Composite resonances and their impact on the electroweak chiral Lagrangian

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In this talk we study the low-energy effective couplings generated by strongly-coupled electroweak models that contain heavy composite resonances. Invariance under $SU(2)_L \times SU(2)_R$ is a key ingredient in the construction of the resonance action. For simplicity, in these proceedings we focus our attention on the impact of a heavy colourless vector $V$, which transforms as a triplet under the custodial group. More precisely, we study the couplings that are relevant for the vector form-factors of the $L + R$ current into two electroweak Goldstones and into two Standard Model fermions, which contribute to the oblique parameters $S$ and $T$ and the anomalous $Z \to f \bar{f}$ couplings, respectively. Our predictions are compatible with bounds from direct and indirect searches for $M_V \gtrsim 1.5$ TeV. Finally, although we consider an antisymmetric tensor formalism to describe the vector resonance, we derive the equivalent action in the Proca four-vector representation and show that the predictions for low-energy couplings and form-factors are identical, as expected.

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1. Impact of heavy resonances on the low-energy electroweak effective theory

So far the Large Hadron Collider (LHC) has not found any trace of beyond the Standard Model (BSM) states with masses below 1 TeV. Likewise, no significant deviation has been observed in the low-energy interactions between Standard Model (SM) particles. Effective field theories are then the natural approach. In this talk [1, 2] we discuss the possibility of strongly-coupled BSM scenarios with the approximate custodial symmetry invariance of the SM, exact in the SM scalar sector. We develop an invariant Lagrangian under \( \mathcal{G} = SU(2)_L \times SU(2)_R \), which spontaneously breaks down to the custodial subgroup \( \mathcal{H} = SU(2)_{L+R} \) and generates the electroweak (EW) would-be Goldstone bosons \( \phi^a \), described a unitary \( 2 \times 2 \) matrix \( U(\phi) \). In these (non-linear) EW chiral Lagrangian with a light Higgs (ECLh), the low-energy amplitude \( \mathcal{M} \) has an expansion in powers of infrared scales \( p \) (external momenta and SM masses) of the form (e.g., for \( 2 \rightarrow 2 \) processes) [2, 3, 4, 5, 6],

\[
\mathcal{M} \sim \frac{p^2}{v^2} + \left( \frac{a_k}{p^2} \right)_{\text{NLO (tree)}} - \left( \frac{\Gamma_k}{16\pi^2} \ln \frac{p}{\mu} + \ldots \right) \frac{p^4}{v^2} + \mathcal{O}(p^6). \quad (1.1)
\]

The EW effective theory (EWET) Lagrangian operators can be sorted out based on their chiral dimension:

\[
\mathcal{L}_{\text{EWET}} = \mathcal{L}_2 + \mathcal{L}_4 + \ldots \quad (1.2)
\]

where the operators in \( \mathcal{L}_2 \) are of \( \mathcal{O}(p^4) \) [2, 3, 4, 5]. Covariant derivatives and masses are \( \mathcal{O}(p) \) [7] and each fermion field scales like \( \mathcal{O}(p^{1/2}) \) in naive dimensional analysis (NDA) [2, 4, 5, 8]. The \( \mathcal{G} \)-invariant operators in \( \mathcal{L}_{\text{EWET}} \) are built with the Goldstone tensors \( U(\phi) \), functions \( \mathcal{F}_k \) of the Higgs singlet \( h \), its derivatives \( \partial_{\mu} \ldots \partial_{\mu} h \), the gauge fields and the SM fermions \( \psi \) [8, 9, 10, 11, 12, 13]. From the chiral counting point of view \( \mathcal{L}^{\text{SM}} \) would be \( \mathcal{O}(p^2) \) but its underlying renormalizable structure makes all \( \Gamma_k = 0 \) and ensures the absence of higher-dimension divergences [6, 14]. The most important contributions to a given process are given by the operators of lowest chiral dimension. The leading order (LO) contribution is \( \mathcal{O}(p^2) \) and is given by tree-level diagrams with only \( \mathcal{L}_2 \) vertices. Likewise, the one-loop contribution with only \( \mathcal{L}_2 \) vertices is \( \mathcal{O}(p^4) \); it is suppressed in (1.1) with respect to the LO by a factor \( \frac{p^2}{\Lambda_{\text{NL}}^2} \), with \( \Lambda_{\text{NL}}^2 \sim 16\pi^2 v^2 \Gamma_k^{-1} \gtrsim 3 \text{ TeV} \) (with \( v = (\sqrt{2}G_F)^{-1/2} \approx 246 \text{ GeV} \)). This suppression factor is related to the non-linearity of the ECLh and \( \Lambda_{\text{NL}} \rightarrow \infty \) when the Higgs can be embedded in a complex doublet \( \Phi \) [6].

