THE $T_z = \pm 1 \rightarrow 0$ AND $\pm 2 \rightarrow \pm 1$ MIRROR GAMOW–TELLER TRANSITIONS IN $pf$-SHELL NUCLEI

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Gamow–Teller (GT) transitions are the most common weak-interaction processes in the Universe. They play important roles in various processes of nucleosynthesis, for example, in the rapid proton-capture process (rp-process). In the $pf$-shell region, the rp-process runs through neutron-deficient nuclei with $T_z = -2, -1,$ and 0 mainly by means of GT and Fermi transitions, where $T_z$ is the $z$ component of isospin $T$ defined by $T_z = (N - Z)/2$. Under the assumption of isospin symmetry, mirror nuclei with reversed $Z$ and $N$ numbers, and thus with opposite signs of $T_z$, have the same structure. Therefore, symmetry is also expected for the GT transitions starting from and ending up in mirror nuclei. We have been studying the $T_z = -2 \rightarrow -1$ and $-1 \rightarrow 0$ GT transitions in $\beta$ decays, while those from stable $T_z = +2$ and $+1$ nuclei by means of hadronic $(^3\text{He},t)$ charge-exchange (CE) reactions. The results from these studies are compared in order to examine the mirror-symmetry structure in nuclei. In addition, these results are combined for the better understanding of GT transitions in the $pf$-shell region.

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1. Introduction

Gamow–Teller (GT) transitions are caused by the spin–isospin (στ) type interaction with ΔL = 0. Since spin and isospin are unique quantum numbers in nuclei, GT transitions represent an important nuclear response. They are of interest not only in nuclear structure physics [1], but also in nuclear astrophysics [2].

GT transitions are studied by β decay and charge-exchange (CE) reactions [1]. The β decay has a direct access to the absolute value of the GT transition strength B(GT) from the study of half-life (T_{1/2}), β-decay Q value (Q^β value), and branching ratio [3]. We see that the inverse of the partial half-life t_j, representing the GT transition strength to the j^th state in the final nucleus, is proportional to the B_j(GT) value

\[ 1/t_j = (1/K) \lambda^2 f_j B_j(GT). \]  

Similarly, for the Fermi transition, we get

\[ 1/t_F = (1/K) f_F B(F), \]  

where K is 6143.6(17) [4], \( \lambda = g_A/g_V = -1.270(3) [5] \), and \( f_j \) and \( f_F \) are phase-space factors (f-factors). The value of \( f_j \) can be calculated from the \( Q_j^β \) value of the j^th excited state. Note, however, \( f_j \) decreases quickly as \( Q_j^β \) decreases. Therefore, β-decay study, in particular the study by means of delayed-γ measurements, can usually access the states up to the half of the \( Q_j^β \) value and the “sensitivity limit” is discussed in Ref. [6].

In contrast, CE reactions, such as (p, n) or (3He, t) reactions at intermediate beam energies (more than \( \approx 100 \text{ MeV/nucleon} \)) and 0°, can selectively excite GT states (states excited by GT transitions) up to the high-excitation energy (\( E_x \)) region in the final nucleus. It has been found that there is a close proportionality between the cross sections at 0° and the transition strengths B(GT) in these CE reactions [7]

\[ \sigma^{GT}(0) \simeq \hat{\sigma}^{GT}(0) B(GT), \]  

where \( \hat{\sigma}_{GT}(0) \) is the unit cross section for the GT transition at 0° (more precisely, at the momentum transfer \( q = 0 [7] \)). Owing to this proportionality, CE reactions are useful tools to study the relative B(GT) strengths up to high excitation energies. Note that the \( \hat{\sigma}_{GT}(0) \) value can be best determined using the B(GT) values from β decays as references. Therefore, studies of β decays and CE reactions are complementary.

In recent (3He, t) measurements performed at RCNP, Osaka [8], energy resolutions of \( \approx 30 \text{ keV} \) have been achieved. This is an improvement of about one order of magnitude compared to the pioneering (p, n) reactions.
As a result, it became possible to identify one-to-one correspondence of GT transitions studied in CE reactions and $\beta$ decays, and thus, a detailed comparison of them is now possible [1].

