In this paper we describe ForCa-G, an experiment that aims at measuring short range forces between a surface and particles in its vicinity. Using atoms trapped in a vertical standing wave, we perform a measurement of the local gravitational potential using atom interferometry techniques.

1 Introduction

Measuring short range forces is one of the new experimental challenges of modern physics. While electromagnetic, strong and weak forces are very well unified within the standard model, gravity remains elusive and stands on its own within general relativity. A great variety of unifying conjecture a deviation to Newton’s law on small scales, some even picture Lorentz violation at those scales. They however predict neither range nor magnitude, and must hence be trialed and constrained by experiments.

Originally proposed in 2007, the ForCa-G experiment has advanced a great deal since then and is now able to measure local potentials with great resolution. We present here the principle of the experiment and give stability measurements for the local gravitational field we probe. We also put forth novel phenomena arising from the necessary developments in our apparatus.

2 Measuring local forces

Our experiment is an atomic interferometer that performs local measurements of vertical potentials. For atoms far from any surface, we only measure the local earth gravitational field. This potential is used to trial test the performances of our apparatus.

In order to do so, we trap $^{87}$Rb atoms in an optical vertical lattice. The dynamic on their
The external degree of freedom is described by the following Hamiltonian:

\[ \hat{H} = \frac{\hat{p}^2}{2m} + \frac{U_0}{2} (1 - \cos(k_1 \hat{z})) + mg\hat{z}, \]

(1)

where \( m \) is the mass of the \(^{87}\text{Rb} \) atom, \( U_0 \) the depth of the lattice, \( k_1 \) the lattice wave-number and \( g \) the earth acceleration constant.

Such a Hamiltonian naturally bears as solutions the Wannier-Stark eigenstates \( |W_m\rangle \), where \( m \) is the lattice well index. Such states are localised in space. This feature can be interpreted as the translational-symmetry breaking of the Bloch states by tilting the lattice — there is no longer constructive interference site to site. The eigenvalues \( E_m \) possess an interesting property, they are separated by multiples of the Bloch energy:

\[ E_{m+\Delta m} = E_m + \Delta m \times \frac{mg\lambda_1}{\text{Bloch energy}}, \]

(2)

where \( \lambda_1 \) is the lattice period. This very naturally yields the Bloch frequency \( \hbar \nu_B = mg\lambda_1 (\nu_B = 568.505 \text{ Hz in our experiment}) \). It is important at this point to understand that measuring the \( \nu_B \) is equivalent to measuring the local earth gravitational acceleration constant \( g \). The endgame of our experiment revolves around measuring this frequency and extracting the information contained within.

The internal atomic state is approximated by a two-level system: the hyperfine structure of \(^{87}\text{Rb} \), where \( |g\rangle = |5^2S_{1/2}, F = 1, m_F = 0\rangle \) and \( |e\rangle = |5^2P_{3/2}, F = 2, m_F = 0\rangle \). These states are long-lived and separated in energy by \( \hbar \nu_{\text{HFS}} (\nu_{\text{HFS}} = 6.835 \text{ GHz in our experiment}) \).

A recap of all these energetic features, internal or external, can be found on figure 1.

![Energy levels of a two-level particle trapped in a vertical (tilted by gravity) lattice. The index m can be seen as the lattice site at the centre of the wave function.](image)

**Figure 1** – Energy levels of a two-level particle trapped in a vertical (tilted by gravity) lattice. The index \( m \) can be seen as the lattice site at the centre of the wave function.

### 3 Experimental Setup

Our experiment consists of cooling \(^{87}\text{Rb} \) atoms and trapping them in a shallow vertical optical lattice (figure 2). Once trapped, atomic interferometry is performed to measure the Bloch frequency.

