PROBING DEFORMED COMMUTATORS WITH MACROSCOPIC HARMONIC OSCILLATORS

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According to different quantum gravity models, gravitational effects at the Planck scale appear in the form of generalized commutation relations. In this framework, the dynamics of an isolated harmonic oscillator becomes intrinsically nonlinear and shows a dependence of the oscillation frequency on the amplitude. Here we analyze the free decay of micro and nano-oscillators, to place new upper limits to the parameters quantifying the commutator deformation.

1 Introduction and model

General relativity and quantum physics are expected to merge at the Planck scale, defined by distances of the order of $L_P = 1.6 \times 10^{-35}$m and/or extremely high energies of the order of $E_P = 1.2 \times 10^{19}$GeV. Since the study of particles collisions around the Planck energy is well beyond the possibilities of current and foreseeable accelerators, high-energy astronomical events (e.g. γ-ray bursts) have been considered as the privileged natural system to unveil quantum gravitational effects. This common view has been enriched in the last years thanks to a number of studies proposing that signatures of the Planck-scale physics could manifest also at low energies$^{1-5}$. It is indeed widely accepted that, when gravity is taken into account, deviations from standard quantum mechanics are expected. Such deviations are likely to be derived from a deformed canonical commutator between position $q$ and momentum $p$, that in its most common
Here we describe an experiment conceived to test this hypothesis, and set limits to the deformation parameter $\beta_0$. Our work is based on two assumptions. First, we suppose that Eq. (1) holds for the operator $q$ describing a measured position in a macroscopic harmonic oscillator, and its conjugate momentum $p$. In terms of the usually normalized operators, $Q = q/\sqrt{\hbar/(m\omega_0)}$ and $P = p/\sqrt{\hbar/m\omega_0}$, defined for an oscillator with mass $m$ and resonance angular frequency $\omega_0$, the commutation relations are therefore

$$[Q, P] = i(1 + \beta P^2),$$

where $\beta = \beta_0 (\hbar m \omega_0 / m_P c^2)$ ($m_P = E_P / c^2$ is the Planck mass) is a further dimensionless parameter that we assume to be small ($\beta \ll 1$). Such assumption will be shown to be consistent with the experimental results. The second hypothesis is the validity of the Heisenberg equations for the temporal evolution of an operator $\hat{O}$, i.e. $d\hat{O}/dt = [\hat{O}, \hat{H}] / i\hbar$, where $\hat{H}$ is the Hamiltonian $\hat{H} = \hbar/2 (Q^2 + P^2)$.

In particular, the standard Heisenberg evolution equations are applied to the operators $P$ and $Q$. The solution is

$$Q(t) = Q_0 \left[ \sin(\tilde{\omega}t) + \frac{\beta}{8} Q_0^2 \sin(3\tilde{\omega}t) \right],$$

where

$$\tilde{\omega} = \left( 1 + \frac{\beta}{2} Q_0^2 \right) \omega_0.$$

It is valid at the first order in $\beta Q_0^2$, and implies two relevant effects with respect to the harmonic oscillator: the appearance of the third harmonic and, less obvious, a quadratic dependence of the frequency shift on the oscillation amplitude. In case of small damping with relaxation time $\tau$, the dynamics is described by a modified version of Eq. (3) with the replacements $\tilde{\omega} t \rightarrow \Phi(t)$, implying $\tilde{\omega}(t) = d\Phi/dt$, and $Q_0 \rightarrow Q_0 \exp(-t/\tau)$.

## 2 Experiment

We have exploited three kinds of oscillators, spanning a wide range of masses around the Planck mass $m_P = 22\mu g$. The measurements are performed by exciting an oscillation mode and monitoring a possible dependence of the oscillation frequency and of the third harmonic distortion on the oscillation amplitude, during the free decay. The first device is a “double paddle oscillator” (DPO) made from a 300 µm thick silicon plate. Thanks to its shape, for two particular balanced oscillation modes, the oscillator is supported by the outer frame with negligible energy dissipation and it can therefore be considered as isolated from the background. The sample is kept in a temperature stabilized vacuum chamber and vibrations are excited and detected capacitively. We have monitored the mode oscillating at frequency of 5636 Hz with a mechanical quality factor of $1.18 \times 10^5$ (at room temperature) and mass $m = 0.033 \text{ g}$.