The $T_z = \pm 1 \rightarrow 0$ and $T_z = \pm 2 \rightarrow \pm 1$ symmetry GT transitions that can be studied by means of strong and weak interactions are shown by upward and downward arrows, respectively, in Figs. 1 and 2 for $pf$-shell nuclei. The $T_z = +1 \rightarrow 0$ and $T_z = +2 \rightarrow +1$ GT transitions can be measured in $(^3\text{He}, t)$ CE reactions on stable target nuclei, while the $T_z = -1 \rightarrow 0$ and $T_z = -2 \rightarrow -1$ transitions in $\beta^+$ decays of proton-rich nuclei.

Fig. 1. (Color online) The pair of $T_z = \pm 1 f$-shell mirror nuclei and $T_z = \pm 1 \rightarrow 0$ mirror transitions (a pair of arrows). Solid lines drawn in the proton-rich side indicate the main rp-process path. Accurate $Q^\beta$ and $T_{1/2}$ values are needed in higher $A$ neutron-deficient nuclei (painted in black).

Fig. 2. (Color online) The pair of $T_z = \pm 2 pf$-shell mirror nuclei and $T_z = \pm 2 \rightarrow \pm 1$ mirror transitions (a pair of arrows). Solid lines drawn in the proton-rich side indicate the main rp-process path.
Note that $T_z = -1$ and $-2$ $pf$-shell nuclei are situated on the expected path of the rapid proton-capture process (rp-process). They are important for the nucleosynthesis of neutron-deficient nuclei [9]. The GT transition strengths starting from these nuclei are the parameters in various models of nuclear astrophysics.

2. $T_z = \pm 1 \rightarrow 0$ mirror GT transitions in $f$-shell nuclei

We studied the $T_z = \pm 1 \rightarrow 0$ mirror GT transitions from the $0^+ \text{g.s.}$ of the even–even $f$-shell nuclei, i.e., for the $A = 42$, 46, 50, and 54 systems (see Fig. 1). Note that in these GT transitions, the identical $J^\pi = 1^+$ excited states (GT states) in $T_z = 0$ final nuclei are reached.

The ($^3\text{He},t$) spectra for the $T_z = +1$ initial target nuclei $^{42}\text{Ca}$ [10], $^{46}\text{Ti}$ [11], $^{50}\text{Cr}$ [12], and $^{54}\text{Fe}$ [13] are shown in Fig. 3. In these spectra,
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the ground states of the $T_z = 0$ final nuclei $^{42}\text{Sc}$, $^{46}\text{V}$, $^{50}\text{Mn}$, and $^{54}\text{Co}$ are all $T = 1$, $0^+$ isobaric analog states (IAS) of the g.s. of the initial nuclei. The analysis of angular distributions shows that most of the prominent states are of $\Delta L = 0$ nature, suggesting that they, except for the known IAS, are excited by GT transitions. The vertical scales are normalized by the heights of the IAS peaks carrying all Fermi transition strength of $B(F) = 2$. Since the ratios of the Fermi and GT unit cross sections are more or less the same for all of these nuclei, it is expected that the peak heights of GT states in these spectra are proportional to $B(\text{GT})$ values associated with the excitations of these states [14].

The mirror $T_z = -1 \to 0$ $\beta$ decays of $^{42}\text{Ti}$, $^{46}\text{Cr}$, $^{50}\text{Fe}$, and $^{54}\text{Ni}$ were studied by measuring delayed $\gamma$ rays in order to obtain the feeding ratios up to higher excitations and accurate half-lives [6]. Since particle (i.e., proton) separation energies ($S_p$) of the final $T_z = 0$ nuclei are relatively high ($S_p = 4.3$–$5.4$ MeV), the measurement of delayed $\gamma$ rays is effective for the study of GT transitions.