In these proceedings [1, 2] we focus our attention on the tree-level next-to-leading order (NLO) contributions. They are \( \mathcal{O}(p^4) \) and are provided by tree-level diagrams with one \( \mathcal{L}_4 \) vertex with low-energy coupling \( a_k \) (LEC) and an arbitrary number of \( \mathcal{L}_2 \) vertices. They get contributions from tree-level heavy resonance exchanges. At low energies, these \( \mathcal{O}(p^4) \) terms in (1.1) are typically suppressed with respect to the LO amplitude, \( \mathcal{O}(p^2) \), by a factor \( a_k p^2 / v^2 \approx p^2 / M_R^2 \) [1, 2, 15, 16].

\[1\] Ref. [14] provides a geometrical interpretation in terms of the curvature of metric of the internal weak space of the Higgs. In the flat-space limit one has \( \Lambda_{\text{NL}} \rightarrow \infty \). Linear-Higgs scenarios with a complex Higgs doublet \( \Phi \) correspond to this case. True “non-linear models” are defined by a non-zero curvature, not by their (non-linear) representation.
At high energies, one must include both the light dof (SM particles) and the possible composite resonances as active degrees of freedom (dof) in the Lagrangian [1, 2, 17]:

\[ \mathcal{L} = \mathcal{L}_{\text{non-res}} + \mathcal{L}_R, \]

where \( \mathcal{L}_{\text{non-res}} \) contains only SM fields and \( \mathcal{L}_R \) is the part of the Lagrangian that also contains resonances [1]. The part of the interaction Lagrangian \( \mathcal{L}_R \) relevant for our analysis of the L4 LECs is given by the terms linear in the resonance fields, \( \Delta \mathcal{L}_R = R \mathcal{O}_R [\chi, \psi] \) [1, 2, 15, 16, 17], with \( \chi, \psi \) referring to the light bosonic (fermionic) fields. The tensor \( \mathcal{O}_R [\chi, \psi] \) that couples the heavy resonance \( R \) to the light dof is going to provide the first correction to the low-energy ECLh by means of diagrams where one has a heavy resonance propagator \( \sim 1/M_R^2 \) exchanged between two vertices with \( \mathcal{O}_R [\chi, \psi] \). This gives an EWET operator of \( \mathcal{O}(p^4) \). At low energies, resonance operators with tensors \( \mathcal{O}[\chi, \psi] \) of a higher order in \( p \) or containing two or more \( R \) fields contribute only to \( \mathcal{L}_f \) with \( d > 4 \).

The tree-level contribution to \( \mathcal{L}_{\text{EWET}} [\chi, \psi] \) is given by the underlying high-energy action \( S[\chi, \psi, R] \) with the resonance fields \( R \) evaluated at the classical solution \( R_\text{cl} [\chi, \psi] \) of their equations of motion (EoM). Solving the resonance EoM and expanding their solutions in powers of momenta for \( p \ll M_R \), one can write the heavy fields as local operators of the EWET dof [15]. This prediction for the contribution to the low-energy ECLh can be complemented through the consideration of ultraviolet-completion hypotheses (sum-rules [18, 19], unitarity [16], asymptotic form-factor counting rules [20],...). This imposes constraints on the resonance couplings that then turn into predictions for the low-energy theory.