#### 3.1 Cooling atoms

A standard 3 dimensional Magneto-Optical Trap (MOT) loaded from a 2 dimensional MOT is systematically used as first stage cooling. Properly tuned, a few millions atoms are brought to a 2 \( \mu \text{K} \) temperature within 1 second. From this stage atoms are transferred in the vertical lattice.
3.2 The MixTrap

The lattice is created by using a back-reflected 532 nm laser. To yield exploitable depths and reduce position-induced jitter, the waist is set to 700 µm. The power sits around 7W. The depths brought forth by such values are typically of 3 to 4\(E_r\) (kinetic energy gained/lost by the absorption/emission of one photon on the \(^{87}\text{Rb}\) D2 transition traditionally called "recoil energy").

The lattice beam being blue-detuned, Rb atoms are repelled by the maxima of light intensity. In the axial direction this naturally creates the lattice. However, in the radial direction, the atoms eventually fall out. To avoid this deleterious effect and to constrain the atoms on the maximum intensity of the lattice, we superimpose a thinner red-detuned 1064nm beam with a 145 µm waist.

The combination of those to beams is referred to as the MixTrap.

3.3 Measuring the Bloch frequency \(\nu_B\) with atomic Ramsey-Raman spectroscopy

The atoms being trapped in the MixTrap, their wave-functions are naturally described by Wannier-Stark states, energetically separated by \(\hbar\nu_B\). Let \(|m, i\rangle\) describe the state of an atom, where \(m \in \mathbb{Z}\) is the site index of the Wannier-Stark wave-function and \(i = g, e\) is the internal state of the atom (see figure 1).

Two counter-propagating Raman beams with 10 mW power and 2.6 \(\mu\)m waists are used to open the interferometer. This configuration allows for two photons with momentum \(h\kappa_{eff} = 2\hbar k_r\) to be transferred from the Raman lasers to the system and allows coupling between neighbouring Wannier-Stark states. By controlling the detuning between the two Raman beams precisely, transitions can be addressed with sufficient selectivity.

The strength of the coupling depends naturally on the laser intensity, but more critically on the depth of the lattice: too deep traps allow for poor spatial overlap of Wannier-Stark states and significantly reduce coupling. The Rabi frequency between two states separated by \(\Delta m\) can
be written as:

$$\Omega_{\Delta m} = \Omega_0 \times \langle m | e^{-ik_0x^2} | m + \Delta m \rangle,$$

where $\Omega_0$ is free space coupling between hyperfine states. The wave function, once the interferometer has been opened with the first Raman pulse, is written as:

$$|\Psi(t)\rangle = \frac{1}{\sqrt{2}} \left( |m, g\rangle + e^{2\pi i m t} |m + \Delta m, e\rangle \right),$$

where $\nu_t = \nu_{\text{HFS}} + \Delta m \nu_B$. After a Ramsey time $T$, the interferometer is closed with a second Raman pulse, bearing the same characteristics. By detecting populations of atoms in $|g\rangle$ $N_g$ and in $|e\rangle$ $N_e$, we compute the population ratio $\rho = N_e / (N_e + N_g)$, and obtain Ramsey fringes in frequency space separated by $1/T$. By locating the central fringe and adequately measuring its position, a multiple of the Bloch frequency $\nu_{\Delta m} = \Delta m \times \nu_B$ is retrieved. However, this frequency bears an offset that is imparted to clock shifts, such as second order Zeeman effect or differential light-shifts induced by the confinement beams. In order to remove then, one symmetrically measures the central fringe frequency at $\pm \Delta m$ and at $-\Delta m$, and computes the half difference $\nu_B = \frac{1}{2\Delta m} (\nu_{\Delta m} - \nu_{-\Delta m})$.

A recap of this methodology can be found on figure 3.

Figure 3 – (left and centre) Wave-functions are spatially separated by a Raman $\pi/2$ pulse, their respective phases evolve during Ramsey time $T$. They are then recombined with the second Raman $\pi/2$ pulse.

(right) Ramsey fringes observed at the output of the interferometer, here $\Delta m = +6$ and $T = 100\text{ms}$.