The measurements were performed using the RISING setup consisting of 15 Euroball Cluster Ge detectors surrounding the DSSSD detectors [6]. The DSSSDs were used for the detection of the implanted ions and the following $\beta$ particles. Due to the good detection efficiency of the RISING setup ($\approx 12.5\%$ at 1 MeV), high-energy delayed $\gamma$ rays up to 3 MeV could be seen [Fig. 4 (b)] at the energies corresponding to the GT states observed in the $^{46}\text{Ti}(^{3}\text{He},t)^{54}\text{V}$ measurement [see Fig. 4 (a)]. A good symmetry is suggested for the $T_z = \pm 1 \to 0$ GT transitions. GT transition strengths $B(\text{GT})$ were derived from the obtained high precision $\beta$-decay half-lives, excitation energies and $\beta$ branching ratios.

Fig. 4. (a) The detail of the $0^\circ$, $^{46}\text{Ti}(^{3}\text{He},t)^{46}\text{V}$ spectrum. Major states excited with $\Delta L = 0$ are indicated by energies in MeV. (b) Delayed $\gamma$-ray spectrum at RISING, GSI, measured in coincidence with the $^{40}\text{Cr} \beta$ decay. The $\gamma$-ray peaks and CE-reaction peaks are at the same energies.
2.1. Merged analysis combining mirror $\beta$ decay and charge-exchange reaction

Accurate values of the total half-life $T_{1/2}$, with uncertainties of $\approx 0.5\%$, have been obtained in these $\beta$-decay studies (e.g. the $T_{1/2}$ is 224.3(13) ms in the $^{46}$Cr $\rightarrow ^{46}$V $\beta$ decay [6]). For the determination of the $B$(GT) values in the mirror $^{46}$Ti($^3$He, $t$)$^{46}$V reaction, we introduce the “merged analysis” [12] in order to make the best use of this good accuracy. In this analysis, the relative GT strengths obtained in the ($^3$He, $t$) reaction are combined with the total half-life $T_{1/2}$ and $f$-factors from the $\beta$-decay study, where a good symmetry is assumed for the strengths of analogous $T_z = \pm 1 \rightarrow 0$ GT transitions.

The “merged analysis” starts with the formula connecting the total $\beta$-decay half-life $T_{1/2}$ and the partial half-life $t_F$ of the Fermi transition and $t_j$s of GT transitions

$$\left(1/T_{1/2}\right) = \left(1/t_F\right) + \sum_{j=GT} \left(1/t_j\right). \tag{4}$$

The inverse of the half-life represents the transition strength. Therefore, this formula shows that the total $\beta$-decay strength given on the left-hand side is the sum of the strengths of the Fermi and GT transitions, where it is assumed that the contribution from forbidden transitions is negligible. Applying Eqs. (1) and (2), one can eliminate both $t_F$ and $t_j$, and we get

$$\frac{1}{T_{1/2}} = \frac{1}{K} \left[ B(F)f_F + \sum_{j=GT} \lambda^2 B_j(GT)f_j \right], \tag{5}$$

where $f_F$ and $f_j$ can be calculated if the $Q^\beta$ value is known, $B(F) = |N-Z|$, and the relative strengths proportional to $B_j$(GT) can be studied in the ($^3$He, $t$) reaction [see Eq. (3)]. Therefore, if the total half-life $T_{1/2}$ of the $\beta$ decay is known accurately, the relative strengths of the $B_j$(GT) studied in the ($^3$He, $t$) reaction can be converted into absolute values.

2.2. Comparison of $T_z = \pm 1 \rightarrow 0$ mirror GT transitions in the $A = 46$ system

The studies of $T_z = -1 \rightarrow 0 \beta$ decays up to the $E_x \approx 4$–$5$ MeV region in $f$-shell nuclei [6] allowed a detailed comparison of the $B$(GT) values with those from the $T_z = +1 \rightarrow 0$ GT transitions derived from high-resolution ($^3$He, $t$) reactions on mirror target nuclei. As we see in Fig. 3 (b), the main part of the $B$(GT) strength is concentrated in the low $E_x$ region of up to 4 MeV in the $^{46}$Ti($^3$He, $t$)$^{46}$V reaction. Therefore, we can well compare the mirror GT transitions in the $A = 46$ system.
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The $B$(GT) values obtained in the $T_z = \pm 1 \rightarrow 0$ GT transitions are compared in the upper panel of Fig. 5. We see that the $B$(GT) values of individual pair of transitions have some differences, in particular, for weak transitions. However, the cumulative sums of the $B$(GT) distributions shown in the lower panel of Fig. 5 are very similar in the energy region up to $E_x = 4$ MeV, where the $\beta$ decay could access [6]. It should be noted that the $B$(GT) values from the ($^3$He, $t$) reaction are derived using the “merged analysis”. Therefore, these $B$(GT) values are essentially normalized by using the absolute $B$(GT) values from the $\beta$-decay study, in particular those for the transitions to low-lying states.

