are obtained from the on primordial nucleosynthesis. In chapter 3, we describe the thermal evolution and the effects on photon decoupling in DACDM model and we give the constraint on cosmological parameters from SNIa and CMB temperature fluctuation. Summary and discussion are given in chapter 4.
Fig. 1.5: Illustration of the estimation of first objects formation epoch and their masses obtained from the relation between the virial temperature and redshift in standard CDM (medium dotted line), SACDM (thin dotted lines) and a decaying-\Lambda model (thick dotted lines). This cooling diagram are characterized by the time scales of the free-fall $t_{ff}$, cooling $t_{cool}$ and Hubble expansion $t_H$. Three regions for the formation of the first objects are indicated by $t_H < t_{cool}$, no formation occurs; $t_{ff} < t_{cool} < t_H$, the molecule clouds become the first objects though it is not effective; and $t_H > t_{cool}$, the formation occurs most efficiently. We should note that the formation of the first objects should be determined by the detailed hydrodynamic simulations, where we can compare the theoretical prediction with future observation (From [36]).
Chapter 2

Big-bang nucleosynthesis in the Brans-Dicke cosmology with a decaying $\Lambda$ term

We investigate the big-bang nucleosynthesis in a Brans-Dicke model with a decaying $\Lambda$ term using the Monte-Carlo method and likelihood analysis. We find that not only the cosmic expansion rate differs appreciably from that of the standard model, but also the produced abundances of helium, deuterium and lithium are consistent with the observed ones within the uncertainties in nuclear reaction rates when the baryon to photon ratio $\eta = (5.47 - 6.64) \times 10^{-10}$, which is in agreement with the value deduced from WMAP [23, 48].

2.1 Historical remark

The standard model of big-bang nucleosynthesis (SBBN) has succeeded in explaining the origin of the light elements $^4\text{He}$, D, and $^7\text{Li}$. Although the value of the baryon-to-photon ratio $\eta$ has been derived from the observations of the Wilkinson Microwave Anisotropy Probe (WMAP) [49] to be $\eta_{10} = 6.1^{+0.3}_{-0.2}$, the value seems to be inconsistent with the results of SBBN [50]. Contrary to the excellent concordance with $\eta$ of WMAP for D, the abundance of $^4\text{He}$ by SBBN is rather low compared to that from WMAP. Therefore, non-standard models of BBN have been proposed with the Friedmann model modified [51].

For non-standard models, scalar-tensor theories have been investigated

\footnote{Big-bang nucleosynthesis in a modified Brans-Dicke cosmology has been investigated for the first time numerically by Ref. [61].}
CHAPTER 2. BIG-BANG NUCLEOSYNTHEISIN IN THE BRANS-DICKE COSMOLOGY WITH A DECAYING $\Lambda$ TERM

For a simple model with a scalar field $\phi$, it is shown that a Brans-Dicke (BD) generalization of gravity with torsion includes the low-energy limit string effective field theory [57]. Related to the cosmological constant problem, a Brans-Dicke model with a varying $\Lambda(\phi)$ term [BD$\Lambda$] has been presented, and also investigated from the point of inflation theory [58]. Moreover, it is found that the linearized gravity can be recovered in the Randall-Sundrum brane world [59]. Furthermore, scalar-tensor cosmology is constrained by a $\chi^2$ test for the WMAP spectrum [60] where the present value of the coupling parameter $\omega_0 = \omega(\phi_0)$ is bounded to be $\omega_0 > 50 \ (4\sigma)$ and $\omega_0 > 1000 \ (2\sigma)$ in the limit to BD cosmology.

BBN has been studied in BD$\Lambda$ [61, 62]. The relation between BBN and scalar-tensor gravity is investigated with the inclusion of $e^+e^-$ annihilation in the equation of state, where the present value of the scalar coupling has been constrained [63]. Therefore, it is worthwhile to check the validity of BD$\Lambda$ related to the recent observations. Here, we investigate to what extent BBN in the BD$\Lambda$ model can be reconciled with $\eta$ from WMAP.

2.2 Brans-Dicke cosmology with a decaying-$\Lambda$ term

The field equations for BD$\Lambda$ are written as follows [61]:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + g_{\mu\nu} \Lambda = \frac{8\pi}{\phi} T_{\mu\nu} + \frac{\omega}{\phi^2} (\phi_{,\mu;\nu} - \frac{1}{2} g_{\mu\nu} \phi_{,\alpha} \phi^{,\alpha})$$
$$+ \frac{1}{\phi} (\phi_{,\mu;\nu} - g_{\mu\nu} \Box \phi), \tag{2.1}$$

$$R - 2\Lambda - 2\frac{\partial \Lambda}{\partial \phi} = \frac{\omega}{\phi^2} \phi_{,\mu} \phi^{,\mu} - \frac{2\omega}{\phi} \Box \phi, \tag{2.2}$$

where $\omega$ is the coupling constant.

The equation of motion is obtained with use of the Friedmann-Robertson-Walker metric:

$$ds^2 = -dt^2 + a(t)^2 \left\{ \frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right\}, \tag{2.3}$$

where $a(t)$ is the scale factor and $k$ is the curvature constant. Let $x$ be a scale factor normalized to its present value, i.e., $x = a/a_0$, then we get from the
CHAPTER 2. BIG-BANG NUCLEOSYNTHESIS IN THE BRANS-DICKE COSMOLOGY WITH A DECAYING \( \Lambda \) TERM

Fig. 2.1: Time evolution of the scale factor for BD\( \Lambda \) (\( \mu = 0.7 \), \( t_0 = 13.7 \) Gyr, and \( \eta_{10} = 6.1 \)) and Friedmann model.

\((0,0)\) component in Eq. (2.1)

\[
\left( \frac{\dot{x}}{x} \right)^2 + \frac{k}{3} - \frac{\Lambda}{3} \frac{\phi^2}{x^2} - \frac{\dot{x}}{x} \frac{\dot{\phi}}{\phi} = \frac{8\pi}{3} \frac{\rho}{\phi},
\]

(2.4)

where \( \rho \) is the energy density.

We assume the simplest case of the coupling between the scalar and matter fields:

\[
\Box \phi = \frac{8\pi}{2\omega + 3} \mu T_{\nu}^{\nu},
\]

(2.5)

where \( \mu \) is a constant. Assuming a perfect fluid for \( T_{\mu\nu} \), Eq. (2.5) reduces to

\[
\frac{d}{dt}(\dot{\phi}x^3) = \frac{8\pi \mu}{2\omega + 3} (\rho - 3p)x^3,
\]

(2.6)

where \( p \) is the pressure.
A particular solution of Eq. (2.2) is obtained from Eqs. (2.1) and (2.5):

$$\Lambda = \frac{2\pi (\mu - 1)}{\phi} \rho_{m0} x^{-3},$$  \hspace{1cm} (2.7)

where $\rho_{m0}$ is the matter density at the present epoch.

The gravitational “constant” $G$ is expressed as follows

$$G = \frac{1}{2} \left( 3 - \frac{2\omega + 1}{2\omega + 3\mu} \right) \frac{1}{\phi}. \hspace{1cm} (2.8)$$

The radiation density $\rho_r$ contains the contributions from photons, neutrinos, electrons and positrons at $t \leq 1$ s. The total energy density is given as

$$\rho = \rho_m + \rho_r, \quad \rho_r = \rho_{rad} + \rho_{\nu} + \rho_{e^\pm}. \hspace{1cm} (2.9)$$

Here the energy density of matter varies as $\rho_m = \rho_{m0} x^{-3}$. The radiation density $\rho_r = \rho_{r0} x^{-4}$ except $e^+ e^-$ epoch where $e^+ e^-$ annihilation changes the relation $T_r \sim x^{-1}$. We assume that the pressure satisfies $p = \rho/3$, which is legitimate only for relativistic particles. Then, Eq. (2.6) is integrated to give

$$\dot{\phi} = \left( \frac{8\pi\mu}{2\omega + 3} \rho_{m0} t + B \right) \frac{1}{x^3}, \hspace{1cm} (2.10)$$

where $B$ is a constant [61]. Although the relation $p = \rho/3$ does not hold during the epoch of $e^+ e^-$ annihilation, as pointed out by Damour and Pichon (1999), the inclusion of $\phi$ measures small deviation from SBBN in our interest, so that our solution (2.10) can reasonably describe the evolution of $\phi$ except during the annihilation epoch. We consider that $B$ affects the evolution of $x$ significantly from the early epoch to the present compared to the contribution from $e^+ e^-$ annihilation. As a consequence, the neutron to proton ratio is seriously affected by the initial value of $B$. Considering the important contribution of $\rho_{e^\pm}$ to BBN, we examine the effects of $e^+ e^-$ annihilation in Sec. 2.4. Hereafter we use the normalized values: $B^* = B/(10^{-24} \text{ g s cm}^{-3})$ and $\eta_{10} = 10^{10} \eta$. The coupled equations (2.4), (2.7) and (2.10) can be solved numerically with the specified quantities: for macroscopic quantities, $G_0 = 6.672 \times 10^{-8} \text{ dyn cm}^2 \text{ g}^{-2}$ [64], $H_0 = 71 \text{ km} \text{ s}^{-1} \text{ Mpc}^{-1}$ [49], and $T_{r0} = 2.725 \text{ K}$ [65]; for microscopic quantities, the number of massless neutrino species is 3 and the half life of neutrons is 885.7 s [66]. Although we adopt $\omega = 500$, the epoch of the appreciable growth of $|\dot{G}/G|$ is $t < 10^3$ s regardless of the value $\omega$ [61]. Therefore,
even if we adopt a value $\omega > 500$ [67], we can arrive qualitatively at the same conclusion by changing the parameters $\mu$ and $B^*$. We impose the condition $|\dot{\phi}/\phi|_0 = |\dot{G}/G|_0 < 10^{-13} \text{ yr}^{-1}$ which is the most severe observational limit [68].

BD$\Lambda$ is an extension of the original form of BD and reduces to the Friedmann model when $\phi = \text{constant}$, $\mu = 1$ and $\omega \gg 1$. We have $\Lambda < 0$ if $\mu < 1$, and $\phi G > 0$ if both $\mu > 3$ and $\omega \gg 1$. Figure 2.1 shows the evolution of the scale factor for BD$\Lambda$ with the relevant parameters in the present study and for the Friedmann model. Note that the difference in the expansion rate at $t < 10$ s in BD$\Lambda$. In particular, around $t = 5$ s, the curve $x$ in BD$\Lambda$ crosses that of the Friedmann model, which will have noticeable effects on BBN. Since $\Lambda$ is proportional to $\rho_m/\phi$, $\mu$ affects the evolution of the scale factor around the present epoch. In our BD$\Lambda$ model, if $|B^*|$ increases, the expansion rate increases at $t < 10 - 100$ s. The change in $G$ between the recombination and the present epoch is less than 0.05 (2$\sigma$) from WMAP [60], which is consistent with BD$\Lambda$ since $|(G - G_0)/G_0| < 0.005$ at $t > 1$ yr.

2.3 Primordial nucleosynthesis in Brans-Dicke model with $\Lambda$ term

Before numerical calculation of BBN in BD$\Lambda$ model, let us explain about the theory of standard BBN. For $T > 10^{10}$ K, the universe is the thermal
equilibrium among elementary particles (see Appendix A), and the weak interactions occur as follows:

\[ n + e^+ \leftrightarrow p + \bar{\nu}_e, \]
\[ n + \nu_e \leftrightarrow p + e^- \]

where \( n, p, e^-, e^+, \nu_e \) and \( \bar{\nu}_e \) are neutrons, protons, positrons, electron-neutrinos and its anti-neutrinos, respectively. When temperature is \( \sim 10^{10} \) K, the weak interaction freeze out. The ratio of neutrons and proton at this epoch is crucial for the light element abundances. When temperature reach to \( \sim 10^9 \) K, the light element synthesis begin from following reaction

\[ p + n \rightarrow D + \gamma. \]

Light element up to \( ^7\text{Be} \) are synthesized by nuclear reaction network shown in Fig. 2.2. The amount of light element depends on the baryon density in the universe. Figure 2.3 illustrates abundances of \(^4\text{He}, \ D/H, \) and \(^7\text{Li}/\text{H} \) with \( 1\sigma \)
uncertainty of nuclear reaction rate. By comparing the theoretical and observational abundances, we can measure the baryon density in the universe. On the other hand, baryon density is measured by CMB observation. However, observational abundances of $^4$He and $^7$Li/H is inconsistent with abundance of D/H and $\eta$ obtained by WMAP as seen in Fig. 2.3.

Let move on to the BBN in BD$\Lambda$ model. In BD$\Lambda$ model, the expansion rates differ from the standard Friedmann cosmology shown in Fig. 2.1. That affect the synthesis of light elements in the early era, because the neutron to proton ratio is sensitive to the expansion rate. Then we examine the light element abundances in BD$\Lambda$ model. For the BBN calculation, we use the reaction rates [69] based on NACRE [70]. We adopt the observed abundances of $^4$He, D/H, and $^7$Li/H as follows:

$$Y_p = 0.2391 \pm 0.0020, \quad [71],$$
$$D/H = 2.78^{+0.44}_{-0.38} \times 10^{-5} \quad [72],$$
$$^7\text{Li}/H = (2.19 \pm 0.28) \times 10^{-10} \quad [73].$$

Since the results of WMAP constrain cosmological parameters, we calculate
the abundance of $^4\text{He}$, D and $^7\text{Li}$ paying attention to the value $\eta_{10} = 6.1$. First, we carry out the BBN calculations with use of the adopted experimental values of nuclear reaction rates given in NACRE. Figure 2.4 illustrates $^4\text{He}$, D/H and $^7\text{Li}/H$ for $\mu = 0.7$. The abundance of $^4\text{He}$ is very sensitive to both $B^*$ and $\mu$; it increases if $|B^*|$ or $\mu$ increases. On the other hand, D and $^7\text{Li}$ are more sensitive to $\mu$ than $B^*$ as seen from Fig. 2.5. To obtain the reasonable regions for $\mu$ and $B^*$, we calculate the $\chi^2$ for $\eta_{10} = 6.1$

$$
\chi^2 = \sum_{i=1}^{3} \frac{(Y_{\text{obs},i} - Y_{\text{th},i})^2}{\sigma_{\text{obs},i}^2},
$$

where $Y_{\text{obs}}$ and $\sigma_{\text{obs}}$ is observational abundances and uncertainty, $Y_{\text{th}}$ is theoretical abundances, $i = 1, 2$ and 3 indicates $^4\text{He}$, D/H and $^7\text{Li}/H$, respectively. Figure 2.6 illustrates the confidence region of 1, 2 and 3$\sigma$ confidence regions. As a result, $^4\text{He}$, D/H and $^7\text{Li}/H$ are consistent with $\eta$ obtained from WMAP in the range $0.4 \leq \mu \leq 0.9$ and $-5.0 \leq B^* \leq 5.0$ at 95.4% confidence levels.

Next, we perform the Monte-Carlo calculations to obtain the upper and lower limits to individual abundance using the uncertainties in the nuclear reaction rates [69]. Figure 2.7 illustrates $^4\text{He}$, D/H and $^7\text{Li}/H$ with $2\sigma$ uncertainties for $B^* = -2.5$ and $\mu = 0.7$, one of best fit parameter sets from Fig.
Fig. 2.6: Constraints on the $\mu - B^*$ plane corresponded to 1, 2 and 3\textsigma confidence regions from observational abundance and baryon density obtained by WMAP.

2.6. The light-shaded areas denote the regions of observed abundances, and the dark-shaded area indicates the limit obtained from WMAP. While the obtained values of $^4\text{He}$ and D are consistent with $\eta$ by WMAP, the lower limit in $^7\text{Li}$ is barely consistent.