For the measurements at intermediate mass we have used a silicon wheel oscillator, made on the 70 µm thick device layer of a SOI wafer and composed of a central disk kept by structured beams, balanced by four counterweights on the beams joints that so become nodal points (Fig. 1b). On the surface of the central disk, a multilayer SiO$_2$/Ta$_2$O$_5$ dielectric coating forms an high reflectivity mirror. The design strategy allows to obtain a balanced oscillating mode (its resonance frequency is 141 797 Hz), with a planar motion of the central mass (significantly reducing the contribution of the optical coating to the structural dissipation) and a strong isolation from the frame. The oscillator is mechanically excited using a piezoelectric ceramic glued on the sample mount. The surface of the core of the device works as end mirror in one arm of a stabilized Michelson interferometer, that allows to measure its displacement. The
quality factor surpasses $10^6$ at the temperature of 4.3 K, kept during the measurements. The meaningful mass is $m = 20 \mu g$. Finally, the lighter oscillators is a $L = 0.5 \, \text{mm}$ side, 30 nm thick, square membrane of stoichiometric silicon nitride, grown on a $5 \, \text{mm} \times 5 \, \text{mm}$, 200 µm thick silicon substrate\textsuperscript{13}. The physical mass of the membrane is 20 ng. We have performed the measurements in a cryostat at the temperature of 65 K and pressure of $10^{-4}$ Pa, where the oscillation frequency is 747 kHz and the quality factor is $8.6 \times 10^{5}$. Excitation and readout are performed as in the experiment with the wheel oscillators.

The frequency shift of the oscillation as function of the amplitude is obtained both by directly fitting the exponentially decaying oscillation with the expected expression, in some cases after a preliminary frequency down-conversion performed with a lock-in amplifier (see Fig. 1a for an example), and by completely frequency down-converting the oscillating signal (down to dc) with hardware and software lock-in amplifiers, then calculating the frequency as first derivative of the residual phase. Similarly, the third harmonic content is deduced both from the fit of time series, and by simultaneous recording of first an third harmonic of the signal with two separated lock-in amplifiers (Fig. 1b). For both indicators, the two methods give comparable results.

Table 1: Maximum relative frequency shifts measured for different oscillators, and corresponding oscillation amplitudes.

<table>
<thead>
<tr>
<th>Mass (kg)</th>
<th>Frequency (Hz)</th>
<th>Max. ampl. (nm)</th>
<th>$1/Q$</th>
<th>Max. $\Delta \omega/\omega_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3.3 \times 10^{-8}$</td>
<td>$5.64 \times 10^5$</td>
<td>600</td>
<td>$8 \times 10^{-6}$</td>
<td>$4 \times 10^{-7}$</td>
</tr>
<tr>
<td>$2 \times 10^{-8}$</td>
<td>$1.42 \times 10^5$</td>
<td>55</td>
<td>$1 \times 10^{-6}$</td>
<td>$6 \times 10^{-8}$</td>
</tr>
<tr>
<td>$2 \times 10^{-11}$</td>
<td>$7.47 \times 10^5$</td>
<td>47</td>
<td>$1.2 \times 10^{-6}$</td>
<td>$3 \times 10^{-6}$</td>
</tr>
</tbody>
</table>

We remark that the frequency stability of the oscillating signal is typically significantly better than the linewidth, as shown in Table 1. We note that $\Delta \omega/\omega_0 \ll 1$ is required in order for the model to be valid (indeed, it implies $\beta Q_0 \ll 1$). In all cases, for large enough excitation, the frequency shows indeed a parabolic dependence on the oscillation amplitude. This feature can be attributed to structural nonlinearity which is intrinsic in all the oscillators. The parabolic coefficients (with its uncertainty) can be used to determine upper limits to the deformation parameters $\beta$ and, actually, $\beta_0$.

3 Results and discussion

Our results are summarized in Fig. 2, where we also report some previously existing limits to the deformation parameter $\beta_0$. In our experiments we have considered a wide range of masses around the Planck mass. We believe our analysis to be particularly meaningful in this regime, as strong deviations from classical Newtonian mechanics arise as soon as the momentum
is of the order of $m_p c$. This is not only true for planetary motion, but even for Kg-scale mechanical oscillators. In relation to this point, we remark that the present approach involves just the expectation values of position and momentum operators. A more powerful route to the search of quantum gravitational effects should focus on specific quantum features of a system. For instance, quantum fluctuations of the spacetime metric and/or spacetime discreteness are expected to significantly affect the evolution of higher order momenta. This motivates a future experiment, based on quantum macroscopic oscillators.

![Figure 2](image_url)

Figure 2 – The parameter $\beta_0$ quantifies the deformation to the standard commutator between position and momentum, or the scale $\sqrt{\hbar g}$ below which new physics could come into play. Full symbols report its upper limits obtained in this work, as a function of the mass. Blue dots: from the amplitude-dependence of the oscillation frequency; red stars: from the third harmonic distortion. Dotted lines are guides for the eyes. Dashed lines reports some previously estimated upper limits, obtained in mass ranges outside this graph. At lower mass, in green: from high resolution spectroscopy on the hydrogen atom, considering the ground state Lamb shift (upper line) and the 1S-2S level difference (lower line). At higher mass, in magenta: from the AURIGA detector, in yellow: from the lack of violation of the equivalence principle.

References