2. Phenomenological example: vector form-factors

Let us illustrate this with a basic example. We consider a colourless triplet vector resonance \( V \) in a composite theory with the same symmetries of the scalar sector of the SM – invariance under parity and charge conjugation –, with its high energy interaction provided by the Lagrangian [1, 2],

\[ \Delta \mathcal{L}^{(A)}_V = \langle V_{\mu \nu} \mathcal{O}^{\mu \nu}_V \rangle, \quad \mathcal{O}^{\mu \nu}_V = \frac{F_V}{2 \sqrt{2}} \epsilon^{\mu \nu \lambda} \frac{i G_V}{2 \sqrt{2}} [u^\mu, u^\nu] + \frac{e V}{2} \left( \nabla^\mu J^V_\mu - \nabla^\nu J^V_\nu \right) / v^2, \tag{2.1} \]

with \( \langle ... \rangle \) for the matrix trace, \( u_\mu = i u(D_\mu U)^\dagger u \), the combinations \( f^{\mu \nu}_\pm = u^\dagger W^{\mu \nu} u \pm u^\dagger \hat{B}^{\mu \nu} u \) of the left and right field-strength tensors \( W^{\mu \nu} \) and \( \hat{B}^{\mu \nu} \), respectively, and \( U = u^2 = \exp \{ i q x (\sigma^a / v) \} \) [21, 22]. The precise definition of the covariant derivatives \( D_\mu \) and \( \nabla_\mu \) can be found in [21, 22]. The tensor \( J^V_\mu = - \text{Tr}_D \{ \xi \xi \gamma^\mu \} \) introduces the fermionic vector current in a covariant way, with \( \xi = u \psi_R + u^\dagger \psi_L \), given by the \( SU(2)_{R,L} \) doublets \( \psi_{R,L} = \frac{1}{2} (1 \pm \gamma_5) \psi \), with \( \psi = (t, b)^T \) (other SM doublets can be also added [61]) and the Dirac trace \( \text{Tr}_D \). The superscript \( (A) \) refers to the antisymmetric tensor formulation employed for the spin–1 resonance [15]. The full Lagrangian may contain additional operators not relevant for the form-factors analyzed in this talk [2]. Integrating out \( V \) one gets a contribution to the EWET, which at lowest order is given by

\[ \Delta \mathcal{L}_{\text{EWET}}^{(V)} = \frac{\langle \mathcal{O}^{\mu \nu}_V \rangle^2}{2 M_V^2} - \frac{\langle \mathcal{O}^{\mu \nu}_V \mathcal{O}_V^{\nu \mu} \rangle}{M_V} = - i \frac{F_V G_V}{4 M_V^2} \left( f^{\mu \nu}_+ [u_\mu, u_\nu] \right) - \frac{F_V e V}{\sqrt{2} M_V^2} \langle f^{\mu \nu}_+ \nabla_\mu J^V_\nu / v^2 \rangle + \ldots \tag{2.2} \]
with the dots standing for other effective operators not relevant in these proceedings. For the Higgsless part, one has $\mathcal{F}_3 = a_2 - a_3$ in Longhitano’s notation of \cite{9, 10}. In what follows, we will focus on the Higgsless sector and $\mathcal{F}_3, \mathcal{F}_X, F_V, G_V$ and $\xi^V_1$ simply represent coupling constants.

The resonance Lagrangian (2.1) provides the vector form-factors of the $L + R$ current into two-Goldstones and into two-fermions \cite{23, 21, 22}:

$$F^V_{\phi\phi}(q^2) = 1 + \frac{F_V G_V}{v^2} \frac{q^2}{M^2_V - q^2}, \quad F^V_{jj}(q^2) = 1 - \frac{\sqrt{2} F_V c_1^V}{v^2} \frac{q^2}{M^2_V - q^2},$$

with momentum transfer $q^\mu$. The square form-factors $|F^V_\mu(s)|^2$ contribute to the $S$-parameter at one-loop through the Peskin-Takeuchi sum-rule on the left-right correlator $\Pi_{\mu\nu}^{\gamma B}$ \cite{19}. If one requires that these form-factors give a ultraviolet-convergent contribution to the sum-rule, they must vanish at $q^2 \to \infty$ and one obtains short-distance (SD) constraints \cite{16, 23, 21, 22} and predictions for the LECs \cite{1, 2, 16}:

$$F_V G_V = v^2 \quad \Rightarrow \quad \mathcal{F}_3 = (a_2 - a_3) = - \frac{F_V G_V}{2 M^2_V} \quad \text{SD constr.} = - \frac{v^2}{2 M^2_V}.$$  