Fig. 5. (Color online) Upper panel: the $B$(GT) values from the $^{46}$Cr $\rightarrow$ $^{46}$V $\beta$ decay (solid triangles) are compared with those obtained from the $^{46}$Ti($^3$He, $t$)$^{46}$V reaction (solid circles). The sensitivity limit in the $\beta$-decay study is shown by the dots. Lower panel: the accumulated $B$(GT) values from the $^{46}$Cr $\rightarrow$ $^{46}$V $\beta$ decay (solid triangles) are compared with those obtained from the $^{46}$Ti($^3$He, $t$)$^{46}$V reaction (solid circles).

Since the agreements of $B$(GT) values are confirmed, the $B$(GT) distribution can be now extended up to high $E_x$ region by the ($^3$He, $t$) reaction. Therefore, one can obtain an overall picture of the “absolute” $B$(GT) strengths for the “full range” of excitation energies that can be reached in a ($^3$He, $t$) CE reaction using the $\beta$-decay $B$(GT) values as standards.
3. Comparison of $T_z = \pm 2 \rightarrow \pm 1$ mirror transitions

It is more intriguing to compare $T_z = \pm 2 \rightarrow \pm 1$ mirror transitions. We have been studying these mirror GT transitions for the even–even $pf$-shell nuclei. Starting from the $0^+ \text{ g.s.}$ of the $T_z = \pm 2$ even–even nuclei, the final $J^\pi = 1^+$ excited states in $T_z = \pm 1$ mirror nuclei are reached (see Fig. 2).

3.1. Study of GT transitions in $(^3\text{He},t)$ reaction and mirror $\beta$ decay

The $(^3\text{He},t)$ spectra for the $T_z = +2$ initial target nuclei $^{44}\text{Ca}$ [15], $^{48}\text{Ti}$ [16], $^{52}\text{Cr}$ [17], and $^{56}\text{Fe}$ [18] are shown in Fig. 6. The vertical scales are again normalized by the heights of the IAS peaks carrying all of the Fermi transition strength of $B(F) = 4$. Therefore, it is expected that the peak heights of GT states in these spectra are proportional to $B(GT)$ values exciting the states.

Although, all of these GT excitations are of $T_z = +2 \rightarrow +1$ nature, the strength distributions are rather different. As discussed for the $T_z = +1 \rightarrow 0$ GT excitations [10, 14], the difference can be attributed to the different shell structure of $pf$-shell nuclei and the active residual interactions. In a simple shell-model picture, we notice that in $^{44}\text{Sc}$ [Fig. 6 (a)] situated just above the $N = Z = 20$, $LS$-closed shell, the configurations of GT excitations have, to a large extent, the nature of "proton ($\pi$)-particle–neutron ($\nu$)-particle" of $\pi f_{7/2} - \nu f_{7/2}$ and $\pi f_{5/2} - \nu f_{7/2}$, where the attractive isoscalar-type (IS-type) residual interaction is active, and thus, the GT strength is pulled down to the lower $E_x$ region. On the other hand, in $^{52}\text{Mn}$ [Fig. 6 (c)] situated just below the $N = Z = 28$, $j$-closed shell, the configurations of GT excitations have, to a large extent, the nature of "$\pi$-particle–$\nu$-hole" of $\pi f_{7/2} - \nu f_{7/2}^{-1}$ and $\pi f_{5/2} - \nu f_{7/2}^{-1}$, where the repulsive isovector-type (IV-type) residual interaction is active [10, 14]. Therefore, the GT strength is pushed up to higher $E_x$ region and a GT resonance (GTR) structure centered at 9–10 MeV is formed. A similar GTR structure is also observed in $^{56}\text{Co}$ [Fig. 6 (d)].