2.4 Effects of $e^+e^-$ annihilation on BBN

In the previous sections, we have assumed the equation of state $p = \rho/3$ in Eq. (2.6) to obtain Eq. (2.10) at the epoch of $e^+e^-$ annihilation. Let us discuss the effects of $e^+e^-$ annihilation on the evolution of the scalar field and the scale factor due to the deviation from the relation $p = \rho/3$. The electron-positron pressure and energy density are written with the variable $\zeta = m_e/k_B T_r$ as follows,

$$p_e = \frac{2m_e^4}{\pi^2\hbar^3} \sum_{n=1}^{\infty} (-1)^{n+1} \left( \frac{1}{n\zeta} \right)^2 K_2(n\zeta),$$

(2.11)
CHAPTER 2. BIG-BANG NUCLEOSYNTHESIS IN THE BRANS-DICKE COSMOLOGY WITH A DECAYING $\Delta$ TERM

Fig. 2.7: Light-element abundances vs. $\eta$ in BDA for $\mu = 0.7$ and $B^* = -2.5$. Dashed lines show $\pm 2\sigma$ uncertainties in nuclear reaction rates. The dark-shaded area indicates the constraint by WMAP and light-shaded areas denote regions of observational abundances.

$$\rho_e = 3 \rho_e + \frac{2m_e^4}{\pi^2 h^3} \sum_{n=1}^{\infty} (-1)^{n+1} \left( \frac{1}{n} \right) K_1(n \zeta),$$

(2.12)

where $h$ is Planck’s constant in units of $2\pi$, $k_B$ is Boltzmann’s constant, and $m_e$ is the electron rest mass. $K_i (i = 1$ and 2) are modified Bessel functions of order $i$ [e.g. Ref.[63]]. In the numerical calculations, the summations in Eqs. (2.11) and (2.12) are taken over $n = 1 - 10$. We can obtain the scale factor by integrating Eq. (2.4) with the aid of Eq. (2.6). To see the effects of $e^+e^-$, we take the form:

$$\dot{x} x^3 = \frac{8\pi \mu}{2\omega + 3} \int^t (\rho_e - 3p_e) x^3 dt + B.$$  

(2.13)

A direct comparison is made for the evolution of the scalar field. The results are shown in Fig. 2.8, where the solid line indicates the case of Eq. (2.13) with $B^* = -2.43$ and $\mu = 0.7$, and the broken line is the case of Eq. (2.10) with $B^* = -2.50$ and $\mu = 0.7$. These sets of parameters yield the same macroscopic
quantities given in Sec. . Although we can appreciate the slight difference at \( t < 10^3 \) s, it remains small during and after the stage of BBN. The effects on the evolution of the scale factor are minor and the change in \( Y_p \) is found to be at most 0.1% compared to that obtained in Sec. 2.3. We can conclude that since the effects of \( B \) in the range \(-10 \leq B^* \leq 10\) is much larger then those of \( e^+e^- \) annihilation, the deviation from the relation \( p = \rho/3 \) due to \( e^+e^- \) does not change our results qualitatively. However, we note that even small differences in \( Y_p \) may affect the detailed statistical analysis combined with theoretical and observational uncertainties performed in the previous sections.
2.5 Concluding remarks

We have carried out BBN calculations in the $\mu - B^*$ plane and obtain the ranges $0.4 \leq \mu \leq 0.0$ and $-5.0 \leq B^* \leq 5.0$ that are consistent with both the abundance observations and $\eta$ obtained from WMAP.

To evaluate the uncertainties in theory and observation, we calculate normalized likelihood distributions in BBN [74, 36]. In Fig. 2.9, we show the likelihood functions for $^4$He, D and $^7$Li. The combined distributions, $L_{47} = L_4 \cdot L_7$ and $L_{247} = L_2 \cdot L_4 \cdot L_7$ are shown in Fig. 2.10. We obtain the 95% confidence limit of $\eta$: $5.47 \leq \eta_{10} \leq 6.64$.

The consistency holds within 1σ errors for $^4$He and D, and 2σ for $^4$He, D and $^7$Li. Although new reaction rates recently published [75] will change the errors to some extent in the likelihood analysis, our conclusion holds qualitatively.

Our previous studies [62] showed $1 < \mu < 3$ if $\Lambda > 0$ for large values of $\omega$. In the present case, the $\Lambda$ term becomes negative in Eq. (2.7) for $\mu < 1$: this would not conflict with available observations and/or basic theory [76]. Alternatively, if we consider $\Lambda = \Lambda_0 + \Lambda(\phi)$ with $|\Lambda(\phi_0)/\Lambda_0| < 0.01$, then the cosmological term becomes consistent with the present observations. Although the evolutionary path in the early universe can deviate from the Friedmann model [61], parameters in BD $\Lambda$ must be searched in detail for values of $\omega > 500$ to obtain quantitative results of BBN. It is shown that negative energies are present in scalar-tensor theories, although it is not clear how to identify them definitely [77].

To avoid the apparent inconsistency for SBBN, effects of neutrino degeneracy, changes in neutrino species or other new physical processes have been included in models [51]. In our model, we need only a scalar field that could be related to string theory [57]. The original BD cosmology ($\mu = 1$) would be limited severely by the more accurate observation of light elements and/or future constraints for $\eta$ as shown in the present investigation.
CHAPTER 2. BIG-BANG NUCLEOSYNTHESIS IN THE BRANS-DICKE COSMOLOGY WITH A DECAYING $\Lambda$ TERM

Fig. 2.9: Likelihood function as a function of $\eta_{10}$ for $^4$He ($L_4$), D ($L_2$) and $^7$Li ($L_7$). The vertical lines indicate the upper and lower limit to $\eta$ by WMAP.

Fig. 2.10: Combined likelihood function for two $^4$He and $^7$Li ($L_{247}$) and three-elements D, $^4$He$^7$Li ($L_{47}$). We note that the combined likelihood function is consistent with the WMAP data, because theoretical prediction of deuterium by BBN agrees very well with the observational data of D/H and the prediction of $\eta$ by WMAP.
Chapter 3
Thermal evolution with decaying cosmological term in the context of Friemann universe.

In this chapter, we investigate the effects of a decaying-$\Lambda$ term on the cosmic thermal evolution during photon decoupling. In previous chapter, the $\Lambda$ term is assumed to be proportional to $\rho_{nr}$ that is, decays in proportion to $a^{-3}$. Here, we assume it to be $a^{-m}$ from the more general point of view, where we note that there exists possibility to change our conclusion. for non-standard cosmology such as the Bran-Dicke type one.

3.1 Cosmological Evolution

Einstein equation is written as follows,

\[ R_{\mu}^\nu - \frac{1}{2} g_{\mu}^\nu R = 8\pi GT_{\mu}^\nu. \]  

(3.1)

Energy momentum tensor $T_{\mu}^\nu$, right hand side of Eq.(3.1), is assumed to be perfect fluid

\[ T_{\mu}^\nu = \text{diag}(\rho, p, p, p). \]  

(3.2)

Using the Friedmann-Robertson-Walker metric,

\[ ds^2 = -dt^2 + a^2 \left( \frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right), \]

the Equation (3.1) and the energy-momentum conservation law $\nabla_\mu T_{\nu}^\mu = 0$, are written as follows:

\[ \left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} \rho - \frac{k}{a^2}, \]  

(3.3)
CHAPTER 3. THERMAL EVOLUTION WITH DECAYING COSMOLOGICAL TERM IN THE CONTEXT OF FRIELMANN UNIVERSE.

\[
\dot{\rho} = -3 \frac{\dot{a}}{a} (\rho + p),
\]

(3.4)

where \(a\) is the cosmic scale factor, \(k\) is the curvature and \(G\) is the gravitational constant. Equation (3.3) is the cosmic expansion rates and called as the Friedmann equation. We choose units such that \(c = 1\). The total energy density \(\rho\) and the pressure \(p\) after electron-positron annihilation are written as

\[
\rho = \rho_m + \rho_\gamma + \rho_\nu + \rho_\Lambda, \quad p = p_\gamma + p_\nu + p_\Lambda,
\]

(3.5)

where the subscripts \(m, \gamma, \nu\) and \(\Lambda\) indicate the non-relativistic matter (baryon plus cold dark matter), photon, neutrino and cosmological term, respectively.

The equation of state \(p/\rho\) for individual components is written as

\[
p/\rho = \begin{cases} 
1/3 & \text{relativistic particles}, \\
0 & \text{non-relativistic particles}, \\
-1 & \text{cosmological term}.
\end{cases}
\]

(3.6)

Here the energy density of matter and that of neutrino vary as \(\rho_m = \rho_{m0} a^{-3}\) and \(\rho_\nu = \rho_{\nu0} a^{-4}\), where the subscript 0 means the present value.

From Eqs. (3.4), (3.5) and (3.6), we get the evolution equation of the photon energy density after the epoch of electron-positron pair-annihilation:

\[
\frac{d\Omega_\gamma}{da} + 4 \frac{\Omega_\gamma}{a} = - \frac{d\Omega_\Lambda}{da},
\]

(3.7)

with density parameter \(\Omega_i\)

\[
\Omega_i = \frac{\rho_i}{\rho_{\text{crit}}}, \quad \rho_{\text{crit}} = \frac{3H_0^2}{8\pi G},
\]

where \(H_0\) is the Hubble constant in unit of km/sec/Mpc. We note that although Eq. (3.7) holds after BBN epoch, it has no relation to the present investigation; we are interested in the epoch of \(z < 10^4\).

In D\(\Lambda\)CDM model, evolution of photon is affected as seen in Eq.(3.7). In this work, we assume a functional form of \(\Lambda\) as follows [22, 35]:

\[
\Omega_\Lambda = \Omega_{\Lambda1} + \Omega_{\Lambda2} a^{-m},
\]

(3.8)

where \(\Omega_{\Lambda1}, \Omega_{\Lambda2}\) and \(m\) are constants. Note the present value of \(\Omega_\Lambda\) is expressed by \(\Omega_{\Lambda0} = \Omega_{\Lambda1} + \Omega_{\Lambda2}\). Formalism of this paper is based on previous studies by Refs.[35, 36]. Equation (3.7) in their works include not only photons.
but also three species massless neutrino. In this paper, we assume that a decaying-\(\Lambda\) affects on photon only because the evolution of photon fluctuation differs from that of massless neutrino and we need to separate photon from massless neutrino. Integrating Eq. (3.7) with (3.8), we obtain the photon energy density as a function of \(a\):

\[
\Omega_\gamma = \begin{cases} 
[\Omega_{\gamma 0} + \alpha (a^{4-m} - 1)] a^{-4} & (m \neq 4), \\
(\Omega_{\gamma 0} + 4\Omega_{\Lambda 2} \ln a) a^{-4} & (m = 4),
\end{cases}
\]  

(3.9)

where \(\alpha \equiv m\Omega_{\Lambda 2}/(4-m)\), \(\Omega_{\gamma 0} = 2.471 \times 10^{-5} h^{-2} (T_{\gamma 0}/2.725 \text{ K})^4\) is the present photon energy density, \(h\) is the normalized Hubble constant \((H_0 = 100 h \text{ km/sec/Mpc})\) and \(T_{\gamma 0}\) is the present photon temperature. Second terms are new ones in D\(\Lambda\)CDM model.

Figure 3.1 shows the evolution of energy densities in D\(\Lambda\)CDM model with the following cosmological parameters: \(\Omega_{\Lambda 2} = 10^{-4}, m = 1.2, h = 0.73\) [17], \(T_{\gamma 0} = 2.725 \text{ K}\) [65], \(\Omega_{\Lambda 0} = 0.763, k = 0\), and three species of massless neutrinos. From the results, a decaying-\(\Lambda\) affects the evolution of photon. The cosmological term decreases from the early time to the present. Since the first term in Eq. (3.8) dominates near the present epoch, the \(\Lambda\) term is nearly constant for low-\(z\). On the other hand, \(\rho_\gamma\) is affected by the decaying \(\Lambda\).

To see the effects of the decaying \(\Lambda\) on \(\rho_\gamma\), we calculate the photon temperature from Eq. (3.9). Following the Stefan-Boltzmann’s law, \(\rho_\gamma \propto T_\gamma^4\), the photon temperature evolves as follows [37]:

\[
T_\gamma = \frac{T_{\gamma 0}}{a} \times \left\{ \begin{array}{ll}
[1 + \alpha (a^{4-m} - 1)]^{1/4} & (m \neq 4), \\
[1 + 4\Omega_{\Lambda 2}/\Omega_{\gamma 0} \ln a]^{1/4} & (m = 4).
\end{array} \right.
\]  

(3.10)

If \(\Omega_{\Lambda 2}\) and/or \(m\) is very large, the total energy density becomes negative and they put constrain \(m\Omega_{\Lambda 2} \leq 10^{-3}\) [35]. In our analysis, the photon temperature becomes negative at some epoch of \(a < 1\). By excluding this kind of solution, we obtain the upper limits on \(\Omega_{\Lambda 2}\) and \(m\) from Eq. (3.10):

\[
\alpha < \Omega_{\gamma 0} \quad (m < 4).
\]  

(3.11)

In the case of \(m \geq 4\), we assume \(T_\gamma > 0\) until primordial nucleosynthesis.
Fig. 3.1: Evolution of energy densities of photon, neutrino, matter and $\Lambda$ as a function of the scale factor in D$\Lambda$CDM with $(\Omega_{\Lambda 2}, m) = (10^{-4}, 1.2)$ [$\Omega_{\Lambda 1} = \Omega_{\Lambda} - \Omega_{\Lambda 2}$].

Epoch, $a = 10^{-10}$, and obtain the limits

$$\Omega_{\gamma 0} \geq 92\Omega_{\Lambda 2}, \quad (m = 4)$$

$$\Omega_{\gamma 0} > -10^{10(4-m)}\alpha, \quad (m > 4)$$

From Eqs. (3.11), (3.12) and (3.13), we can find that upper bound of $\Omega_{\Lambda 2}$ decrease as $m$ increase. On the other hand, for $\Omega_{\Lambda 2} < 0$ or $m < 0$, we find that $T_\gamma$ becomes negative at $a > 1$. Therefore we set the conditions $\Omega_{\Lambda 2} \geq 0$ and $m \geq 0$.

Figure 3.2 illustrates the evolution of the photon temperature in D$\Lambda$CDM with $m < 4.0$ and the same parameters as in Fig. 3.1. It can be seen that $T_\gamma$ in D$\Lambda$CDM is lower compared to that in S$\Lambda$CDM. Therefore molecules may be produced significantly at earlier epoch [36]. The evolution of photon temperature in D$\Lambda$CDM model with $m \geq 4$ is shown in Fig. 3.3. In this case,
we can find the difference in $T_\gamma$ between S$\Lambda$CDM and D$\Lambda$CDM model at about $z > 100$.

For $0 < m < 4$, the photon evolves as $T_\gamma \propto a^{-1}$ at early epoch and the slope of $T_\gamma$ against $a$ decreases due to the contribution of $a^{-m/4}$ near the present epoch. For $m > 4$, opposite results occur. As the results, effects of a decaying-$\Lambda$ with $m < 4$ and $m \geq 4$ affects at around present epoch and after hydrogen recombination era as shown in Fig. 3.3.

In S$\Lambda$CDM model, the temperature of photon and neutrino is $T_\gamma/T_\nu = (11/4)^{1/3}$ after electron-positron annihilation because $T_\gamma$ and $T_\nu$ evolve as $\propto a^{-1}$. In D$\Lambda$CDM model, since a decaying-$\Lambda$ alter the evolution of the photon, the ratio of photon to neutrino is change by time. Nonetheless we set as the initial condition $T_\gamma/T_\nu = (4/11)^{1/3}$ at $a = 10^{-10}$. The second column in Tab. 3.2 shows the neutrino temperature at $z = 0$. The neutrino temperature $z = 0$ is
lower than that in SΛCDM. Recent studies put the constrains on properties the cosmic neutrino background such as the neutrino species or masses (e.g. [17, 85]), but there are no observation or experiment about temperature (or the energy density) of the neutrino. Therefore, DΛCDM seems to have no problems related to the lower neutrino temperature.

CMB temperature at $z = 0$ is measured accurately by the Far Infrared Absolute Spectrophotometer of COBE satellite: $T = 2.725 ± 0.002$ K at 2σ C.L. [65]. On the other hand, CMB temperature observation at $z > 0$ are reported in various literatures. Consistency between CMB temperature and redshift, $T \propto (1 + z)^{-1}$, was discussed in Ref. [78].