(2.4)

For $M_V > 1.5 \text{ TeV}$ one finds the bound

$$-1.3 \cdot 10^{-2} < \mathcal{F}_3 = (a_2 - a_3) < 0.$$  

(2.5)

One can obtain analogous bounds for the LEC $\mathcal{F}_X = v^2/(2 M^2_V)$ by demanding a similar SD behaviour $F^V_{jj}(q^2) \xrightarrow{q^2 \to \infty} 0$ to the fermion form-factor, which would give $\sqrt{2} F_V \xi^V_1 = -v^2$.

### 2.1 $F^v_{\phi\phi}$ form-factor: S-parameter

The impact of the bosonic form-factor $F^v_{\phi\phi}$ on the oblique parameters $S$ and $T$ was studied in a dispersive one-loop resonance analysis \cite{23, 21, 22}, where the lightest triplet vector ($V$) and axial-vector ($A$) resonances were taken into account. Therein, the contribution from the Goldstone and Higgs absorptive channels was incorporated. In particular the $F^v_{\phi\phi}(q^2)$ determined the contribution from the $\phi\phi$ and $B\phi$ cuts to the $S$ and $T$ parameter, respectively \cite{22}. We studied asymptotically-free strongly coupled theories, where $\Pi_{\mu\nu}^{\gamma B}$ satisfies the two Weinberg Sum Rules (WSRs), and scenarios with weaker ultraviolet (UV) conditions (only the 1st WSR applies) such as Conformal \cite{24} or Walking \cite{25} Technicolour, obtaining the 68% confidence level determinations \cite{22}:

$$0.97 < \kappa_W = M^2_V/M^2_A < 1, \quad M_V > 5 \text{TeV} \quad (1\text{st} \& 2\text{nd WSR}),$$

$$0.84 < \kappa_W < 1.30, \quad M_V > 1.5 \text{TeV} \quad (\text{only 1st WSR, for } 0.5 < M_V/M_A < 1),$$

where $\kappa_W$ denotes the $hWW$ (and $h\phi\phi$) coupling in SM units ($\kappa_W^{\text{SM}} = 1$).

### 2.2 $F^v_{jj}$ form-factor: $Z \to f\bar{f}$ anomalous couplings

The $v_f$ and $a_f$ constants that parametrize the $Z \to f\bar{f}$ decay have the form \cite{26},

$$v_f = T^f_3 - 2 Q_f \sin^2 \theta_W + (\delta g_{Zf}^R + \delta g_{Zf}^L), \quad a_f = T^f_3 + (\delta g_{Zf}^R - \delta g_{Zf}^L).$$

(2.7)
with $T_j^i = +1/2$, $T_j^h = -1/2$, the electric charge $Q_j$, the weak angle $\theta_W$ and the new physics parametrized through the $\delta_{SRL}$, given in our low-energy description by

$$|\delta_{SRL}^Z| = |\mathcal{F}^X| \cos(2\theta_W) m_Z^2/v^2,$$

(2.8)
in agreement with current bounds of $\mathcal{O}(10^{-3})$ [27] for the fermion coupling $\mathcal{F}^X \phi \sim v^2/(2M^2) < 1.3 \cdot 10^{-2}$ that one gets from the previous resonance coupling estimate $\sqrt{2} f_{V} c_{\phi}^V = -v^2$, the bound $M_V > 1.5$ TeV [22] and the experimental value $\cos(2\theta_W) m_Z^2/v^2 = 0.07$.