In the $pf$-shell region, $T_z = -2 \rightarrow -1$ GT and Fermi transitions can be studied in $\beta^+$ decays. In the final $T_z = -1$ nuclei, proton separation energies ($S_p$) are usually small. Therefore, measurements of $\beta$-delayed proton decay and $\beta$-delayed $\gamma$ decay should be combined in order to fully reconstruct the $\beta$-decay scheme. In addition, mass production of these $T_z = -2$ proton-rich exotic nuclei is now possible only in fragment separator (FRS) facilities, and produced exotic nuclei are usually implanted deep inside the stop detectors (often DSSSD detectors are used). Thus, the energy signal from a proton decay is smeared by the continuous kinetic energy of positrons. In addition, $\beta$-delayed proton decay and $\beta$-delayed $\gamma$ decay can compete. Therefore, inputs from mirror $(^3\text{He},t)$ reactions that are free from these disturbances...
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\[ E = 140 \text{ MeV/nucleon} \]

\[ \Theta = 0^\circ \]

\[ \text{Counts} \]

\( E_x \) (MeV)

\begin{align*}
(a) & \quad ^{44}\text{Ca}(^3\text{He},t)^{44}\text{Sc} \\
& \quad 0.667, 1^+ \\
& \quad 2.779, \text{IAS} \\
& \quad 3.152, 1^+ \\
& \quad 3.641, 1^+ \\
(b) & \quad ^{48}\text{Ti}(^3\text{He},t)^{48}\text{V} \\
& \quad 0.421, 1^+ \\
& \quad 2.289, 1^+ \\
& \quad 3.019, \text{IAS} \\
& \quad 3.866, 1^+ \\
& \quad 3.941, 1^+ \\
(c) & \quad ^{52}\text{Cr}(^3\text{He},t)^{52}\text{Mn} \\
& \quad 0.546, 1^+ \\
& \quad 2.651, 1^+ \\
& \quad 3.050, \text{IAS} \\
& \quad 4.376, 1^+ \\
(d) & \quad ^{56}\text{Fe}(^3\text{He},t)^{56}\text{Co} \\
& \quad 1.720, 1^+ \\
& \quad 2.790, 1^+ \\
& \quad 3.605, \text{IAS} \\
& \quad 4.355, \text{IAS} \\
\end{align*}

Fig. 6. (Color online) High energy-resolution spectra of \(^3\text{He},t\) reaction on \( A = 44–56, \) \( T_z = +2 \) even–even target nuclei. The vertical scales are normalized so that the heights of all GT peaks are approximately proportional to the \( B(\text{GT}) \) values. Owing to the high energy-resolution of \( \Delta E \approx 30 \text{ keV} \), fine structures are observed for GT strength distributions. The strength distributions are rather different in different \( A \) nuclei. For the qualitative explanation, see the text.

can provide a good reference for the better understanding of the \( \beta \) decay of \( T_z = -2 \) and of more exotic nuclei. The importance of such a combined analysis can be seen in the report describing the finding of “\( \beta \)-delayed \( \gamma \)-proton decay” in the decay of the \( T_z = -2 \) nucleus \(^{56}\text{Zn} \) [19].

Since the information from \( \beta \) decay is rather limited, we use the “\( R^2 \) systematics” [20, 21] for the derivation of \( B(\text{GT}) \) values from \( T_z = +2 \rightarrow +1 \) \(^3\text{He},t\) reactions. This systematics is based on the finding that the \( R^2 \) value defined by the ratio of GT and Fermi unit cross sections
\[ R^2 = \frac{\hat{\sigma}^{GT}}{\hat{\sigma}^F} = \frac{\sigma_j^{GT}(0)}{B_j(GT)} \frac{\sigma_j^F(0)}{B(F)}, \]