Songaila et al. reported the detection of absorption from the first fine-structure level of neutral carbon atoms in a cloud at $z = 1.776$ toward the
CHAPTER 3. THERMAL EVOLUTION WITH DECAYING
COSMOLOGICAL TERM IN THE CONTEXT OF FRiEMANN UNIVERSE.

![Graph showing temperature and scale factor](image)

Fig. 3.4: Upper panel: comparison of observational and theoretical photon temperature in D$\Lambda$CDM with $\Omega_{A2} = 10^{-4}$. Lower panel: the ratios of $T_\gamma$ to that of S$\Lambda$CDM. We note that the observed redshift translate a scale factor by $a = 1/(1 + z)$.

quasar Q1331+170 and they reported [79]

$$T_\gamma = 7.4 \pm 0.8 \text{ K at } z = 1.776.$$  

Cui et al. obtained the spectrum of molecular hydrogen associated with the damped Ly$\alpha$ system toward the same quasar and estimated the CMB temperature to be [80]

$$T_\gamma = 7.2 \pm 0.8 \text{ K at } z = 1.77654$$

which is consistent with the observation by Ref. [79].  

Ge et al. presented detections of absorption from the ground state and excited states of carbon in the damped Ly$\alpha$ system at $z = 1.9731$ of the QSO 0013–004. They estimated the CMB temperature [81]

$$T_\gamma = 7.9 \pm 1.0 \text{ K at } z = 1.9731.$$
Srianand et al. reported the detection of absorption lines from the first and second fine-structure levels of neutral carbon atoms in an isolated cloud of gas at $z = 2.3371$. They found the upper and lower limits of the CMB temperature $[82]$

$$6.0 \text{ K} \leq T_\gamma \leq 14.0 \text{ K} \text{ at } z = 2.3371.$$  

From the analysis of the C$^+$ fine-structure population ratio in the damped Ly$\alpha$ system at $z = 3.025$ toward the quasar Q0347–3819, the CMB temperature was measured to be $[83]$

$$T_\gamma = 12.1^{+1.7}_{-3.2} \text{ K at } z = 3.025.$$  

Battistelli et al. have deduced $T_\gamma$ using data of the Coma cluster (A1656, $z = 0.0231$) and of A2163 ($z = 0.203$) over four bands at radio and microwave frequencies. As the results, they estimated $[84]$

$$T_\gamma = 2.789^{+0.080}_{-0.065} \text{ K at } z = 0.0231, \quad T_\gamma = 3.377^{+0.101}_{-0.102} \text{ K at } z = 0.203.$$  

Comparison of the temperature evolution in D$\Lambda$CDM model with these observational results is shown in Fig.3.4. When $m$ (or $\Omega_{A2}$) take large value, $T_\gamma$ in D$\Lambda$CDM model is not consistent with temperature observation. Puy has only put constraints on $m - \Omega_{A2}$ plane from the temperature observation $[37]$:

$$|m| \leq 1, |\Omega_{A2}| \leq 10^{-4}. \quad (3.14)$$  

These limits are obtained by comparing observational temperature included $1\sigma$ error and Eqs. (3.10). Figure 3.5 shows constraints on $m - \Omega_{A2}$ plane from observational temperature using the same analysis in Ref. [37], and a theoretical request of $T_\gamma > 0 \ (3.11)$, $3.12$ and $3.13$). Constraints from $T_\gamma$ at $z > 1$ is similar as shown in the top of Fig. 3.5 and constraints from $T_\gamma$ at $z < 1$ has large uncertainty shown in bottom of Fig. 3.5. Limits of $m$ or $\Omega_{A2}$ from the observed temperature is consistent with excluded region by $3.11$, $3.12$ and $3.13$. Constraints $3.14$ is only obtained from the data of Ref. [84] and it is not enough. Therefore we put further severe constraints by CMB anisotropy.
Fig. 3.5: Illustration of constraints on $m - \Omega_{\Lambda 2}$ plane from observational temperatures. The black-solid line shows the upper limits of parameters by Eqs. (3.11), (3.12), and (3.13), and other lines indicate the upper limits obtained by same analysis as [37]. Upper panel: constraints from the temperature at $z > 1$. Lower panel: constraints from the temperature at $z < 1$. 
CHAPTER 3. THERMAL EVOLUTION WITH DECAYING COSMOLOGICAL TERM IN THE CONTEXT OF FRiEMANN UNIVERSE.

Table 3.1: Effect of a decaying-Λ on the neutrino temperature, redshift and temperature at photon decoupling, and ionization fraction at $z = 0$.

<table>
<thead>
<tr>
<th>parameter $(\Omega_{A2} = 10^{-4})$</th>
<th>$T_\nu \text{ K at } z=0$</th>
<th>redshift $z_{dec}$</th>
<th>$T_{dec} \text{ [K]}$</th>
<th>electron fraction at $z = 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m = 0.0$</td>
<td>1.945</td>
<td>1087</td>
<td>2965</td>
<td>$1.62 \times 10^{-4}$</td>
</tr>
<tr>
<td>$m = 0.5$</td>
<td>1.774</td>
<td>1188</td>
<td>2957</td>
<td>$1.35 \times 10^{-4}$</td>
</tr>
<tr>
<td>$m = 1.0$</td>
<td>1.416</td>
<td>1480</td>
<td>2939</td>
<td>$8.70 \times 10^{-5}$</td>
</tr>
<tr>
<td>$m = 1.2$</td>
<td>1.022</td>
<td>2043</td>
<td>2921</td>
<td>$4.56 \times 10^{-5}$</td>
</tr>
</tbody>
</table>

3.2 Thermal evolution at photon decoupling epoch

Fig. 3.6: Illustration of the ionization fraction as a function of $1 + z$ with and without a decaying-Λ.

In this section, we examine the effects of the decaying-Λ term on the photon last scattering. We calculate ionization fraction in DΛCDM model using RECFAST code [86] modified by Dubrovich & Grachev [87]. Figure 3.6 shows the ionization fraction in the DΛCDM model with $\Omega_{A2} = 10^{-4}$ and the helium mass fraction $Y_p = 0.24$. As the result, the universe becomes neutral earlier in DΛCDM model because of the lower photon temperature as seen in Fig. 3.2.
CHAPTER 3. THERMAL EVOLUTION WITH DECAYING
COSMOLOGICAL TERM IN THE CONTEXT OF FRIEMANN UNIVERSE.

Fig. 3.7: Illustration of visibility function $g$ as function of scale factor. Peak of $g$ show epoch of photon decoupling. When $m$ increase, photon decoupling occur at earlier epoch.

Next, we estimate the epoch of photon decoupling by using visibility function $g(t)$ which has peak at the epoch of photon decoupling and its width corresponding to the thickness of the last-scattering surface [88]:

$$g(t) = -\tau e^{-\tau}, \quad \tau = \sigma_T \int n_e dt,$$

where $\sigma_T$ is the Thomson-scattering cross-section and $n_e$ is number density of the free electrons that varies as $a^{-3}$. Figure 3.7 illustrates visibility function as a function of scale factor in DΛCDM model. In this result, epoch of photon decoupling shifts to higher-\(z\) as shown in second column \(z_{dec}\) of Tab. 3.2. If we take the upper limits obtained from Eq. (3.11), \((\Omega_{\Lambda2}, m) = (10^{-4}, 1.2)\), the photon decoupling occurs at \(z_{dec} = 2040\) that is earlier by \(\Delta z_{dec} \sim 950\) compared to SΛCDM [16]. The decaying-\(\Lambda\) affects not only photon decoupling era, but also photon temperature at the last scattering surface. We found that the temperature is about 0.1% lower than that in SΛCDM model, as shown in third of Tab. 3.2.
3.3 SNIa constraint

Usually cosmological models with the $\Lambda$ term (or dark energy) is tightly constrained by the luminosity-redshift relation of type Ia supernovae (SNIa). This is because the cosmological term affect significantly the cosmic expansion rate at low-$z$. In this section, we constrain $\Omega_{\Lambda 0}$ in $\Lambda$CDM using the data of the magnitude-redshift relation of SNIa.

3.3.1 Cosmological test from SNIa

The relation between the total photon intensity luminosity of SNIa, $L$, and the received energy flux $f$ is written by luminosity distance $d_L$

$$f = \frac{L}{4\pi d_L^2},$$

where the luminosity distance is written by

$$d_L = (1 + z) r.$$  \hspace{1cm} (3.15)

The radius $r$ is obtained from line element with the Friedmann-Robertson-Walker metric, $ds^2 = 0$. Using following relation,

$$\int_0^t \frac{dt}{a(t)} = \int \frac{dr}{\sqrt{1 - kr^2}} = \begin{cases} 
    r, & k = 0 \\
    k^{-1/2} \sin^{-1} (\sqrt{k}r), & k > 0 \\
    |k|^{-1/2} \sinh^{-1} (\sqrt{|k|}r), & k < 0
\end{cases}$$

Defining Hubble parameter $H = \dot{a}/a$ and $a = 1/(1 + z)$, Eq. [3.15] is written as

$$d_L = (1 + z) \times \begin{cases} 
    \int \frac{dz}{H(z)}, & \text{for } k = 0 \\
    k^{-1/2} \sin \xi, \quad \xi = \sqrt{k} \int \frac{dz}{H(z)}, & \text{for } k > 0 \\
    |k|^{-1/2} \sinh \xi, \quad \xi = \sqrt{|k|} \int \frac{dz}{H(z)}, & \text{for } k < 0
\end{cases}$$  \hspace{1cm} (3.16)

Apparent magnitude is written by received energy flux $f$:

$$m = -2.5 \log_{10} f + \text{constant.}$$  \hspace{1cm} (3.17)

And absolute magnitude $M$ is defined as the magnitude the source would have at a distance of 10 parsec [32.616 light years, or $3 \times 10^{14}$ kilometres]:

$$M = -2.5 \log L + \text{constant.}$$  \hspace{1cm} (3.18)
Defining distance moduli \( \mu = m - M \), the magnitude-redshift relation of SNIa is given from Eqs. (3.17) and (3.18):

\[
\begin{align*}
\mu_{th} &\equiv m_B - M = 5 \log d_L + 25, \\
d_L &\equiv \frac{1 + z}{H_0 \sqrt{|\Omega_{k0}|}} \times \xi \left( |\Omega_{k0}| \int_0^z \frac{dz'}{f(z')} \right), \\
f(z) &\equiv \sqrt{\Omega_\gamma (z) + \Omega_\nu (1 + z)^4 + \Omega_m (1 + z)^3 + \Omega_{k0} (1 + z)^2 + \Omega_\Lambda (z)},
\end{align*}
\]

where \( m_B, M \) and \( d_L \) are the apparent magnitude, the absolute magnitude and the luminosity distance, respectively, \( \Omega_{k0} = -k/3H_0^2 \) is the curvature parameter, and \( \xi(x) = \sin x, x, \text{ or } \sinh x \) according as \( \Omega_{k0} \) is negative, zero or positive. \( \Omega_\gamma \) and \( \Omega_\Lambda \) are given by Eqs. (3.9) and (3.8).

### 3.3.2 SNIa constraint

Figure 3.8 compares two cosmological models, D\( \Lambda \)CDM (\( \Omega_{\Lambda 2} = 10^{-3}, m = 0.03 \)) and cold dark matter model (\( \Omega_m = 1 \)), with SNIa observation data by High-z Supernova Search Team [7] and Supernova Legacy Survey [8]. Note that \( \Omega_\gamma \) and \( \Omega_\nu \) are negligible at \( z < 2 \).

To obtain the best fit parameters, we calculate the likelihood function \( \mathcal{L} \):

\[
-\chi^2 = 2 \ln \mathcal{L} = \sum_i \frac{(\mu_{obs,i} - \mu_{th,i})^2}{\sigma_i^2},
\]

where \( \mu_{obs} \) and \( \sigma \) are the observed values and their uncertainties that include the velocity dispersions. Since \( \Omega_\Lambda + \Omega_{\Lambda 1} + \Omega_{\Lambda 2} \) at present, three parameters of \( \{\Omega_{\Lambda 1}, \Omega_{\Lambda 2}, m\} \) are independently taken into account.

For the parameters \( \Omega_{\Lambda 2} = 10^{-2} \) and \( m = 0.03 \), the constraint on \( \Omega_\Lambda \) with flat universe is shown in Fig. 3.9. We get \( 0.654 \leq \Omega_\Lambda \leq 0.758 \) at the 95.4 \% confidence regions and this value is consistent with the cosmological constant from the astronomical observations [7, 8, 17]. We conclude that the decaying \( \Lambda \) has negligible effects on the cosmological parameters at \( z < 2 \), because the best fitted value of \( \Omega_\Lambda \sim 0.7 \) should not be changed.
CHAPTER 3. THERMAL EVOLUTION WITH DECAYING COSMOLOGICAL TERM IN THE CONTEXT OF FRIEMANN UNIVERSE.

\[ (\Omega_m, \Omega_{\Lambda}) = (0.26, 0.74) \]
\[ (\Omega_m, \Omega_{\Lambda}) = (1.00, 0.00) \]

Riess et al. (2004)
Astier et al. (2005)

Fig. 3.8: Magnitude-redshift relation. The solid line is the theoretical curve in $\Lambda$CDM with $(\Omega_{\Lambda}, m) = (10^{-2}, 0.03)$ and the dotted line is the model with $\Lambda = k = 0$.

3.4 Effects on CMB anisotropy and constraints by Markov Chain Monte Carlo analysis

The COBE satellite discovered the CMB anisotropy as explained in Chap. 1. On the other hand, it had been known that the CMB angular power spectrum depends on the cosmological parameters. Results by COBE could not determine the parameters because of the insufficient resolution.\(^5\)

The CMB anisotropy observed by WMAP satellite constrains the cosmological model to very high accuracy. Comparison of observational results by COBE (with other observations released before WMAP) and WMAP satellite are shown in Fig. 3.10.

In this section, we investigate the consistency of $\Lambda$CDM model with CMB

\(^5\)COBE had an angular resolution of 7 degrees across the sky, 14 times larger than the Moon’s apparent size. WMAP has angular resolution of $< 0.25$ in the highest frequency (90 GHz) channel
CHAPTER 3. THERMAL EVOLUTION WITH DECAYING COSMOLOGICAL TERM IN THE CONTEXT OF FRIEMANN UNIVERSE.

Fig. 3.9: Probability distribution of $\Omega_\Lambda$ from the magnitude-redshift relation of SNIa with $(\Omega_\Lambda, m) = (10^{-2}, 0.03)$. The 95.4% confidence limits are indicated by the shaded regions.

power spectrum observation and give the limits to the model parameters.

3.4.1 Cosmological perturbation theory – Overview –

First of all, we construct the Boltzmann equations of photon, massless and massive neutrinos, cold dark matter and baryon from the cosmological perturbation theory [89].

Let us calculate the Einstein equation with the metric perturbation. Note that $(\cdot)$ in Sec.3.4.1 means derivative with respect to conformal time $\tau$.

The line element in the synchronous gauge with flat space is written as

$$ds^2 = a^2(\tau) \left[-d\tau^2 + (\delta_{ij} + h_{ij}) \, dx^i dx^j\right]. \quad (3.22)$$

The metric perturbation $h_{ij}$ is decomposed in the following,

$$h_{ij} = \frac{h}{3} \delta_{ij} + h^\parallel_{ij} + h^\perp_{ij} + h^T_{ij} \quad (3.23)$$

where $h = h_{ii}$ is the trace part, the second and third terms correspond to the vector component and $h^T_{ij}$ is the tensor mode. $h^\parallel_{ij}$ can be written in terms of
Fig. 3.10: Angular power spectrum, $l(l + 1)C_l/2\pi$, obtained by observation released before WMAP first year (top panel) and WMAP (bottom) [101].