3. Equivalent Proca four-vector representation

Through an appropriate duality transformation in the generating functional it is possible to rewrite the underlying resonance Lagrangian $\mathcal{L}^{(A)}$ in (2.1) as a Proca Lagrangian $\mathcal{L}^{(P)}$ in terms of four-vector field $\tilde{V}_\mu$ and its field strength tensor $\tilde{V}_{\mu\nu} = \nabla_\mu \tilde{V}_\nu - \nabla_\nu \tilde{V}_\mu$. A similar procedure [2, 16, 28] can be applied to models where the resonances are introduced as gauge fields [29]. In the process, additional non-resonant operators with only light dof are generated, which guarantee a proper UV behaviour. [16, 23, 28]. On-shell, this duality can be read as $V^{\alpha\beta} = \tilde{V}^{\alpha\beta}/M_V$ and $\nabla_\mu V^{\rho\mu} = -M_V \tilde{V}^\mu$. In our particular case, the duality transformation [2, 28] changes the antisymmetric tensor Lagrangian (2.1) into

$$\mathcal{L}^{(A)} \rightarrow \mathcal{L}^{(P)} = \langle \tilde{V}_{\mu\nu} \left( f_{\tilde{V}} \frac{f_{\tilde{V}}}{2\sqrt{2}} f_{\tilde{V}}^\mu [u^\mu, u^\nu] \right) + \tilde{V}_{\mu} ( \xi_{\tilde{V}} J_{\nu}/v^2 ) \rangle - \langle ( f_{\tilde{V}} \frac{f_{\tilde{V}}}{2\sqrt{2}} f_{\tilde{V}}^\mu + i g_{\tilde{V}} [u^\mu, u^\nu] )^2 \rangle, \quad (3.1)$$

with $f_{\tilde{V}} = F_{\tilde{V}}/M_V$, $g_{\tilde{V}} = G_{\tilde{V}}/M_V$ and $\xi_{\tilde{V}} = c_{\tilde{V}}^2 M_V$. In the low-energy limit $p \ll M_V$, Eq. (3.1) leads to the same EWET,

$$\mathcal{L}_{\text{EWET}} = -i \frac{f_{\tilde{V}} g_{\tilde{V}}}{4} \langle f_{\tilde{V}}^{\mu\nu} [u^\mu, u^\nu] \rangle - \frac{f_{\tilde{V}} \xi_{\tilde{V}}}{\sqrt{2} M_V} \langle f_{\tilde{V}}^{\mu\nu} \nabla_\mu \tilde{V}_\nu/v^2 \rangle + \ldots \quad (3.2)$$

The same agreement is found for the two form-factors previously obtained in (2.3):

$$F_{\psi\phi}^{\psi\psi}(q^2) = 1 + \frac{f_{\tilde{V}} g_{\tilde{V}}}{v^2} q^2 + \frac{f_{\tilde{V}} g_{\tilde{V}}}{v^2} q^4 M_V^{-2}, \quad F_{\xi\phi}^{\psi\psi}(q^2) = 1 - \frac{\sqrt{2} f_{\tilde{V}} \xi_{\tilde{V}}}{v^2} q^2 M_V^{-2} - q^2 \quad (3.3)$$

4. Conclusions

The EWET couplings can be predicted in terms of resonance parameters; different resonance quantum numbers lead to different patterns for the LECs [1, 15, 17]. Further assumptions about the UV structure of the underlying theory can be used to refine the predictions [1, 22]. In this talk we have provided a couple of examples (oblique parameters $S$ and $T$ and the anomalous $Z f \bar{f}$ couplings) to show that composite resonances with masses of a few TeV ($M_R \sim 4 \pi v \approx 3$ TeV) are compatible with present direct and indirect searches. The $SU(2)_L \times SU(2)_R$ chiral invariance of the ECLh leads to an appropriate low-energy suppression of tree-level NLO corrections by factors $a_k p^2/v^2 \sim p^2/M_R^2$ with respect to the LO prediction, $\mathcal{O}(p^2)$ [1, 15, 16]. Finally, we have shown the equivalence between the antisymmetric tensors $V^{\mu\nu}$ and Proca four-vectors $\tilde{V}^{\alpha}$ representations for spin-1 fields [16, 28].

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References