is a smooth function of mass number \( A \). The \( A \) dependence of \( R^2 \) was systematically studied and a gradual increase in \( R^2 \) was observed with increasing \( A \) (e.g. \( R^2 \) of 7.8(4) is estimated for the \( A = 44 \) system [20, 21]). Since the Fermi transition strength is concentrated in the IAS, the complete sum rule value of \( B(F) = N - Z = 4 \) can be assumed. The cross sections \( \sigma_j^{GT}(0) \) and \( \sigma_j^F(0) \) are the experimentally obtained \( j \)th GT and the Fermi excitations extrapolated to the point of momentum transfer \( q = 0 \) by DWBA calculations (see e.g. Ref. [10, 15]). Then, the \( B(GT) \) value of each state can be derived once the \( R^2 \) value for a specific \( A \) system is derived from the systematics.

We discuss here the similarities (and also the differences) of the \( T_z = \pm 2 \rightarrow \pm 1 \) mirror GT transitions taking the \( A = 44 \) system as an example.

### 3.2. Half-life estimate for the \( ^{44}{\rm Cr} \rightarrow ^{44}{\rm V} \beta \) decay

First, let us examine how well the \( \beta \)-decay half-life \( T_{1/2} \) of \( ^{44}{\rm Cr} \) can be reproduced from the distribution of the Fermi and GT transition strengths studied in the \( ^{44}{\rm Ca}(^{3}\text{He},t)^{44}{\rm Sc} \) reaction using the concept of merged analysis. Equation (5) shows that once the \( B(GT) \) values are derived for individual \( T_z = +2 \rightarrow +1 \) GT transitions in a \( (^{3}\text{He},t) \) reaction, the \( T_{1/2} \) values of the mirror \( T_z = -2 \rightarrow -1 \) \( \beta \) decay can be deduced under the assumption of isospin symmetry. Here again, we assume that \( B(F) = 4 \) in both \( T_z = \pm 2 \rightarrow \pm 1 \) mirror transitions. The \( f \)-factor for each transition can be calculated using the \( Q^\beta \) of the \( T_z = -2 \rightarrow -1 \) \( \beta \) decay and the \( E_x \) value of each state obtained in the \( T_z = +2 \rightarrow +1 \) \( (^{3}\text{He},t) \) reaction.

The \( Q^\beta \) value for the \( ^{44}{\rm Cr} \rightarrow ^{44}{\rm V} \beta \) decay is given, unfortunately, with a large uncertainty [10.970(230) MeV] in Ref. [22]. Since the \( f \)-factor decreases rapidly with the excitation energy (the \( f \)-factor is approximately proportional to \( (Q^\beta_j)^5 \) for the \( j \)th excited state) [23], the GT transitions to the higher excitation region contribute less to the \( T_{1/2} \) value. Taking the contributions of GT transitions up to 7 MeV into account, a \( T_{1/2} \) value of 40(5) ms was obtained, where \( f \)-factors were calculated following Ref. [23]. The uncertainty of 15% in the \( T_{1/2} \) value is due to large uncertainties in the \( f \)-factors, that further originate from the large uncertainty of the \( Q^\beta \) value of the \( ^{44}{\rm Cr} \rightarrow ^{44}{\rm V} \beta \) decay. We find that the derived \( T_{1/2} \) value is in agreement within errors with that of the \( \beta \)-decay value of 42.8(6) ms given in Refs. [22, 24].
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It is noted that our $T_{1/2}$ value was derived using the $B(F)$ and $B(GT)$ strength distribution in the $T_z = +2 \to +1$ transition. The agreement of the $T_{1/2}$ values suggests that the $B(GT)$ strengths derived in the $(^3\text{He}, t)$ reaction, in particular those for the transitions to the low-lying GT states that make larger contributions to the second term of Eq. (5), are in agreement with the $^{44}\text{Cr} \to ^{44}\text{V}$ $\beta$-decay values.