Chapter 3. Thermal Evolution with Decaying Cosmological Term in the Context of Friedmann Universe

A scalar field \( \mu \)

\[
h_{ij}^\parallel = \left( \partial_i \partial_j - \frac{1}{3} \delta_{ij} \nabla^2 \right) \mu. \tag{3.24}
\]

where \( \nabla^2 = \partial^2 / \partial x_i^2 \). On the other hand, \( h_{ij}^\perp \) is written in divergence-less of vector \( A \) as

\[
h_{ij}^\perp = \partial_i A_j + \partial_j A_i, \quad \partial_i A_i = 0.
\]

Then we introduce two scalar fields \( h \) and \( \eta \) in \( k \) space and Eq. (3.24) is transformed to

\[
h_{ij}^\parallel (\mathbf{x}, \tau) = \int d^3k e^{ikx} \left( \hat{k}_i \hat{k}_j - \frac{1}{3} \delta_{ij} \right) \left[ h(\mathbf{k}, \tau) + 6\eta(\mathbf{k}, \tau) \right], \quad \mathbf{k} = k\hat{k} \tag{3.25}
\]

The two scalar fields \( h \) and \( h^\parallel \) characterize the scalar mode of the metric perturbations. We write the scalar mode of \( h_{ij} \) as the Fourier integral,

\[
h_{ij}(\mathbf{x}, \tau) = \int d^3k e^{ikx} \left[ \hat{k}_i \hat{k}_j h(\mathbf{k}, \tau) + \left( \hat{k}_i \hat{k}_j - \frac{1}{3} \delta_{ij} \right) 6\eta(\mathbf{k}, \tau) \right] \tag{3.26}
\]

Now let us calculate the necessary components in Einstein equation with Eq. (3.22),

\[
R^\mu_\nu - \frac{1}{2} \delta^\mu_\nu R = -8\pi G T^\mu_\nu. \tag{3.27}
\]

First each components of the Christoffel symbol

\[
\Gamma^\alpha_\mu_\nu = g^{\alpha\beta} \Gamma^\beta_\mu_\nu = \frac{1}{2} g^{\alpha\beta} \left( \partial_\mu g_{\nu\beta} + \partial_\nu g_{\beta\mu} - \partial_\beta g_{\mu\nu} \right)
\]

are written as follows,

\[
\begin{align*}
\Gamma^0_0_0 &= \frac{\dot{a}}{a} \\
\Gamma^0_0_j &= \frac{\dot{a}}{a} \delta_{ij} + \frac{\dot{a}}{a} h_{ij} + \frac{1}{2} \dot{h}_{ij} \\
\Gamma^i_0_j &= \frac{\dot{a}}{a} \delta^{ij} + \frac{1}{2} \delta^{ij} \\
\Gamma^i_j_k &= \frac{1}{2} g^{il} \left( \partial_j h_{kl} + \partial_k h_{lj} - \partial_l h_{kj} \right) \tag{3.28}
\end{align*}
\]

Next we write the Ricci tensor

\[
R^\mu_\nu = \partial_\nu \Gamma^\alpha_\mu_\alpha - \partial_\nu \Gamma^\alpha_\mu_\alpha + \Gamma^\alpha_\mu_\beta \Gamma^\beta_\nu_\alpha - \Gamma^\alpha_\mu_\alpha \Gamma^\beta_\nu_\beta, \quad R^\mu_\nu \equiv g^{\mu\alpha} R^\alpha_\nu
\]
CHAPTER 3. THERMAL EVOLUTION WITH DECAYING
COSMOLOGICAL TERM IN THE CONTEXT OF FRiEMANN UNIVERSE.

as follows:

\[
R_0^0 = -a^{-2} \left[ 3 \left( \frac{\dot{a}}{a} - \left( \frac{\dot{a}}{a} \right)^2 \right) + \frac{1}{2} \ddot{h} + \frac{1}{2} \frac{\dot{a}}{a} \dot{h} \right]
\]

\[
R_0^i = a^{-2} \left[ \frac{1}{2} \dot{h}_{,i} - \frac{1}{2} \dot{h}_{,ij,j} \right]
\]

\[
R^i_j = -a^{-2} \left[ - \left( \frac{\ddot{a}}{a} + \frac{\dot{a}}{a} \right)^2 \delta_{ij}
\right.

\left. + \frac{1}{2} \dot{h}_{,ij} - \frac{1}{2} \dot{h}_{ij} - \frac{\dot{a}}{a} \dot{h}_{ij} - \frac{\ddot{a}}{a} \dot{h}_{ij} - \frac{1}{2} \dot{h}_{ik,ik} + \frac{1}{2} \delta_{ij} \left( h_{jk,ik} + h_{ik,kj} - h_{ij,ki} \right) \right].
\] (3.29)

From Eq. (3.29), we can obtain the Ricci scalar \( R = R^\mu_\mu \)

\[
R = - \frac{6 \dot{a}}{a^2} + \frac{1}{a^2} \left[ -3 \frac{\dot{a}}{a} \ddot{h} - \ddot{h} - h_{ik,ik} + h_{ii} \right].
\] (3.30)

The energy momentum tensor \( T^\mu_\nu \) under the assumption of the perfect fluid with the perturbation included:

\[
T_0^0 = - (\bar{\rho} + \delta \rho),
\]

\[
T_0^i = (\bar{\rho} + \bar{p}) v_i = -T_0^i,
\]

\[
T^i_j = (\bar{\rho} + \delta p) \delta^i_j + \Sigma^i_j,
\] (3.31)

where \( \bar{\rho}, \delta \rho, \bar{p}, \delta p, v_i \) and \( \Sigma^i_j \) is background and perturbed part of the energy density, background and perturbed part of the pressure, velocity of fluids and anisotropic stress, respectively. From the background part of Eq. (3.31), we obtain

\[
\left( \frac{\dot{a}}{a} \right)^2 = \frac{8 \pi G}{3} a^2 \bar{\rho},
\] (3.32)

\[
\frac{d}{dt} \left( \frac{\dot{a}}{a} \right) = - \frac{4 \pi G}{3} a^2 \left( \bar{\rho} + 3 \bar{p} \right).
\] (3.33)

Equation (3.32) is the Friedmann equation Eq. (3.3) [Note that dot in Eq. (3.3) is a cosmic time \( t; dt = a(\tau) d\tau \).]

From Eqs. (3.28), (3.29) and (3.27), we obtain the perturbed equations of Einstein equation

\[
k^2 \eta - \frac{1}{2} \frac{\dot{a}}{a} \dot{h} = -4 \pi a^2 G \delta T^0_0.
\] (3.34)

\[
k^2 \eta = 4 \pi a^2 G (\bar{\rho} + \bar{p}) \theta.
\] (3.35)

\[
\ddot{h} + 2 \frac{\dot{a}}{a} \dot{h} - 2 k^2 \eta = -8 \pi a^2 G \delta T^i_i
\] (3.36)

\[
\dddot{h} + 6 \ddot{h} + \frac{\dot{a}}{a} \left( \dddot{h} + 6 \dddot{h} \right) - 2 k^2 \eta = -24 \pi a^2 G (\bar{\rho} + \bar{p}) \sigma,
\] (3.37)
where $\theta$ and $\sigma$ are defined by

$$(\dot{\rho} + \ddot{\rho}) \theta = i k^j \delta T_j^0$$

$$(\dot{\rho} + \ddot{\rho}) \sigma = - \left( \dot{k}_i \cdot \dot{k}_j - \frac{1}{3} \delta_{ij} \right) \Sigma^i_j.$$  

The energy conservation law

$$T^\mu{}_{;\mu} = \partial_\mu T^\mu{}_{\nu} + \Gamma^\nu_{\alpha\beta} T^\alpha{}_{\beta} + \Gamma^\nu_{\alpha\beta} T^\nu{}^{;\alpha\beta} = 0$$  \tag{3.38}

implies the following equations

$$(\dot{\rho} + \delta \dot{\rho}) = -3 \frac{\dot{a}}{a} (\rho + \delta \rho + \ddot{\rho} + \delta \ddot{\rho}) - \left( \partial_i v_i + \frac{3}{2} \dot{h} \right) (\dot{\rho} + \ddot{\rho}),$$  \tag{3.39}

$$\dot{v}_i = - \frac{\dot{a}}{a} (1 - 3w) v_i, \quad \frac{\dot{w}}{1 + w} v_i - \frac{ik^j \delta p/\delta \rho}{1 + w} \delta - \frac{\partial_j \Sigma^j_i}{\rho + \ddot{\rho}}.$$  \tag{3.40}

where $w$ is the EOS of the fluid, $w \equiv p/\rho$. From the background part of Eq. (3.39), we obtain the evolution equation of the energy density Eq. (3.4) :

$$\dot{\rho} = -3 \frac{\dot{a}}{a} (\dot{\rho} + \ddot{\rho}).$$

Perturbed parts of Eqs. (3.39) and (3.40) are written as

$$\dot{\delta} = -3 \frac{\dot{a}}{a} \left( \frac{\delta p}{\delta \rho} - w \right) \delta - (1 + w) \left( \theta + \frac{\dot{h}}{2} \right),$$  \tag{3.41}

$$\dot{\theta} = - \frac{\dot{a}}{a} (1 - 3w) \theta - \frac{\dot{w}}{1 + w} \theta - \frac{k^2 \delta p/\delta \rho}{1 + w} \delta - k^2 \sigma,$$  \tag{3.42}

where $\delta$ is defined as $\delta \equiv \delta \rho/\ddot{\rho}$, $\theta$ defined in Eq. (3.42) is simply the divergence of the fluid velocity: $\theta \equiv i k^j v_j$. Equations (3.41) and (3.42) are Euler and continuity equations, respectively. These equations describe the uncoupled single fluid. If a fluid have interaction with another ones, we need to modify the equation.

### 3.4.2 Energy momentum tensor in a Phase space

The conjugate momenta $P_i$ is written as $P_i = m U_i$, where $U_i = dx^i/\sqrt{-ds^2}$ and $m_i$ is the particle mass. $P_i$ is expressed by the proper momenta $p^i$,

$$P_i = a \left( \delta_{ij} + \frac{1}{2} h_{ij} \right) p^j.$$
CHAPTER 3. THERMAL EVOLUTION WITH DECAYING COSMOLOGICAL TERM IN THE CONTEXT OF FRİE芒NN UNIVERSE.

The general expression of the energy momentum tensor is written by the distribution function and four dimensional momentum $P_0$

$$T_{\mu\nu} = \int dP_1 dP_2 dP_3 (-g)^{-1} \frac{P_{\mu} P_{\nu}}{P_0} f(x^i, P_j, \tau), \quad (3.43)$$

where $P_0 = -\epsilon = -\sqrt{q^2 + m^2 a^2}$ and $g$ is determinant of $g_{\mu\nu}$ and is written as $(-g)^{-1/2} = a^{-4} \left(1 - \frac{\hbar^2}{2}ight)$. The distribution function as the zeroth-order and the perturbed piece are written as

$$f(x^i, P_j, \tau) = f_0 \left(1 + \psi(x^i, q, n_j, \tau)\right). \quad (3.44)$$

The zeroth-order phase-space distribution $f_0$ is the Fermi-Dirac distribution for fermions and the Bose-Einstein distribution for bosons.

Equation (3.43) in polar coordinate is written as

$$T_{\mu\nu} = \frac{1}{a^2} \int q^2 dq d\Omega \frac{P_{\mu} P_{\nu}}{\epsilon} f_0 \left(1 + \psi(x^i, q, n_j, \tau)\right), \quad (3.45)$$

where a $d\Omega$ is solid angle, and we obtain each component of Eq. (3.45):

$$T^0_0 = -\frac{1}{a^4} \int q^2 dq d\Omega \sqrt{q^2 + m^2 a^2} f_0 \left(1 + \psi(x^i, q, n_j, \tau)\right)$$

$$T^0_i = -\frac{1}{a^4} \int q^2 dq d\Omega q_i f_0 \psi$$

$$T^i_j = \frac{1}{a^4} \int q^2 dq d\Omega \frac{q^2 n_i n_j}{\sqrt{q^2 + (ma)^2}} f_0 \left(1 + \psi\right). \quad (3.46)$$

The distribution function in phase space evolves according to the Boltzmann equation:

$$\frac{Df}{d\tau} = \frac{\partial f}{\partial \tau} + \frac{dx^i}{d\tau} \frac{\partial f}{\partial x^i} + \frac{dq}{d\tau} \frac{\partial f}{\partial q} + \frac{dn_i}{d\tau} \frac{\partial f}{\partial n_i} = \left(\frac{\partial f}{\partial \tau}\right)_{\epsilon}, \quad (3.47)$$

where the right hand side of Eq. (3.47) is the collision term which denotes the interaction. When we consider the Fermi-Dirac or the Bose-Einstein distribution, $f_0$ is time independent. Then the first term of Eq. (3.47) is written as

$$\frac{\partial f}{\partial \tau} = f_0 \frac{\partial \psi}{\partial \tau}. \quad (3.48)$$

The left part of the second term corresponds to the equation of motion,

$$\frac{\partial x^i}{\partial \tau} = \frac{a}{m} P^i \quad (3.49)$$
CHAPTER 3. THERMAL EVOLUTION WITH DECAYING COSMOLOGICAL TERM IN THE CONTEXT OF FRIEDEMANN UNIVERSE.

Here $a$ is written as follows:

$$\frac{1}{a} = \frac{dx^0}{dt} = \frac{P^0}{m}.$$  

Then

$$\frac{\partial x^i}{\partial \tau} = \frac{P^i}{P^0} = \left(\delta_{ij} - \frac{h_{ij}}{2}\right) \frac{q^i}{\epsilon}. \quad (3.50)$$

The second of second term in Eq. (3.47) can be written,

$$\frac{\partial x^i}{\partial \tau} \frac{\partial f}{\partial x^j} = \frac{q^i}{\epsilon} f_0 \frac{\partial \Psi}{\partial x^j}. \quad (3.51)$$

From geodesic equation,

$$P^0 \frac{dP^\mu}{d\tau} + \Gamma^\mu_{\alpha\beta} P^\alpha P^\beta = 0, \quad (3.52)$$

we obtain the third term of Eq. (3.47) as follows,

$$\dot{q} = -\frac{1}{2} h_{ij} q_i \hat{n}_j. \quad (3.53)$$

The third term of Eq. (3.47) is neglected because it corresponds to the second order perturbation: $dn^i/d\tau$ and $\partial f/\partial n^i = f_0 \partial \Psi/\partial n^i$ are both the first order.

Finally, from Eqs. (3.48), (3.51) and (3.53), Eq. (3.47) is written as follows

$$\frac{\partial \Psi}{\partial \tau} + \frac{i q}{\epsilon} (k \cdot \hat{n}) \Psi + \left(\frac{\dot{\eta} + 6\eta}{2} (k \cdot \hat{n})\right) \frac{\partial \ln f_0}{\partial \ln q} = \frac{1}{f_0} \left(\frac{\partial f}{\partial \tau}\right). \quad (3.54)$$

Cold dark matter

CDM does not have interaction with other energy component. Setting $\theta_{CDM} = \sigma_{CDM} = w_{CDM} = \dot{w}_{CDM} = 0$ in Eq. (3.41), we get

$$\dot{\theta}_{CDM} = \frac{\dot{h}}{2}. \quad (3.55)$$

Massless neutrino

Usually the background part of energy density can be obtained by integrating distribution function:

$$\bar{\rho}_\nu = -\bar{T}_{0\nu}^0 = \frac{1}{a^4} \int q^2 dq d\Omega q f_0. \quad (3.56)$$
CHAPTER 3. THERMAL EVOLUTION WITH DECAYING COSMOLOGICAL TERM IN THE CONTEXT OF FRIEMANN UNIVERSE.