3.4 Allan standard deviation

In order to quantify stability when measuring such a quantity, we use the Allan deviation\textsuperscript{11}, a 2-sample estimator repeated in time. We measure a stability $\sigma_\alpha(\tau) = 1.8 \times 10^{-6}$, obtained from the data on figure 4. Since our setup can be seen as a gravimeter, we have $\frac{\sigma_\alpha}{\nu_B} = \sigma_g$. While modest compared to the best atom interferometry based gravimeters\textsuperscript{18} which typically yield a $\sim 5 \times 10^{-9}$, our apparatus remains very competitive in the trapped regime\textsuperscript{17}. This accuracy corresponds to $1\text{mHz}$ in 1 second and $0.1\text{mHz}$ in 100 seconds resolution on the Bloch frequency.

We will now address the question of how we can harness this stability to measure short range forces.

4 Short range forces in the $\mu$m regime

As mentioned earlier, our experiment’s ultimate goal is to measure short range forces with unprecedented accuracy in the micro-meter regime. At the ranges we wish to probe, we expect two forces: gravity and the Casimir-Polder interaction. Our resolution is vastly sufficient to measure these two effects, and we plan at the very least to set new constraints on the existence of a deviation to Newton’s law and at best to measure such a deviation.
4.1 Casimir-Polder

For two conducting plates, quantification of the vacuum virtual states generates an attractive force that pulls the surfaces together. The Casimir-Polder arises from the same phenomena: instead of two plates, it now takes place between an electric dipole and a conducting surface. An approximate derivation of such a potential gives:

\[ U_{CP} = \frac{3\hbar \alpha_0}{4\pi d^3}, \]

where \( \alpha_0 \) is the atomic polarisability, \( c \) the Einstein constant and \( d \) the distance between the perfectly conducting plate and the dipole.

4.2 Gravitational force and possible deviations

The second force — though orders of magnitude smaller — is the Newtonian potential. Interestingly, in the range we are interested in probing, unifying models predict a deviation to this force. As mentioned earlier, they predict neither range nor magnitude and must be therefore tested. To this effect, we consider a Yukawa type potential to test their bounds:

\[ U(r) = \frac{GMm}{r} \left( 1 + \alpha e^{-\lambda r} \right), \]

where \( \alpha \) and \( \lambda \) constrains respectively magnitude and range of the deviation. The lower they are constrained, the better. Despite its Yukawa form, it does not contain any physics, but just a mean to parametrise.

5 Paving the way to short range force measurements

The next step in our experiment is to introduce the dielectric mirror — used for the lattice laser back-reflection — in the vacuum cell and move the atoms to its vicinity. As the atoms are moved closer to the surface, they will become affected by the aforementioned short range forces. In practice, this means that to the Bloch frequency \( \nu_B \), will be added \( \nu_{CP} \) for the Casimir-Polder potential and by \( \nu_G \) for the gravitational pull generated by the mirror on the atom.
The shifts $\nu_{CP}$ have been calculated. The precise methodology can be found in $^{19,20}$. In figure 5, we can observe that at the 12th site, the shift is of 2 Hz. Let it be reminded that this is 3 orders of magnitude larger than our resolution at 1 second, and 4 orders of magnitude at 100 seconds.

However the setup that has been described earlier is not adequate for such measurements...

![Figure 5](image)

Figure 5 – Energy shift due to the Casimir polder interaction, expressed in units of recoil frequency. The Casimir-Polder potential is calculated for an atom perfectly located on a lattice site (red) and more realistically for an atom described by a wave-function spread on several sites (black). Adapted from $^{19}$.

5.1 Achieving higher densities and the added benefits

Loading the atoms from a cold $^{87}$Rb cloud is the most straightforward way to populate sites in the MixTrap. However, the size of the cloud is around 1 mm, and the spatial density is low. If loaded from those conditions, the atoms populate 4000 sites, with around 10 atoms per site. While sufficient to perform measurements of the gravitational acceleration constant $g$, those conditions are not satisfactory for short range force probing: since the Raman transition frequencies will be shifted by the additional forces close to the mirror, only one site can be addressed at a time. One can easily see that making measurements with the signal to noise ratio inherent to 10 atoms per site is far-fetched.