3.3. $T_z = \pm 2 \to \pm 1$ mirror GT and Fermi transitions in $A = 44$ system

In the $A = 44, T = 2$ system, GT and Fermi transitions from the $J^\pi = 0^+$ g.s. of the $T_z = \pm 2$ nuclei $^{44}\text{Ca}$ and $^{44}\text{Cr}$ to $1^+$ states (GT states) and the $0^+, T = 2$ IAS in the $T_z = \pm 1$ nuclei $^{44}\text{Sc}$ and $^{44}\text{V}$, respectively, are analogous. In order to examine the similarity of these mirror transitions, we compare the $^{44}\text{Ca}(^3\text{He}, t)^{44}\text{Sc}$ spectrum with the $\beta$-delayed proton spectrum studied in the $^{44}\text{Cr} \beta$ decay [24]. For this purpose, we deduce the “$^{44}\text{Cr} \beta$-decay spectrum” starting from the $(^3\text{He}, t)$ spectrum shown in Fig. 6(a) assuming the mirror symmetry of the $T_z = \pm 2 \to \pm 1$ GT transitions. The “$\beta$-decay spectrum” here is a hypothetical $\beta$-ray energy spectrum under a supposition that $\beta$ rays carry all available energy in the decay to individual states (for the detail, see Ref. [15]).

First, we take into account the suppression of the GT and Fermi transition strengths to higher excited states in the mirror $^{44}\text{Cr} \beta$ decay due to the decrease in the $f$-factor [see in Eq. (1) and (2)]. Therefore, (i) we multiply the $f$-factor, that decreases with the increasing $E_x$ of the final state, by the $^{44}\text{Ca}(^3\text{He}, t)^{44}\text{Sc}$ spectrum given in Fig. 6(a).

In addition, we notice that the coupling constants for the $\sigma \tau$ and $\tau$ operators, that cause GT and Fermi transitions, respectively, are different by the factor of $\lambda^2$ on the $\beta$-decay side. Furthermore, the difference in the unit cross sections, i.e., $R^2$ value, should be considered on the CE reaction side. Taking these differences of Fermi and GT excitation strengths into account, (ii) the Fermi transition strength should be multiplied by a factor of $R^2/\lambda^2$ ($\approx 4.9$) to the spectrum obtained in step (i).

In the study of the $T_z = -2 \to -1$ $^{44}\text{Cr} \beta$ decay, a delayed proton spectrum brings crucial information due to the low $S_p$ value of 2.08(19) MeV [22] in $^{44}\text{V}$. Except for the $\beta$ decay to the 0.677 MeV, $1^+$ state that corresponds to the 0.667 MeV state seen in Fig. 6, the major part of the $^{44}\text{Cr} \beta$ decay can be studied from the $\beta$-delayed proton spectrum. The spectrum from Ref. [24] is shown in Fig. 7(a), where the resolution was $\approx 150$ keV. Therefore, the obtained “$\beta$-decay spectrum” should be further given a width of 150 keV in order to compare with the delayed-proton spectrum.
Fig. 7. (Color online) (a) The delayed charged-particle spectrum from the $^{44}$Cr $\beta$ decay \cite{24}. The low-energy bump indicated by “low-\textit{E} BG” originates in the emission of $\beta^+$ particles, while other peaks are from the delayed-proton emission, where the pile-up of $\beta$-particle energy broadens the peaks. (b) The $\beta$-decay spectrum deduced from $^{44}$Ca($^3\text{He},t$)$^{44}$Sc (with 150 keV width).

The finally obtained spectrum is indicated by the solid line in Fig. 7 (b), where the energy scale is displaced by 1.8 MeV compared to Fig. 7 (a) to get a good agreement in the peak positions. The vertical scales are adjusted so that the 1.74 MeV peak in the delayed-proton spectrum [Fig. 7 (a)] and the 3.67 MeV peak (in reality consisting of a few GT states) in the “$\beta$-decay spectrum” [Fig. 7 (b)] have approximately the same height.