The perturbed energy density, the energy flux and the anisotropic stress for the massless neutrino can be found from in Eq. (3.46)

\[ \delta \rho_\nu = -\delta T_{0,\nu}^0 = \int q^2 dq d\Omega q f_0 \Psi = 3 \delta \rho_\nu, \]
\[ \delta T_{i,\nu}^0 = \frac{1}{a^2} \int q^2 dq d\Omega n_i f_0 \Psi, \]
\[ \Sigma_{i,\nu}^j = T_j - \frac{1}{3} \delta_i^j T_k^k = \frac{1}{a^4} \int q^2 dq d\Omega \left( n_i n_j - \frac{1}{3} \delta_{ij} \right) f_0 \Psi. \]  

(3.57)

To obtain the Boltzmann equation for massless neutrino, we expand the angular dependence of the perturbation in a series of Legendre polynomials \( P_l(\hat{k} \cdot \hat{n}) \),

\[ F_\nu(\hat{k}, \hat{n}, \tau) = \int \frac{q^2 dq q f_0 \Psi}{\int q^2 dq q f_0} = \sum_{l=0}^{\infty} (-i)^l (2l + 1) F_{\nu l}(\hat{k}, \tau) P_l(\hat{k} \cdot \hat{n}). \]  

(3.58)

From Eqs. (3.56), (3.57) and (3.58), the perturbation \( \delta, \theta \) and \( \sigma \) can be written as follows

\[ \delta_\nu = \frac{\int q^2 dq d\Omega q f_0 \Psi}{\int q^2 dq d\Omega q f_0} = \frac{1}{4\pi} \int d\Omega F_\nu(\hat{k}, n, \tau) \]
\[ \theta_\nu = \frac{i}{4} \int \frac{q^2 dq d\Omega q (\hat{k} \cdot \hat{n}) f_0 \Psi}{\int q^2 dq d\Omega q f_0} = \frac{3i}{16} \int d\Omega (\hat{k} \cdot \hat{n}) F_\nu \]
\[ \sigma_\nu = -\frac{3}{16\pi} \int d\Omega \left( \left( \hat{k} \cdot \hat{n} \right)^2 - \frac{\delta_{ij}}{3} \right) F_\nu = -\frac{1}{8\pi} \int d\Omega P_2 F_\nu \]  

(3.59)

From Eq. (3.58) and the orthonormality of the Legendre polynomially,

\[ \int_{-1}^1 P_m(x) P_n(x) dx = \begin{cases} 0 & \text{for } m \neq n, \\ \frac{2}{2m+1} & \text{for } m = n, \end{cases} \]  

(3.60)

the perturbation Eqs. (3.59) take the form

\[ \delta_\nu = F_{\nu 0}, \quad \theta_\nu = \frac{3}{4} F_{\nu 1}, \quad \sigma_\nu = \frac{1}{2} F_{\nu 2}. \]  

(3.61)

where \( P_0(x) = 1, P_1(x) = x \) and \( P_2(x) = (3x^2 - 1)/2 \).

Integrating Eq. (3.54) over \( q^2 dq q f_0 \) and dividing that by \( \int q^2 dq q f_0 \), we obtain the Boltzmann equation for massless neutrino in k-space:

\[ \frac{\partial F_\nu}{\partial \tau} + i (\hat{k} \cdot \hat{n}) F_\nu = -\frac{2}{3} \dot{h} - \frac{4}{3} \left( \dot{h} + 6 \ddot{\eta} \right) P_2(\hat{k} \cdot \hat{n}). \]  

(3.62)
CHAPTER 3. THERMAL EVOLUTION WITH DECAYING COSMOLOGICAL TERM IN THE CONTEXT OF FRIEDMANN UNIVERSE.

Finally, integrating Eq. (3.62) multiplied by $P_l$ over $\cos \theta$, we obtain following equations:

$$
\delta_\nu = \frac{2}{3} \dot{h} - \frac{4}{3} \theta_\nu, \\
\dot{\theta}_\nu = k^2 \left( \frac{\delta_\nu}{4} - \sigma_\nu \right), \\
\dot{F}_{\nu 2} = -\frac{k}{5} (3 F_{\nu 3} - 2 F_{\nu l}) + \frac{4}{15} \left( \dot{h} + 6 \eta \right), \\
\dot{F}_{\nu l} = \frac{k}{2l+1} \left( -(l+1) F_{\nu (l+1)} + l F_{\nu (l-1)} \right), \quad (l \geq 3).
$$

(3.63)

where we used the relation $(l+1) P_{l+1}(x) = (2l+1) x P_l(x) - l P_{l-1}(x)$.

Massive neutrino

For massive neutrino, we neglect the collision term of the Boltzmann equation as massless neutrino. However, the distribution function has non-zero mass. From Eq. (3.46), the background energy density and pressure are written as follows:

$$
\bar{\rho}_h = -\bar{T}_0^0 = a^{-4} \int q^2 dq d\Omega \epsilon f_0(q), \\
\bar{\rho}_h = \frac{1}{3} \bar{T}_i^i = a^{-4} \int q^2 dq d\Omega \epsilon^{-1} q^2 f_0(q),
$$

(3.64)

where subscript $h$ means hot dark matter and $\epsilon = \sqrt{q^2 + m_h^2}$. The perturbed energy density and pressure, energy flux and anisotropic stress are written as follows.

$$
\delta \rho_h = a^{-4} \int q^2 dq d\Omega \epsilon f_0(q) \Psi, \\
\delta p_h = a^{-4} \int \frac{1}{3} q^2 dq d\Omega \epsilon^{-1} q^2 f_0(q) \Psi, \\
\delta T_{i, h}^0 = a^{-4} \int q^2 dq d\Omega q n_i f_0(q) \Psi, \\
\Sigma_{j, h}^i = a^{-4} \int q^2 dq d\Omega \epsilon^{-1} q^2 \left( n_i n_j - \frac{1}{3} \delta_{ij} \right) f_0(q) \Psi.
$$

(3.65)

Instead of applying Eq. (3.58), we expand $\Psi$ directly in the Legendre series

$$
\Psi(k, \hat{n}, q, \tau) = \sum_{l=0}^{\infty} (-i)^l (2l+1) \Psi_l(k, q, \tau) P_l(k, \hat{n}),
$$
CHAPTER 3. THERMAL EVOLUTION WITH DECAYING COSMOLOGICAL TERM IN THE CONTEXT OF FRIEMANN UNIVERSE.

As a consequence, Eqs. (3.65) are given by

\[
\delta \rho_h = 4\pi a^{-4} \int q^2 dq f_0(q) \Psi
\]

\[
\delta p_h = \frac{4\pi}{3} a^{-4} \int q^2 dq e^{-1} q^2 f_0(q) \Psi
\]

\[
(p_h + \bar{p}_h) \theta_h = 4\pi ka^{-4} \int q^2 dq q f_0 \Psi_1
\]

\[
(p_h + \bar{p}_h) \sigma_h = \frac{8\pi}{3} a^{-4} \int q^2 dq e^{-1} q^2 f_0 \Psi_2
\]

(3.66)

Following the same procedure used for the massless neutrino, the Boltzmann equation is given by the following equations

\[
\dot{\Psi}_0 = -\frac{k}{\epsilon} q \Psi_1 + \frac{1}{6} \hbar \frac{d (\ln f_0)}{d (\ln q)}
\]

\[
\Psi_1 = \frac{1}{3} \frac{kq}{\epsilon} (\Psi_0 - 2 \Psi_2)
\]

\[
\Psi_2 = \frac{1}{5} \frac{kq}{\epsilon} (2 \Psi_1 - 3 \Psi_3) - \frac{\partial \ln f_0}{\partial \ln q} \left( \frac{\hbar}{15} + \frac{2}{5} \right)
\]

\[
\Psi_l = \frac{1}{2l + 1} \frac{kq}{\epsilon} \left( - (l + 1) \Psi_{l+1} + l \Psi_{l-1} \right)
\]

(3.67)

**Photon**

The evolution of photon distribution function is similar to massless neutrino. However the collision term of the Boltzmann equation cannot neglect because the photon suffers the Thomson scattering.

The collision terms of Thomson scattering are written as follows [90],

\[
\left( \frac{\partial F_\gamma}{\partial \tau} \right)_c = n_e \sigma_T \left( -F_\gamma + F_{\gamma 0} + 4 n \cdot v_e - \frac{1}{2} (F_{\gamma 2} + G_{\gamma 0} + G_{\gamma 2}) P_2 \right),
\]

\[
\left( \frac{\partial G_\gamma}{\partial \tau} \right)_c = n_e \sigma_T \left( -G_\gamma + \frac{1}{2} (F_{\gamma 2} + G_{\gamma 0} + G_{\gamma 2}) (1 - P_2) \right),
\]

where \( n_e \) is number density of electron and \( \sigma_T \) is cross section of Tomson scattering.

Expanding \( F_\gamma(k, \tau) \) and \( G_\gamma(k, \tau) \) in the Legendre series as Eq. (3.58) we
obtain the collision terms of photon:

\[
\left( \frac{\partial F_\gamma}{\partial t} \right)_e = ax_e n_e \sigma_T \left( \frac{4i}{k} (\theta_\gamma - \theta_b) P_1 + \left( 9\sigma_\gamma - \frac{1}{2} (G_{\gamma 0} + G_{\gamma 2}) \right) P_2 - \sum_{l \geq 3} (-i)^l (2l + 1) F_{\gamma l} P_l \right),
\]

(3.68)

\[
\left( \frac{\partial G_\gamma}{\partial t} \right)_e = ax_e n_e \sigma_T \left( -\sum_{l=0}^{\infty} (-i)^l (2l + 1) G_{\gamma l} P_l + \frac{1}{2} (F_{\gamma 2} + G_{\gamma 0} + G_{\gamma 2}) (1 - P_2) \right)
\]

(3.69)

Since the left hand side of the Boltzmann equation is the same as the massless neutrino, we obtain the following equations from Eqs. (3.68) and (3.69):

\[ \dot{\delta}_\gamma = -\frac{2}{3} \dot{h} + \frac{4}{3} \theta_\gamma, \]

\[ \dot{\theta}_\gamma = \frac{k^2}{4} \delta_\gamma - k^2 \sigma_\gamma - ax_e n_e \sigma_T (\theta_\gamma - \theta_b), \]

\[ \dot{F}_{\gamma 2} = \frac{4}{15} \dot{h} + \frac{8}{3} \theta_\gamma + \frac{8}{15} \theta_\gamma - \frac{3}{5} k F_{\gamma 3} \]

\[ + \frac{1}{10} ax_e n_e \sigma_T (-9 F_{\gamma 2} + G_{\gamma 0} + G_{\gamma 2}), \]

(3.70)

\[ \dot{F}_{\gamma l} = \frac{k}{2l + 1} ((l + 1) F_{\gamma, l+1} + 1 F_{\gamma, l-1}) - a n_e \sigma_T F_{\gamma, l}, \]

\[ \dot{G}_{\gamma l} = ax_e n_e \sigma_T \left( \frac{1}{2} (F_{\gamma 2} + G_{\gamma 0} + G_{\gamma 2}) \left( \delta_{l0} + \delta_{l2} \right) - G_{\gamma l} \right) \]

\[ - \frac{k}{2l + 1} ((l + 1) G_{\gamma, l+1} - l G_{\gamma, l-1}). \]

Baryon

Baryon is a nonrelativistic particle after neutrino decoupling, therefore its evolution is similar to that of CDM. However, the interaction between the radiation and baryon cannot be neglected. Considering the effects of Thomson scattering, Eqs. (3.41) and (3.42) for baryon are written as

\[ \dot{\delta}_b = -\theta_b - \frac{\dot{h}}{2}, \]

(3.71)

\[ \dot{\theta}_b = \frac{\dot{a}}{a} \theta_b + c_s^2 k^2 \delta_b + \frac{4 \rho_\gamma}{3 \rho_b} ax_e n_e \sigma_T (\theta_\gamma - \theta_b). \]

(3.72)

where baryon sound speed \( c_s \) is written as,

\[ c_s^2 = \frac{\dot{p}_b}{\dot{\rho}_b} = \frac{k_b T}{\mu} \left( 1 - \frac{1}{3} \frac{d \ln T_b}{\ln t} \right), \]

46
CHAPTER 3. THERMAL EVOLUTION WITH DECAYING COSMOLOGICAL TERM IN THE CONTEXT OF FRIEMANN UNIVERSE.

where $\mu$ is mean molecular weight and $T_b$ is the baryon temperature [91]:

$$T_b = -2 \frac{\dot{a}}{a} T_b + \frac{8 \mu \rho_\gamma}{3 m_e \rho_b} a x e n_e \sigma_T (T_\gamma - T_b).$$

Initial condition

We start integration at early time when a $k$ mode is outside of the horizon, $k \tau \ll 1$. At this era, the total energy density is written as $\bar{\rho} = \bar{\rho}_\gamma + \bar{\rho}_\nu$ because this era is radiation dominate. Neglecting $F_{\gamma l}$ at $l \geq 2$ and $G_{\gamma \mu}$ Eqs. (3.70) and (3.63) are written as follows:

- photon

$$\dot{\delta}_\gamma = -\frac{4}{3} \theta_\gamma - \frac{2}{3} \dot{h},$$
$$\dot{\theta}_\gamma = \frac{k^2}{4} \delta_\gamma,$$  \hspace{1cm} (3.73)

- neutrino

$$\dot{\delta}_\nu = -\frac{4}{3} \theta_\nu - \frac{2}{3} \dot{h},$$
$$\dot{\theta}_\nu = \frac{k^2}{4} \delta_\nu,$$  \hspace{1cm} (3.74)
$$\dot{\sigma}_\nu = \frac{4}{15} \theta_\nu + \frac{2}{15} \left( \dot{h} + 6 \dot{\eta} \right).$$  \hspace{1cm} (3.75)

From Eqs. (3.34) and (3.36) at radiation dominant, we obtain the evolution of the metric perturbation as follows:

$$\ddot{h} - \tau^{-1} \dot{h} + 6 \tau^{-2} \left( \delta_\gamma (1 - R_\nu) + \delta_\nu R_\nu \right) = 0$$

where $R_\nu = \bar{\rho}_\nu / \bar{\rho}_{\text{tot}}$. Since we consider scale at $k \tau \ll 1$ and $k^2$-terms is negligible, Eqs. (3.73) and (3.74) are written as

$$\dot{\delta}_\gamma = -\frac{4}{3} \theta_\gamma - \frac{2}{3} \dot{h},$$
$$\dot{\delta}_\nu = -\frac{4}{3} \theta_\nu - \frac{2}{3} \dot{h},$$
$$\dot{\sigma}_\nu = \frac{4}{15} \theta_\nu + \frac{2}{15} \left( \dot{h} + 6 \dot{\eta} \right).$$
$$\dot{\theta}_\gamma = \dot{\theta}_\nu = 0.$$  \hspace{1cm} (3.79)
Then $\theta_\gamma$ and $\theta_\nu$ are constant. From Eqs. (3.76), (3.77) and (3.79), we obtain the forth order equation for $h$:

$$\tau \frac{d^4 h}{d\tau^4} + 5 \frac{d^3 h}{d\tau^3} = 0,$$

and, as the solution,

$$h(\tau) = A (k\tau)^{-2} + B (k\tau)^2 + C (k\tau) + D. \quad (3.80)$$

where $A, B, C$ and $D$ are dimensionless constant. Defining $\delta \equiv \delta_\gamma - R_\nu (\delta_\gamma - \delta_\nu)$ and $\theta \equiv (1 - R_\nu) \theta_\nu + R_\nu$, we obtain

$$\dot{\delta} = -\frac{2}{3} \left( A(k\tau)^{-2} + B(k\tau)^2 - \frac{C}{6}(k\tau) \right)$$

$$\dot{\theta} = -\frac{3}{8} C k$$

And other metric perturbation $\eta$ is written as

$$\eta = 2B + \frac{3}{4} C (k\tau)^{-1}.$$

Here $A$ and $D$ can be eliminated by coordinate conversion [92] and the mode proportional to $C$ in the radiation dominant era decays in the matter dominant era [93]. We obtain

$$h(\tau) = B(k\tau^2).$$

Using all of these results, we obtain the initial condition of perturbations in synchronous gauge as follows:

$$\delta_\gamma = -\frac{2}{3} B (k\tau)^2$$

$$\delta_{CDM} = \delta_b = \frac{3}{4} \delta_\gamma = \frac{3}{4} \delta_\nu,$$

$$\theta_\gamma = \theta_b = -\frac{B}{18} k^4 \tau^3,$$

$$\theta_\nu = \frac{23 + 4R_\nu}{15 + 4R_\nu} \theta_\gamma,$$

$$\sigma_\nu = \frac{4B}{3(15 + 4R_\nu)} (k\tau)^2,$$

$$\eta = 2B - \frac{5 + 4R_\nu}{6(15 + 4R_\nu)} B(k\tau)^2 \quad (3.81)$$

Here the relation $\delta_b = 3\delta_\gamma/4$ comes from the tight coupling approximation (see Appendix B).
3.4.3 Cosmological perturbation theory with a decaying cosmological term

Before discussing effects of a decaying-Λ on the CMB power spectrum, let us formulate the Boltzmann equation for photon in DΛCDM model based on the cosmological perturbation theory.