The solution to this difficulty is optical evaporative cooling. For more details on this technique, we refer the curious reader to the rich literature on this subject $^{1}$. To create the required dipolar trap, we cross two 1064 nm laser beams at a 30° angle. The first beam has a maximum of 10W of power and a waist of 30 µm, the second has a maximum power of 30W and a waist of 120 µm. The technique consists of properly ramping their power in order to achieve lower temperature, higher spatial densities and smaller sizes. In practice we obtain around 40000 atoms with a 300 nK temperature, with density of a few $10^{-11}$ at.cm$^{-3}$. When loaded in the MixTrap, we estimate that a normal distribution of sites with a scale parameter of 4 sites are populated: this means that less than 20 sites are significantly populated. It is now reasonable to expect that measuring the transition frequency can be achieved when only one site is addressed.

When loading the MixTrap from a dipolar trap, the average density over the trapped atoms is much higher (a few $10^{-11}$ at.cm$^{-3}$). When the atoms collide, their spins tend to realign. This phenomenon is called Identical Spin Rotation Effect (ISRE) $^{14,15}$ and protects the coherence of the system over long time scales. We measured coherence times over 3 seconds long (limited only by computer-interfacing difficulties) (figure 6). The subject is still under heavy investigation and at this stage it has not been determined if the frequency offset ISRE yields can be accounted for systematically in the Bloch frequency measurements.
Figure 6 – Long coherence times are observed for high densities. For the highest density (black, \(4.0 \times 10^{-11} \text{at.cm}^{-3}\)) the observed contrast plateau is higher than for lower densities (red, \(1.7 \times 10^{-11} \text{at.cm}^{-3}\)). This is a signature of ISRE.

5.2 Elevating the atoms to the mirror

Once the atoms are cooled in the dipolar trap, they need to be brought close to the surface before being loaded in the MixTrap. In order to do so, we have set up a Bloch elevator. It consists of two 780 nm counter-propagating beams, 50 GHz red-detuned, with 100 mW power each and 1 mm waist. By precisely controlling the frequency difference \(\Delta \nu\) between the two beams, an accelerated lattice is created. In a classical picture it is easily understood that the atoms can be accelerated upwards if the lattice is moved accordingly. In the quantum picture, the atoms undergo Bloch oscillations\(^{12,13}\). A process through which they gain \(2 \hbar k_L\) momentum per oscillation. By adequately ramping the power and the frequency, preliminary tests have shown that the atoms can easily go 40 Bloch oscillations (figure 7). We expect to be able to perform the 300 Bloch oscillations that are deemed necessary for our experiment.

Figure 7 – (left) Power and frequency variations the atoms undergo. (right) Time of flight signals on the two hyperfine levels of \(^{87}\text{Rb}\) atoms. Here, the atoms undergo 40 Bloch oscillations in 3 ms to gain \(80\hbar v\) in velocity.
6 Conclusion

We have presented the principle of our experiment and detailed the apparatus. It was also shown that it has the required specifications to perform measurements of the Casimir-Folder interaction in the micro-meter regime with unprecedented accuracy — we expect better than 1% resolution.

Novel phenomena was exhibited in the trapped regime such as ISRE, which may yield some unexpected advantages in probing short range forces. We also gave proof of principle concerning our ability to bring the atoms close to the surface.

We plan in the next year to introduce the mirror in the vacuum and start measuring short range forces.

Acknowledgements

This research is carried on within the iSense project, which acknowledges financial support from the Future and Emerging Technologies (FET) program within the Seventh Framework Program for Research of the European Commission under FET, Open Grant No. 250072. We also acknowledge financial support from the Ville de Paris Emergence(s) Program, IFRAF, the IDEX PSL (ANR-10-IDEX-0001-02 PSL), and ANR (ANR-13-BS04-0003-01). M.K.Z. thanks the FFCSA, CSC, and CNSF (Grant No. 11205064) for financial support. Helpful discussions with all members of the IACI team are gratefully acknowledged.

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