Note that the strength of the 0.91 MeV peak in Fig. 7 (a), mainly representing the strength of the Fermi transition, is largely suppressed compared to the corresponding 2.80 MeV peak shown by the solid line in Fig. 7 (b). We found that the strength ratio of the three main peaks at 0.91, 1.38, and 1.74 MeV in the delayed-proton spectrum [Fig. 7 (a)] can be reproduced only if the strength of the 2.779 MeV, IAS peak is reduced down to $\approx 30\%$ of the original strength in the “$\beta$-decay spectrum” [the short broken line in Fig. 7 (b)].
We notice that the decay of the $T = 2$ IAS in the $T_z = -1$ nucleus $^{44}$V into a proton having $T = 1/2$ and the $^{43}$Ti having the $T = 1/2$ g.s., and the low-lying states is not allowed by the isospin selection rule. Therefore, it is suggested that the observed proton-decay strength of $\approx 30\%$ of the expected full strength was caused by the $T = 1$, isospin impurity components in the “$T = 2$” IAS [25]. It is also suggested that the remaining $\approx 70\%$ of the IAS strength fed by the $^{44}$Cr $\beta$ decay should have decayed by means of $\beta$-delayed $\gamma$s. For the further discussion, however, a delayed-proton spectrum with a higher quality and also in coincidence with delayed $\gamma$s is needed.

4. Summary and prospects

We have been studying the GT transitions from the $T_z = \pm 1$ mirror $pf$-shell nuclei to the $T_z = 0$ intermediate nuclei and also the $T_z = \pm 2$ nuclei to the $\pm 1$ mirror nuclei, respectively, by means of $\beta^-$-type hadronic $(^3\text{He},t)$ CE reaction and the complementary $\beta^+$ decay. The initial $T_z = -1$ and $-2$ nuclei and also the final $T_z = 0$ and $-1$ nuclei involved in these $\beta^+$ decays are on the rp-process path, and thus, the study is of interest not only for the nuclear structure physics, but also for the nuclear astrophysics.

Under the assumption of isospin symmetry, mirror nuclei with reversed $Z$ and $N$ numbers have the same structure and thus the symmetry is expected for the GT transitions in mirror nuclei, although the Coulomb and isospin breaking forces may deteriorate the symmetry. Since GT transitions caused by the simple $\sigma \tau$ operator are sensitive to the structures of initial and final nuclei, it is expected that we can make a stringent test of the symmetry nature of the mirror structure of nuclei.

The symmetry of $T_z = \pm 1 \rightarrow 0$ GT transitions was discussed in detail for the $T_z = \pm 1$ mirror nuclei $^{46}\text{Ti}$–$^{46}\text{Cr}$ by comparing the results from $(^3\text{He},t)$ reactions and the $\beta$-delayed $\gamma$ decays. It was found that $B(\text{GT})$ values of individual pairs of transitions have some differences. However, the gross features of GT strength distributions are very similar in the energy region up to $E_x = 4$ MeV, where the $\beta$-decay measurements could access.

The study of symmetry in $T_z = +2 \rightarrow +1$ transitions and the corresponding $T_z = -2 \rightarrow -1$ transitions was more difficult. As we have seen for the GT transitions starting from a pair of $T_z = \pm 2$ nuclei $^{44}\text{Ca}$ and $^{44}\text{Cr}$, the $\beta$-decay half-life could be well-predicted from the strength distribution obtained in the $(^3\text{He},t)$ reaction. In addition, the hindrance of the proton decay from the IAS could be studied. However, the detailed discussion on the mirror symmetry was difficult; we need more precise information, such as $Q^\beta$ value and higher resolution in the particle-decay spectrum from the $\beta$-decay measurements.
In order to examine these questions further, we seek more precise $\beta$-decay data for the $T_z = -1$ and $-2$ nuclei. An experiment for the higher mass nuclei in the $pf$-shell region has been performed at RIKEN in May, 2015 [26] and detailed $\beta$-decay measurements for proton-rich $f$-shell nuclei are planned at GANIL [27].

The high-resolution ($^3$He, $t$) experiments were performed at RCNP, Osaka, and the $\beta$-decay studies of $T_z = -1$ nuclei at GSI as a part of RISING campaign. The authors are grateful to the participants in these experiments. This work was supported by MEXT, Japan under Grants Nos. 18540270, 22540310 and 15K05104; the Japan–Spain collaboration program by JSPS and CSIC; the Spanish MICINN under Grants Nos. EPA2008-06419-C02-01 and EPA2011-24553; CPAN Consolider-Ingenio 2010 Program CSD2007-00042.

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