In the case of a decaying-Λ which changes into other energy, the energy conservation of the background part is written as,

\[ \dot{\rho} + \dot{\rho}_\Lambda = -3 \frac{\dot{a}}{a} (1 + w) \rho. \]

(3.82)

On the other hand, the perturbed part of the energy momentum conservation, Eq. (3.38), imply the following equations

\[ \dot{\delta} = -3 \frac{\dot{a}}{a} \left( \frac{\delta p}{\delta \rho} - w \right) \delta - (1 + w) \left( \theta + \frac{\dot{h}}{2} \right) + \frac{\dot{\rho}_\Lambda}{\rho} \delta \]

(3.83)

\[ \dot{\theta} = \frac{\dot{a}}{a} (1 - 3w) \theta - \frac{w - \dot{\omega}}{1 + w} - \frac{k^2 \delta p / \delta \rho}{1 + w} \delta - k^2 \sigma + \frac{\dot{\rho}_\Lambda}{\rho} \theta. \]

(3.84)

We may consider that the decaying-Λ create photon. In this case, Eqs. (3.83) and (3.84) for photon \( w = 1/3 \) are written as follows

\[ \dot{\delta}_\gamma = -4 \frac{\dot{a}}{a} \frac{1}{3} \sigma - \frac{2}{3} \frac{\dot{h}}{h} + \frac{\dot{\rho}_\Lambda}{\rho} \delta_\gamma \]

(3.85)

\[ \dot{\theta}_\gamma = \frac{1}{4} k^2 \delta_\gamma - k^2 \sigma_\gamma + \frac{\dot{\rho}_\Lambda}{\rho} \theta_\gamma. \]

(3.86)

Eqs. (3.85) and (3.86) is the continuity and Euler equation, respectively. Note that we have to consider interaction term in Eq.(3.86) because photon suffers Compton scattering with electron.

In addition, to construct the perturbed evolution of photon, we cannot neglect the contribution of the higher multipole moment. Boltzmann equation for a relativistic particle in \( k \)-space is written as follows [89]

\[ f_0 \frac{\partial \Psi}{\partial \tau} + \Psi \frac{\partial f_0}{\partial \tau} + ik \mu f_0 \Psi + \frac{d \ln f_0}{d \ln q} \left( \eta + \frac{\dot{h}}{2} + 6 \dot{\eta} \right) \mu^2 = \left( \frac{\partial f}{\partial \tau} \right)_{col} \]

(3.87)

where \( \mu = \hat{k} \cdot \hat{n} \) and \( q_i = q n_i \) is 3-dimensional momentum and right hand side of Eq. (3.87) is collision term. The distribution function as the zeroth-order and the perturbed part are written as,

\[ f(x^i, q, n_j, \tau) = f_0(q)(1 + \Psi(x^i, q, n_j, \tau)). \]
Usually, the zeroth-order distribution function of photon \( f_{\gamma 0} \),

\[
f_{\gamma 0}(q) = \frac{1}{h_p} \exp \left( \frac{q}{k_B T_\gamma} \right) - 1
\]

where \( h_p, k_B \) is the Planck and Boltzmann constants. In D\( \Lambda \)CDM model, however, \( f_{\gamma 0} \) could be a function of current as discussed in Ref. [94].

To obtain the Boltzmann equation for photon, we expand the angular dependence of the perturbation in a series of Legendre polynomials \( P_l(k \cdot \hat{n}) \),

\[
F_\gamma(k, \hat{n}, \tau) = \frac{1}{\int q^2 dq f_0(q, \tau)} \sum_{l=0}^{\infty} (-i)^l (2l + 1) F_{\gamma l}(k, \tau) P_l(\mu).
\]

Integrating Eq.(3.87) over \( q^2 dq f_0 \) and dividing that by \( \int q^2 dq f_0 \), we obtain the Boltzmann equation for CMB photon in \( k \)-space:

\[
\dot{F}_\gamma + ik\mu F_\gamma + \frac{4}{3} \left( \dot{\hat{n}} + 6\dot{\hat{n}} \right) P_2(\mu) + \frac{2}{3} \dot{\hat{n}} - \frac{\dot{\rho}_\Lambda}{\rho_\gamma} F_\gamma = \left( \frac{\partial F_\gamma}{\partial \tau} \right)_{col} (3.88)
\]

where collision terms include Thomson scattering in Eqs. (3.68) and (3.69). The last term of left hand side in Eq. (3.88) corresponds to a new one in D\( \Lambda \)CDM model.

Substituting the Legendre expansion for \( F_\gamma \), using the orthonormality of the Legendre polynomially Eq. (3.60), and recursion relation

\[
(l + 1) P_{l+1}(\mu) = (2l + 1) \mu P_l(\mu) - lP_{l-1}(\mu),
\]

we obtain

\[
\dot{\theta}_\gamma = \frac{1}{4} k^2 \delta_\gamma - k^2 \sigma_\gamma + \frac{\dot{\rho}_\Lambda}{\rho_\gamma} \theta_\gamma - a n_e e_\sigma_T (\theta_\gamma - \theta_b) \quad (l = 1) \tag{3.89}
\]

\[
\dot{\sigma}_\gamma = \frac{4}{15} \theta_\gamma - \frac{3}{10} k F_{\gamma 0} + \frac{2}{15} \left( \dot{\hat{n}} + 6\dot{\hat{n}} \right) + \frac{\dot{\rho}_\Lambda}{\rho_\gamma} \sigma_\gamma - \frac{9}{10} a n_e e_\sigma_T \sigma_\gamma + \frac{1}{20} a n_e e_\sigma_T (G_{\gamma 0} + G_{\gamma 2}) \quad (l = 2) \tag{3.90}
\]

\[
\dot{F}_{\gamma l} = \frac{k}{2l + 1} \left( l F_{\gamma ,l-1} - (l + 1) F_{\gamma ,l+1} \right) + \frac{\dot{\rho}_\Lambda}{\rho_\gamma} F_{\gamma l} - a n_e e_\sigma_T F_{\gamma l} \quad (l \geq 3) \tag{3.91}
\]

where

\[
\delta_\gamma = F_{\gamma 0}, \quad \theta_\gamma = \frac{3}{4} k F_{\gamma 1}, \quad \sigma_\gamma = \frac{F_{\gamma 2}}{2}
\]

For \( l = 0 \), we obtain Eq. (3.85).

Figure 3.11 illustrate the evolution of density perturbation of photon [Eq. (3.85)] for the S\( \Lambda \)CDM and D\( \Lambda \)CDM models at \( k = 1.2 \times 10^{-3} \text{ Mpc}^{-1} \) (top), 0.026
Fig. 3.11: Evolution of density perturbation in $\Lambda$CDM model at $k = 1.183 \times 10^{-3}$ (top panel), $0.026$ (middle) and $1.0$ Mpc$^{-1}$ (bottom), corresponded to $l = 1$, $l = 220$ (last scattering surface) and $1.0$ Mpc$^{-1}$ (cluster scale).
Fig. 3.12: Comparison of the angular power spectrum in the decaying $\Lambda$ model with the WMAP observation data\cite{47} and BOOMERanG \cite{111}. The solid line is the result of S$\Lambda$CDM. The dashed, the dot-dashed and the dotted lines are those of D$\Lambda$CDM with $(\Omega,\Lambda, m) = (10^{-4}, 0.5), (10^{-4}, 1.0)$ and $(10^{-4}, 1.2)$, respectively.

From these results, a decaying-$\Lambda$ affects on $\delta$, in the small scale.

Let us define the temperature fluctuation $\Delta \equiv \Delta T/T = \delta/4$. We expand $\Delta$ in terms of spherical harmonics $Y(\theta, \phi)$. That is written as,

$$\Delta(n) = \sum_{l=1}^{\infty} \sum_{m=-l}^{l} a_{lm} Y_{lm}(n),$$

where $n = (\theta, \phi)$. We introduce the power spectrum $C_l$ and defined by

$$\langle a_{lm} a_{l'm'} \rangle = C_l \delta_{lm} \delta_{l'm'}.$$

The correlation function of the CMB anisotropy can be defined

$$C(\theta) = \langle \Delta(n_1) \Delta(n_2) \rangle,$$
CHAPTER 3. THERMAL EVOLUTION WITH DECAYING COSMOLOGICAL TERM IN THE CONTEXT OF FRIEMANN UNIVERSE.

\[ l(l+1) C_l / 2\pi \text{ [\mu K]^2} \]

\[ \log(W L^2) = -5.5 \]
\[ \log(W L^2) = -5.0 \]
\[ \log(W L^2) = -4.9 \]

WMAP (2006)
BOOMERanG (2005)

Fig. 3.13: Comparison of the angular power spectrum in the decaying $\Lambda$ model with the WMAP observation data [47] & BOOMERanG [111]. The solid line is the result of S$\Lambda$CDM. The dashed, the dot-dashed and the dotted lines are those of D$\Lambda$CDM with $(\Omega_{\Lambda}, m) = (3.2 \times 10^{-5}, 3.0), (10^{-5}, 3.0)$ and $(1.3 \times 10^{-5}, 3.0)$, respectively.

where $\theta$ is an angle between the direction between $n_1$ and $n_2$. From the isotropic and homogeneous universe, the spherical harmonics is curriter as given

\[ Y_{l0} = \sqrt{\frac{2l + 1}{4\pi}} P_l(\cos \theta), \]

Then we obtain

\[ C(\theta) = \sum_l \frac{2l + 1}{4\pi} C_l P_l(\cos \theta). \quad (3.94) \]

Actually, the temperature fluctuation is described by $l(l+1)C_l/2\pi$ converged to $l(2l+1)C_l/4\pi$ at higher-$l$.

We calculate the CMB power spectrum by modified CAMB code [95] based on the CMBFAST code [96] and assume the same initial condition in Eqs.
Fig. 3.14: Top panel: Acoustic oscillations. Spring and balls schematically represent fluid pressure and effective mass, respectively [From Ref.[112]]. Bottom: Effects of baryon density on the temperature fluctuation. When $R(\propto \rho_b/\rho_\gamma)$ increase, the relative height of odd peaks and even peaks boosts.
(3.81). Figure 3.12 shows the effects of \( m \) on the angular power spectrum with the following cosmological parameters: the baryon density parameter \( \Omega_b h^2 = 0.0223 \), the CDM density parameter \( \Omega_{CDM} h^2 = 0.104 \), \( k = 0 \), helium mass fraction \( Y_p = 0.24 \), number of neutrino \( N_\nu = 3.04 \) [97] and without the reionization. The effects of \( \Omega_{\Lambda_2} \) with \( m = 3 \) are shown in Fig.3.13. The varying-\( \Omega_{\Lambda_2} \) on CMB power spectrum is similar to Fig. 3.12.

We find that a decaying-\( \Lambda \) modify the CMB power spectrum in three ways. If \( m \) and/or \( \Omega_{\Lambda_2} \) is small, the amplitude of the power spectrum decreases. If we take larger values of \( m \) (or \( \Omega_{\Lambda_2} \)), the first and third peaks of the power spectrum increases because of the large baryon density relative to the photon energy density. Furthermore, the CMB power spectrum shifts toward higher-\( l \), because the photon last scattering occurs at an earlier epoch as seen in Fig.3.7 and Tab.3.2.

Let us explain the boost of the first peak. Before hydrogen recombination, the baryon and photon is tightly coupled [tight coupling approximation]. With tight coupling approximation, the evolution of density perturbation for photons is described by the following evolution equation\(^4\):

\[
(1 + R) \ddot{\delta}_\gamma + \dot{R} \dot{\delta}_\gamma + \frac{k^2}{3} \delta_\gamma = -\frac{2}{3} \dot{h} (1 + R) + \frac{2}{3} \dot{R} h, \quad R \equiv \frac{3 \rho_b}{4 \rho_\gamma} \tag{3.95}
\]

The photon pressure resists gravitational compression of the baryon which sets up oscillations shown in top panel of Fig. 3.14. From a solution of Eq. (3.95), \( \delta_\gamma \) (namely temperature fluctuation \( \Delta T/T \)) evolves as acoustic oscillation by forcing function in the right hand side in Eq. (3.95) [98, 99]. Since the energy density of photon in \( \Lambda CDM \) model is lower than that in \( \Lambda CDM \) model, effective mass \( m_{eff} = 1 + R \) in the right hand side of Eq. (3.95) is large. Then it leads to the increasing amplitude of oscillations. As the results, the decaying-\( \Lambda \) affects the first and third peaks in CMB power spectrum shown in the bottom of Fig.3.14.

To obtain the upper limit of \( \Omega_{\Lambda_2} \) and \( m \), we calculate the likelihood function given by Ref. [100]. Figure 3.15 shows 68.3%, 95.4% and 99.7% confidence limits on the \( m - \Omega_{\Lambda_2} \) obtained by the CMB fluctuation by WMAP first year

\(^4\)Equation (3.95) is described in the synchronous gauge. However, when we explain the tight-coupling approximation, it is usual to write in the conformal Newtonian gauge. See Appendix B.
Fig. 3.15: Constraint on \( m - \Omega_{\Lambda_2} \) plane from WMAP first year. Drawn lines correspond to 1, 2 and 3\( \sigma \) confidence limit. The labeled no Big-Bang region indicate that \( T_\gamma \) is negative at \( a < 1 \).

result [101]. The confidence lines we obtained are parallel with \( \alpha \) defined in §3. In our analysis, limits of \( \Omega_{\Lambda_2} \) is extremely small for \( m > 4 \). We obtain the constraint \( \alpha < 4.9 \times 10^{-6} \) at 95 \% confidence limit. In the value of this upper limit, the photon last scattering occurs to the earlier epoch by \( \Delta z \sim 30 \) compared with \( \Lambda \)CDM model of \( z = 1089 \) [49].

3.4.4 Markov chain Monte Carlo analysis

The CMB angular power spectrum is rather sensitive to other cosmological parameters. For instance baryon and CDM densities affect the amplitude of CMB anisotropy. Therefore, we need to carry out the Markov-Chain Monte Carlo [MCMC] approach [104] to constrain the parameters in \( \Lambda \)CDM. We can constrain the critical parameters : \( \Omega_b h^2, \Omega_{CDM} h^2, h, \) the reionization redshift
CHAPTER 3. THERMAL EVOLUTION WITH DECAYING COSMOLOGICAL TERM IN THE CONTEXT OF FRiEMANN UNIVERSE.

$z_{re}$, the scalar spectral index $n_s$, the amplitude of density fluctuation $A_s$, and two parameters in $\Lambda$CDM [$\Omega_{A2}$ and $m$].

MCMC provides an answer to the difficult problem of simulation from the $n$-dimensional distribution of the unknown quantities that appear in the complex models. Adopted algorithms in our analysis is Metropolis-Hastings algorithms as follows [102, 103]:

1. Initialize the iteration count $j = 1$ and set an arbitrary initial value $\theta^{(0)}$.
2. Move the chain to a new value $\phi$ generated from the density $q(\theta^{j-1})$.
3. Evaluate the acceptance probability of the move $\alpha(\theta^j, \phi)$ given by acceptance probability:
   \[
   \alpha(\theta, \phi) = \min\{1, \frac{\pi(\phi)q(\phi, \theta)}{\pi(\theta)q(\theta, \phi)}\}
   \]
   If the move is accepted, $\theta^j = \phi$. If it is not accepted, $\theta^j = \theta^{j-1}$ and the chain does not move.
4. Change the counter from $j$ to $j-1$ and return to step 2 until convergence is reached.

As the results, the parameters converge to the probability value shown in Fig. 3.16.

To start the MCMC calculations, we assume the priors on the cosmological parameters given as follows:

- $h = 0.72 \pm 0.05$ at 1$\sigma$ C.L. [105] (HST Key Project prior)
- $0.5 \leq n_s \leq 1.5$,
- $\Omega_{\gamma}h^2 = 0.022 \pm 0.0022$ at 1$\sigma$ C.L. [106] (BBN prior),
- $0.01 \leq \Omega_{CDM}h^2 \leq 0.99$,
- $-0.3 \leq \Omega_{k0} \leq 0.3$,
- $0 \leq \Omega_{A0} \leq 1.0$
- $10$ Gyr $< t_0 < 20$ Gyr (age of the universe).
CHAPTER 3. THERMAL EVOLUTION WITH DECAYING COSMOLOGICAL TERM IN THE CONTEXT OF FRIEMANN UNIVERSE.

Table 3.2: Comparison of parameters in $\Lambda$CDM and DACDM. Parameters in DACDM model are obtained by MCMC calculation and these in $\Lambda$CDM model is by LAMBDA site [113]

<table>
<thead>
<tr>
<th>parameter</th>
<th>WMAP first year</th>
<th>WMAP three year</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>DACDM</td>
<td>SACDM</td>
</tr>
<tr>
<td>$\Omega_{A2}$</td>
<td>$&lt; 5.1 \times 10^{-5}$</td>
<td>$&lt; 2.5 \times 10^{-4}$</td>
</tr>
<tr>
<td>$m$</td>
<td>$0.222^{+0.003}_{-0.003}$</td>
<td>$0.024 \pm 0.001$</td>
</tr>
<tr>
<td>$\Omega_{CDM}h^2$</td>
<td>$0.10 \pm 0.04$</td>
<td>$0.12 \pm 0.02$</td>
</tr>
<tr>
<td>$\log_{10} (10^{10} A_s)$</td>
<td>$3.13^{+0.06}_{-0.03}$</td>
<td>$3.19 \pm 0.04$</td>
</tr>
<tr>
<td>$h$</td>
<td>$0.66 \pm 0.08$</td>
<td>$0.72 \pm 0.05$</td>
</tr>
<tr>
<td>$z_{re}$</td>
<td>$16^{+14}_{-13}$</td>
<td>$17 \pm 5$</td>
</tr>
<tr>
<td>$n_s$</td>
<td>$0.96^{+0.18}_{-0.18}$</td>
<td>$0.97 \pm 0.03$</td>
</tr>
<tr>
<td>$\Omega_{\Lambda}$</td>
<td>$0.75^{+0.14}_{-0.18}$</td>
<td>$0.73 \pm 0.04$</td>
</tr>
<tr>
<td>$\Omega_m$</td>
<td>$0.26^{+0.12}_{-0.18}$</td>
<td>$0.24 \pm 0.04$</td>
</tr>
<tr>
<td>$t_0$ Gyr</td>
<td>$14.3^{+2.7}_{-1.7}$</td>
<td>$13.4 \pm 0.3$</td>
</tr>
</tbody>
</table>

In our analysis, we assume that the cosmic reionization occur at $z = z_{re}$ suddenly, i.e. the ionization fraction $x_e$ after photon decoupling is follows:

$$x_e = \begin{cases} 
0 & \text{at } z < z_{re} \\
1 & \text{at } z > z_{re}.
\end{cases}$$

As pointed out in Ref. [49], this assumption may be unphysically. We are not able to obtain the details of reionization history from the present CMB observation and we consider that a complex history during reionization is not likely to alter any our result [17].

First, we put constraint on DACDM model from WMAP first year [101, 107], Cosmic Background Imager (CBI) [108], Very Small Array (VSA) [109], Aber [110] and our results are shown in Figs. 3.17, 3.18 and 3.19. Figure 3.17 shows the likelihood function of $m$ and $\Omega_{A2}$. The 95.4% confidence levels are obtained to be $m \leq 4.3$ and $\Omega_{A2} \leq 5.0 \times 10^{-5}$. It can be seen that the constraint on $\Omega_{A2}$ is more severe compared to the case of $m$. This means that the effects of the decaying $\Lambda$ should be small on the thermal evolution of the universe. Figure 3.18 show the contours of $\Omega_{A2}$ and other cosmological parameter on the $(\Omega_b h^2, \Omega_{CDM} h^2, n_s, z_{re}, \Omega_\Lambda, m)$ plane at the 68.3% and 95.4% confidence levels. Contours on $m - (\Omega_b h^2, \Omega_{CDM} h^2, n_s, z_{re}, \Omega_\Lambda)$ are shown in Fig. 3.19. $\Omega_{A2}$ and $m$ have no degeneracy with other cosmological parameters.
CHAPTER 3. THERMAL EVOLUTION WITH DECAYING COSMOLOGICAL TERM IN THE CONTEXT OF FRIEMANN UNIVERSE.

The best fit parameters in $\Lambda$CDM and $S\Lambda$CDM are shown in the second and third column of Tab. 3.2. Interestingly, the present density of dark matter is smaller, but the baryon density is larger than that in $S\Lambda$CDM. We also find that $\Omega_{m0} + \Omega_{\Lambda0}$ is close to $-1$ which supports the flat universe.

In our analysis from WMAP first year result, the uncertainty of reionization redshift is very large, that is similar to the results in $S\Lambda$CDM model. On the other hand, $z_{re}$ from the newest result of WMAP is more accurate than first year results. Next we carry out the constraint from the recent CMB observation, WMAP [44, 47], BOOMERanG [111], CBI [108] and Acber [110], with same priors in analysis using WMAP first year. Our results are shown in Figs. 3.18 and 3.19. $\Omega_{\Lambda2}$ and $m$ have no degeneracy with other cosmological parameters. The comparison of best fit values from WMAP three years in $\Lambda$CDM and $S\Lambda$CDM model are shown in the fourth and fifth column of Tab.3.2. After all from recent CMB observation, a decaying-$\Lambda$ contribution on the cosmic thermal evolution is very small. Figure 3.22 illustrates likelihood functions of cosmological parameters in $\Lambda$CDM from WMAP first and three year data. Whereas many cosmological parameters are determined more accurately compared with results from WMAP first year, constraints on $\Omega_{\Lambda2}$ somewhat seems to be loose : $m < 4.2$ and $\Omega_{\Lambda} < 1.9 \times 10^{-4}$ at 95.4% confidence limits. On the other hand, the upper limit of $m$ from recent CMB observation has no difference compared to that from WMAP first year. From these results, our analysis by MCMC cannot determine the value of $m$, because effects of large-$m$ are canceled by small-$\Omega_{\Lambda}$. In this respect, we must investigate the properties of $m$ and/or $\Omega_{\Lambda2}$ in the early epoch of the universe from the more fundamental point of view. Otherwise, the more detailed observations concerning the reionization and/or the first objects could give a severe constraint to the variable $\Lambda$ term.
CHAPTER 3. THERMAL EVOLUTION WITH DECAYING COSMOLOGICAL TERM IN THE CONTEXT OF FRIEMANN UNIVERSE.

Fig. 3.16: An example of parameter search by Markov Chain Monte Carlo. The vertical line indicates the parameter $\theta$ and the horizontal axis is the iteration numbers. Calculations began the iteration number $\sim 100$ converge to a certain value which can be determined by statistical analysis such as the $\chi^2$ fitting [102]. In this thesis, all parameters finally determined in this chapter are determined with the same procedure which convergence levels are checked statistically.
Fig. 3.17: Probability distribution function of $m$ (top panel) and $\Omega_{A2}$ (top panel). The vertical lines mean the 95.4% confidence levels of individual parameters.
CHAPTER 3. THERMAL EVOLUTION WITH DECAYING COSMOLOGICAL TERM IN THE CONTEXT OF FRIEMANN UNIVERSE.

Fig. 3.18: Constraints on $\log \Omega_{A2} - \Omega_b h^2$ (left top), $\log \Omega_{A2} - \Omega_{CDM} h^2$ (right top), $\log \Omega_{A2} - n_s$ (left middle), $\log \Omega_{A2} - z_{re}$ (right middle), $\log \Omega_{A2} - \Omega_\Lambda$ (left bottom), and $m - \log \Omega_{A2}$ (right bottom) from WMAP first year results. The curves correspond to the 68.3% and 95.4% confidence levels.
Fig. 3.19: Constraints on $m - \Omega_b h^2$ (left top), $m - \Omega_{CDM} h^2$ (right top), $m - n_s$ (left middle), $m - z_{re}$ (right middle), and $m - \Omega_\Lambda$ (bottom), from WMAP first year results. The curves correspond to the 68.3% and 95.4% confidence levels.
CHAPTER 3. THERMAL EVOLUTION WITH DECAYING COSMOLOGICAL TERM IN THE CONTEXT OF FRiEMANN Universe.

Fig. 3.20: Constraints on $\log \Omega_{A2} - \Omega_b h^2$ [left top], $\log \Omega_{A2} - \Omega_{CDM} h^2$ [right top], $\log \Omega_{A2} - n_s$ [left middle], $\log \Omega_{A2} - z_{re}$ [right middle], $\log \Omega_{A2} - \Omega_{\Lambda}$ [left bottom], and $m - \log \Omega_{A2}$ [right bottom] from WMAP three year results. The curves correspond to the 68.3% and 95.4% confidence levels.
Fig. 3.21: Constraints on \( m - \Omega_b h^2 \) (left top), \( m - \Omega_{CDM} h^2 \) (right top), \( m - n_s \) (left middle), \( m - z_{re} \) (right middle), and \( m - \Omega_\Lambda \) (bottom), from WMAP first three results. The curves correspond to the 68.3% and 95.4% confidence levels.
Fig. 3.22: Illustration of likelihood functions from WMAP first and three year.
Chapter 4

Summary and Discussion

We have investigated the decaying-$\Lambda$ terms that affect the thermal evolution of the universe. Since, we have no deceit observational evidence that restricts the property of $\Lambda$, we have chosen two different models to describe the universe. First is the modified Brans-Dicke model that includes a decaying-$\Lambda$, where the model can affect the cosmic expansion rate during the early epoch in the universe appreciably. Thus, we can use the Big-Bang Nucleosynthesis to limit the parameters included in the model. We find that the model is consistent with the observations of light elements. As a consequence, we can constrain the coupling constant for the parameter $\Lambda$.

Next in the contest of the Friedmann universe, we have investigated the thermal evolution of the universe with the $\Lambda$ term as function of the cosmic scale factor that reduces the photon energy density (D$\Lambda$CDM model). Although the energy density of a $\Lambda$-term is increasing at the early era, it is found that the effects of the decaying $\Lambda$ can be ignored at $z < 2$ from the magnitude-redshift relation of SNIa with a reasonable parameter space. On the other hand, a $\Lambda$ term alters the evolution of the photon energy density (or photon temperature). Depending on parameters in D$\Lambda$CDM model, the photon energy density could be lower and higher compared with that in S$\Lambda$CDM model at $z > 0$. However, the second case should not occur because the photon temperature become negative at some epoch of $z < 0$. The photon decoupling occurs at $z_{dec} = 2040$, that is earlier by $\Delta z_{dec} = 950$ compared to S$\Lambda$CDM.

We investigate the effects of a decaying-$\Lambda$ term on CMB angular power spectrum. We obtain the Boltzmann equation for photons in D$\Lambda$CDM model.
based on the cosmological perturbation theory and find that a $\Lambda$ contribution appears. We found that a decaying-$\Lambda$ alters the CMB angular power spectrum significantly because of follows effects: large baryon energy density relative to photon density cause to boost up first and third peaks, and the early photon decoupling shifts CMB spectrum to higher multipoles. Fixing cosmological parameters such as baryon density or Hubble constant, we obtain the upper limits of $m$ and $\Omega_{\Lambda,2}$ from WMAP first year results: $m\Omega_{\Lambda,2}/(4-m) < 4.9 \times 10^{-6}$. If photon decoupling occur earlier by $\Delta z = 30$ compared to S$\Lambda$CDM model, D$\Lambda$CDM model is consistent with CMB observed by WMAP.

Next, using Markov Chain Monte Carlo analysis, we put constraint on $m$, $\Omega_{\Lambda,2}$ and cosmological parameters: present energy density of baryon and dark matter, Hubble constant, reionization redshift, scalar spectrum index and amplitude of density fluctuation. As the results, we obtain the upper limits of parameters in D$\Lambda$CDM as $m < 4.1$ and $\Omega_{\Lambda,2} < 2.5 \times 10^{-4}$ from WMAP three year results. We cannot find definite evidences of the decaying-$\Lambda$. Interestingly, there are not degeneracy between parameters in D$\Lambda$CDM model and that in S$\Lambda$CDM model. From our constrains, the contribution of a decaying-$\Lambda$ term to cosmic thermal evolution should be extremely small, though the possibility of a D$\Lambda$CDM model cannot be ruled out. Although it was suggested that a decaying-$\Lambda$ affect on the first objects formation, adopted parameter ($\Omega_{\Lambda,2} = 10^{-4}$ and $m = 1.5$) in Ref. [36] is excluded in our analysis.

In this thesis, we assume a cosmological term as function of scale factor for simplification. Even if we parametrize reasonably the evolution of the cosmological term or the equation of state of dark energy, it should be unlikely that dark energy decay into CMB photons appreciably.

On the other hand, we obtain that the reionization occur at $z_{re} = 11$ from WMAP three year, which suggests that a first object could be formed at around this epoch. However, the uncertainty is very large, and it may be difficult to judge that decaying $\Lambda$ term does not exist within the accuracy of the observations. We should note that we assume reionization history can be described by delta-function as in chapter 3.4.4. The next CMB satellite, Plank, is expected to determine reionization history.

Finally, we remark that although the cosmological term seems to be con-
stant after the photon decoupling era, it does not insure the constancy during the early epoch as shown in Brans-Dicke $\Lambda$ universe. Therefore, it would be desirable to construct consistent model from the early universe to the present in the context of gravitational theory, which includes the inflational theory.
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BIBLIOGRAPHY


74


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Appendix A

Thermodynamics in the early Universe

At the early epoch, the universe is in thermal equilibrium state. As a consequence, we can apply the thermodynamics to describe the thermal history of the universe. In standard model, the thermal bath consists of bosons (photons) and fermions (electrons and neutrinos). The distribution function becomes Fermi-Dirac distribution for fermions ($+ \text{ sign}$) or Bose-Einstein distribution for bosons ($- \text{ sign}$):

$$f(p) = \frac{1}{\exp\left(\frac{E(p)-\mu}{k_B T}\right) \pm 1},$$  \hspace{1cm} (A.1)

where $E(p) = (m^2 + p^2)^{1/2}$; $m, k_B$ and $\mu$ is mass of the particle, the Boltzmann constant and chemical potential, respectively.

The number density $n$, energy density $\rho$, pressure $p$ are written in terms of Eq. \(\text{(A.1)}\) as follows:

$$n = g \int \frac{d^3p}{(2\pi \hbar)^3} f(p) = \frac{g}{2\pi^2 \hbar^3} \int dE \frac{E\sqrt{E^2-m^2}}{\exp\left(\frac{E-E/\mu}{k_B T}\right) \pm 1},$$  \hspace{1cm} (A.2)

$$\rho = g \int \frac{d^3p}{(2\pi \hbar)^3} E(p) f(p) = \frac{g}{2\pi^2 \hbar^3} \int dE \frac{E^2\sqrt{E^2-m^2}}{\exp\left(\frac{E-E/\mu}{k_B T}\right) \pm 1},$$  \hspace{1cm} (A.3)

$$p = \frac{g}{(2\pi \hbar)^3} \int d^3p \frac{|p|}{3E} f(p) = \frac{g}{6\pi^2 \hbar^3} \int dE \frac{(E^2-m^2)^{3/2}}{\exp\left(\frac{E-E/\mu}{k_B T}\right) \pm 1},$$  \hspace{1cm} (A.4)

where $g$ is the degree of freedom. When we consider the case of non relat-
On the other hand, for the relativistic particle, Eqs. (A.2), (A.3) and (A.4) are written as

\[ n = g \frac{\zeta(3) (k_B T)^3}{\pi^2 \hbar^3} \times \begin{cases} 1 & \text{Boson} \\ 3/4 & \text{Fermion} \end{cases} \]  

(A.8)

\[ \rho = g \frac{\pi^2 (k_B T)^4}{30 \hbar^3} \times \begin{cases} 1 & \text{Boson} \\ 7/8 & \text{Fermion} \end{cases} \]  

(A.9)

\[ p = \frac{1}{3} \rho \]  

(A.10)

where \( \zeta(s) = \sum_{n=1}^{\infty} n^{-s}, (s > 1) \) is Riemann’s zeta functions.

Since the energy densities of non-relativistic particles are much smaller than that of relativistic particles, the cosmic temperature in the thermal equilibrium is dominated by the relativistic particles:

\[ \rho(T) \simeq \rho_r(T_r) = \frac{\pi^2 k_B^4}{30 \hbar^3} \left( \sum_{\text{boson}} g_b T_b^4 + \frac{7}{8} \sum_{\text{fermion}} g_f T_f^4 \right) = \frac{\pi^2}{30} g_* (k_B T)^4 \]  

(A.11)

where we set \( T \equiv T_r \). In the last term in Eq. (A.11), \( g_* \) is the degree of freedom defined by

\[ g_* = \sum_{\text{boson}} g_b \left( \frac{T_b}{T} \right)^4 + \frac{7}{8} \sum_{\text{fermion}} g_f \left( \frac{T_f}{T} \right)^4. \]  

(A.12)

In the following section, let us explain the thermal history at the early universe.

A.1 Thermal equilibrium epoch

For \( 5 \times 10^9 \lesssim T_r \lesssim 10^{11} \text{ K} \), photons, neutrinos, and electrons (plus positrons) are thermal equilibrium. In this era, the energy density of radiation component is described as follows

\[ \rho_{\text{rad}} = \rho_\gamma + \rho_e + \rho_\nu, \]  

(A.13)
where the subscripts \( \gamma, e \) and \( \nu \) means photons, electron and neutrinos (three species and anti-neutrinos). Integrating Eq. (A.3) we can obtain for each components in Eq. (A.13):

\[
\rho_{\gamma} = 2 \frac{\pi^2}{30} \frac{(k_B T_{\gamma})^4}{h^3 c^5} = a_{\gamma} T_{\gamma}^4, \quad (A.14)
\]

\[
\rho_e = \frac{7}{8} \cdot 2 \frac{\pi^2}{30} \frac{(k_B T_{\nu})^4}{h^3} = \frac{7}{8} a_{\nu} T_{\nu}^4, \quad (A.15)
\]

\[
\rho_{\nu} = \frac{7}{8} \frac{\pi^2}{30} \frac{(k_B T_{\nu})^4}{h^3} = \frac{7}{16} a_{\nu} T_{\nu}^4, \quad (A.16)
\]

where \( \hbar \) is the Planck constant divided by \( 2\pi \) and \( a_{\gamma} \) is the radiation constant defined by

\[
a_{\gamma} \equiv 2 \frac{\pi^2}{30} \frac{k_B^4}{h^3} = 7.564 \times 10^{-5} \text{ erg cm}^{-3} \text{ K}^{-4}.
\]

From the evolution of the radiation energy density, \( \rho_{\gamma} \propto a^{-4}(t) \), and the condition of thermal equilibrium \( T_{\gamma} = T_{\nu} = T_e \), we obtain

\[
\rho_r = \left( \frac{11}{4} + \frac{7}{8} N_{\nu}^{\text{eff}} \right) a_{\nu} \left( \frac{T_{\nu 0}}{a} \right)^4, \quad (A.17)
\]

where \( N_{\nu}^{\text{eff}} \) is effective number of neutrinos and we set \( N_{\nu}^{\text{eff}} = 3 \) from the result of the standard BBN and CMB analysis.

We note that the weak interaction rate \( \Gamma \) is written as

\[
\Gamma \sim \frac{G_F^2 k_B^3 T^5}{\hbar}. \quad (A.18)
\]

Where the Fermi coupling constant \( G_F = 1.2 \times 10^{-5} \text{ GeV}^{-2} \). If the time scale of the weak interaction becomes the same order of the cosmic expansion rate \( H \), the neutrino decoupling occurs. Namely the condition is

\[
\frac{\Gamma}{H} \sim \frac{G_F^2 k_B^3}{2\pi^2} \left( \frac{45 \hbar c^5}{g_e G} \right)^{\frac{1}{2}} T^3 \sim \left( \frac{T}{1.6 \times 10^{10} K_B} \right)^3 \sim 1. \quad (A.19)
\]

### A.2 Electron-positron annihilation epoch

When the temperature in the universe decreases to the corresponding mass of electrons, \( T_e \approx 0.5 \text{ MeV} \approx 5.9 \times 10^9 \text{ K} \), the electron-positron annihilation occurs:

\[
e^+ + e^- \rightarrow \gamma + \gamma. \quad (A.20)
\]
Fig. A.1: Illustrations of evolution of the energy density of photons, neutrinos and electrons before the primordial nucleosynthesis era. The decoupling of electrons around $5 \times 10^9$ K is due to the pair-annihilation, where the small increase in the temperature changes the results of BBN in some degree.

Thereafter the number density of electrons decreases rapidly since the temperature decreases due to the expansion. The number density of electrons and positron is given by (A.2),

$$n_{e\pm} = 2 \int \frac{d^3 p}{(2\pi \hbar)^3} \frac{1}{\exp (E/kT) + 1}.$$  

Therefore the entropy production by Eq. (A.20), the photon temperature increase. Then the total energy density at this era is written as follows:

$$\rho_{tot} \simeq \rho_\gamma + \rho_\nu + \rho_e = a_\gamma T_\gamma^4 + \frac{7}{8} N_{\nu}T_\nu^4 + \rho_e(T_e).$$  \hspace{1cm} (A.21)

where $\rho_e$ is the sum of the energy density of the electrons and positrons, and only numerically calculated.

### A.3 After annihilation

After the decoupling of neutrinos, the relation between the photon and neutrino temperature is given by

$$\frac{T_\nu}{T_\gamma} = \left(\frac{4}{11}\right)^{\frac{1}{3}}.$$  \hspace{1cm} (A.22)
APPENDIX A. THERMODYNAMICS IN THE EARLY UNIVERSE

Then the radiation energy density is written as,

$$
\rho_r = \left[ 1 + \left( \frac{4}{11} \right)^{\frac{4}{3}} \frac{7}{8} N_{\nu}^{\text{eff}} \right] a_r T_\gamma^4 = \left[ 1 + \left( \frac{4}{11} \right)^{\frac{4}{3}} \frac{7}{8} N_{\nu}^{\text{eff}} \right] a_r \left( \frac{T_{\gamma 0}}{a} \right)^4
$$

where the photon temperature at the present epoch $T_{\gamma 0}$ is obtained by COBE satellite: $T_{\text{CMB}} = 2.725 \pm 0.002$ K at 2$\sigma$ [65].

Figure A.1 illustrates the evolution of energy density of photons, neutrinos and electrons. We can find that the electron density decrease rapidly around $T = 10^9$ K and the density of photon increase. Table. A.1 shows the density and pressure of electrons.
Table A.1: Evolution of pressure and density of electrons.

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Appendix B

Tight coupling approximation

In this appendix, we give simple explanation of the tight coupling approximation, where both baryons and photons are treated as single fluid before the hydrogen renomination. For convenience, in this section, we adopt the metric perturbation in the conformal Newtonian gauge.

In the conformal Newtonian gauge, the line element is characterized by two scalar fields $\Phi$ and $\Phi$,

$$
\langle s^2 = -a(\tau) \left[(1 + 2\Psi) d\tau^2 + (1 - 2\Phi) dx^i dx_i, \right].
$$

As the same analysis in §3.4, we obtain the hierarchy of Boltzmann equations for photon written as,

$$
\begin{align}
\dot{\delta}_\gamma &= -\frac{4}{3} \theta_\gamma + 4\dot{\Phi}, \\
\dot{\theta}_\gamma &= k^2 \left(\frac{\delta_\gamma}{4} - \sigma_\gamma\right) - \dot{\tau}_c (\theta_\gamma - \theta_b) + k^2 \Psi, \\
\dot{\sigma}_\gamma &= \frac{4}{15} \theta_\gamma - \frac{3}{10} k F_{\gamma 3} + \frac{\dot{\tau}_c}{10} (-9\sigma_\gamma + G_{\gamma 0} + G_{\gamma 2}), \\
\dot{F}_{\gamma l} &= \frac{k}{2l + 1} \left(lF_{\gamma (l-1)} - (l + 1)F_{\gamma (l+1)}\right) - \dot{\tau}_c F_{\gamma l} \ (l > 3),
\end{align}
$$

where $\dot{\tau}_c \equiv a x_c n_c \sigma_\gamma$. Definition of $\delta, \theta,$ and $\sigma$ is same as in §3.4.1. On the other hand, equations for baryons in the conformal Newtonian gauge are written as,

$$
\begin{align}
\dot{\delta}_b &= -\theta_b + 3\dot{\Phi}, \\
\dot{\theta}_b &= \frac{\dot{a}}{a} \theta_b + c_s^2 k \delta_b + R^{-1} \dot{\tau}_c (\theta_\gamma - \theta_b) + k^2 \Psi,
\end{align}
$$

where $R \equiv 3\bar{p}_b/4\bar{\rho}_\gamma \propto a^{-1}$ and $c_s$ is the sound speed of baryon.
Before recombination, the mean free path of photon is shorter than the horizon of the universe, i.e. $k/\dot{r}_c \ll 1$. Combining Eqs. (B.1) and (B.2), following equation are obtained,

\[ \frac{3}{4} \delta_\gamma - \dot{\delta}_b = -\theta_\gamma + \theta_b. \]  
\[ \delta_\gamma - \dot{\theta}_b = -\dot{\tau} (\delta_\gamma - \theta_b) \left( 1 + \frac{1}{R} \right). \]  
\[ \sigma_\gamma = -\frac{9}{10} \dot{r}_c \sigma_\gamma. \]  
\[ \dot{F}_{\gamma l} = -\dot{r}_c F_{\gamma l}. \]  

Integrating Eqs. (B.4), (B.5), and (B.6) we obtain

\[ \theta_\gamma - \theta_b \propto \exp \left( -\int d\tau \dot{r}_c \left( 1 + \frac{1}{R} \right) \right), \]  
\[ \sigma_\gamma \propto \exp \left( -\frac{9}{10} \dot{r}_c \tau \right), \]  
\[ F_{\gamma l} \propto \exp (-\dot{r}_c \tau). \]  

Since $\dot{r}_c$ is very large, from Eqs. (B.3), (B.7), (B.8), and (B.9), $\delta_\gamma$, $\theta_\gamma$, $\sigma_\gamma$ and $F_{\gamma l}(l > 3)$ can be written to a good approximation as follows:

\[ \theta_b = \theta_\gamma, \]  
\[ \frac{\delta_\gamma}{4} = \delta_b, \]  
\[ \sigma_\gamma = F_{\gamma l} = 0. \]

From Eq. (B.10), we can treat the baryons and photons as the single fluid in the early universe.

Next we consider that the velocity of photons differs from that of baryons exiguitly,

\[ \theta_\gamma - \theta_b = k^2 \dot{r}_c^{-1} d, \]  

where $d$ is arbitrary function. Substituting Eq. (B.11) for the Euler equation of photon,

\[ \dot{\delta}_\gamma = \frac{k^2}{4} \delta_\gamma + k \Psi - kd. \]  

From the Euler equation of baryon,

\[ \dot{\theta}_\gamma = -\frac{\dot{a}}{a} \theta_\gamma + k^2 \left( \Psi + \frac{d}{R} \right). \]
Fig. B.1: (a): Acoustic oscillation: Photon pressure resists gravitational compression of the fluid setting up acoustic oscillations. Spring and balls schematically represent fluid pressure and effective mass, respectively. (b): Bryon drag decreases the sound horizon the increase the gravitational mass, causing more infall and a net zero-point displacement. $\Psi$ indicate the metric perturbation in the conformal Newtonian gauge and $\Theta = \delta_{\gamma}/4$ [99].

Combining Eq. (B.13) with Eq. (B.12), we get

$$(1 + R) \dot{\theta}_{\gamma} = -\frac{\dot{a}}{a} \theta_{\gamma} + (1 + R) k^2 \Psi + \frac{k}{4} R \delta_{\gamma}. $$

From the relation, $\dot{a}/a = \dot{R}/R$, the above equation is written

$$\dot{\theta}_{\gamma} = \frac{\dot{R}}{1 + R} \theta_{\gamma} + k \Psi + \frac{k^2 \delta_{\gamma}}{4(1 + R)}. $$ \hspace{1cm} (B.14)

Differentiated continually equation for photon is applied to Eq. (B.14) and we obtain

$$\ddot{\delta}_{\gamma} - \frac{\dot{R}}{1 + R} \dot{\delta}_{\gamma} + k^2 c_s^2 \delta_{\gamma} = -\frac{4 \dot{R}}{1 + R} \Phi - \frac{4}{3} k^2 \Psi + 4 \ddot{\Phi}. $$ \hspace{1cm} (B.15)
where $c_{\gamma b}$ is the sound speed of baryon plus photon fluid,

$$c_{\gamma b}^2 = \frac{\tilde{p}_\gamma}{\tilde{\rho}_b + \tilde{\rho}_\gamma} = \frac{1}{3 (1 + R)}.$$ 

We can understand that the fluctuation of the energy density of photon evolves as the forces oscillation by forcing function in the right hand side of Eq. (B.15). Now we assume that the time scale of the oscillation is shorter than the cosmic expansion rate, where terms with $\dot{R}$ are negligible. Then Eq. (B.15) is simplified as follows,

$$\ddot{\delta}_\gamma + k^2 c_{\gamma b}^2 \delta = -\frac{4}{3} k^2 \Psi.$$ 

(B.16)

For an analytical solution, we obtain

$$\frac{1}{4} \delta_\gamma + (1 + R) \Psi = a_1 \sin (k r_s) + a_2 \cos (k r_s)$$

(B.17)

where $r_s$ is the sound horizon defined by

$$r_s = \int c_{\gamma b} d\tau \simeq c_{\gamma b} \tau,$$

and $a_1$ and $a_2$ are integration constant given by the initial condition of the conformal Newtonian gauge [89],

$$\frac{\delta_\gamma}{4} = -\frac{\Psi}{2}.$$  

As a consequence, we get

$$a_1 = 0,$$

$$a_2 = \left( R + \frac{1}{2} \right) \Psi$$

(B.18)

and from Eqs. (B.17) and (B.18), we obtain the temperature fluctuation [88]

$$\frac{\Delta T}{T} = \frac{\delta_\gamma}{4} + \Psi = -R \Psi + \left( R + \frac{1}{2} \right) \Psi \cos (k r_s).$$

(B.19)

The baryon infall into potential wells and compress by gravity. However, the photon pressure resists gravitational compression of the baryon. As the results, the photon density perturbation evolves as harmonic oscillator shown in Fig. B.1. a.

If baryon density increase, the effective mass, $m_{eff} = 1 + R$, increase and it causes higher peaks in the acoustic oscillation (Fig. B.1.b).

In D$\Lambda$CDM model, in spite of the large baryon density, energy density of photon at the last scattering surface is lower than that in S$\Lambda$CDM model. As the results, the ratio of first peak to second peak increases shown in Figs. 3.12 and 3.13.
Appendix C

Results of WMAP first and three year

In this chapter, we compare the temperature power spectrum obtained by WMAP first year [101] and newest WMAP result [47]. These data can obtain from LAMBDA website [113].

C.1 WMAP first year

Column 1 mean multipole moment $l$ for the bin

Column 2 mean value of TT power spectrum ($= l(l+1)C_l/\pi$) in the bin, units = uK$^2$

Column 3 ‘Error’ for binned value, as computed from diagonal terms of the Fisher matrix, units = uK$^2$.

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### C.2 WMAP three year

Column 1 mean multipole moment \( l \) for the bin

Column 2 smallest \( l \) contributing to the bin

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APPENDIX C. RESULTS OF WMAP FIRST AND THREE YEAR

Column 3 = largest $l$ contributing to the bin

Column 4 = mean value of TT power spectrum ($= l(l+1)C_l/2\pi$) in the bin,

Column 5 = ‘Error’ for binned value, as computed from diagonal terms of the Fisher matrix

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APPENDIX C. RESULTS OF WMAP FIRST AND THREE YEAR

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Fig. C.1: Comparison of WMAP first and three year data and angular power spectrum obtained from these. Since newest WMAP results can determine the acoustic peaks up to third peak, we can obtain the cosmological parameters more precisely.