Probing and Identifying New Physics Scenarios at International Linear Collider

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Abstract

Numerous non-standard dynamics are described by contact-like effective interactions that can manifest themselves in $e^+e^-$ collisions only through deviations of the observables (cross sections, asymmetries) from the Standard Model predictions. If such a deviation were observed, it would be important to identify the actual source among the possible non-standard interactions as many different new physics scenarios may lead to very similar experimental signatures. Here we study the possibility of uniquely identifying the indirect effects of $s$-channel sneutrino exchange, as predicted by supersymmetric theories with $R$-parity violation, against other new physics scenarios in process $e^+e^- \rightarrow \mu^+\mu^-$ at the International Linear Collider. To evaluate the identification reach on sneutrino exchange, we use as basic observable a double polarization asymmetry, $A_{\text{double}}$. Also, we examine the effects of neutrino and electron mixing with exotic heavy leptons in the process $e^+e^- \rightarrow W^+W^-$ within $E_6$ models, in particular, the possibility of uniquely distinguishing and identifying such effects of heavy neutral lepton exchange from $Z-Z'$ mixing.

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1 Introduction

Numerous new physics (NP) scenarios, candidates as solutions of Standard Model (SM) conceptual problems, are characterized by novel interactions mediated by exchanges of very heavy states with mass scales significantly greater than the electroweak scale. In many cases, theoretical considerations as well as current experimental constraints indicate that the new objects may be too heavy to be directly produced even at the highest energies of the CERN Large Hadron Collider (LHC) and at foreseen future colliders, such as the $e^+e^-$ International Linear Collider (ILC). In this situation the new, non-standard, interactions would only be revealed by indirect, virtual, effects manifesting themselves as deviations from the predictions of the SM. In the case of indirect discovery the effects may be subtle since many different NP scenarios may lead to very similar experimental signatures and they may easily be confused in certain regions of the parameter space for each class of models.

There are many very different NP scenarios that predict new particle exchanges which can lead to contact interactions (CI) which may show up below direct production thresholds. These are compositeness [1], a $Z'$ boson from models with an extended gauge sector, scalar or vector leptoquarks [2], $R$-parity violating sneutrino ($\tilde{\nu}$) exchange [3], bi-lepton boson exchanges [4], anomalous gauge boson couplings (AGC) [5], virtual Kaluza–Klein (KK) graviton exchange in the context of gravity propagating in large extra dimensions, exchange of KK gauge boson towers or string excitations [6], etc. Of course, this list is not exhaustive, because other kinds of contact interactions may be at play.

If $R$-parity is violated it is possible that the exchange of sparticles can contribute significantly to SM processes and may even produce peaks or bumps [3] in cross sections if they are kinematically accessible. Below threshold, these new spin-0 exchanges may make their manifestation known via indirect effects on observables (cross sections and asymmetries), including spectacular decays [7]. Here we will address the question of whether the effects of the exchange of scalar (spin-0) sparticles can be differentiated at linear colliders in process

$$e^+ + e^- \rightarrow \mu^+ + \mu^- \quad \text{(or } \tau^- + \tau^+),$$

(1)

from those associated with the wide class of other contact interactions
mentioned above.\footnote{For details of the analysis and original references, see \cite{8,9}} 

Another important reaction at ILC is 

$$e^+ + e^- \rightarrow W^+ + W^-,$$

which is quite sensitive to NP effects, in particular, both the leptonic vertices and the trilinear couplings to $W^+ W^-$ of the SM $Z$ and of any new heavy neutral boson or a new heavy lepton that can be exchanged in the $s$-channel or $t$-channel, respectively. A popular example in this regard, is represented by $E_6$ models. In particular, an effective $SU(2)_L \times U(1)_Y \times U(1)_Y'$ model, which originates from the breaking of the exceptional group $E_6$, leads to extra gauge bosons. Indeed, in the breaking of this group down to the SM symmetry, two additional neutral gauge bosons could appear and the lightest $Z'$ is defined as

$$Z' = Z'_x \cos \beta + Z'_\psi \sin \beta,$$

where the angle $\beta$ specifies the orientation of the $U(1)'$ generator in the $E_6$ group space. The values $\beta = 0$ and $\beta = \pi/2$ would correspond, respectively, to pure $Z'_x$ and $Z'_\psi$ bosons, while the value $\beta = -\arctan \sqrt{5/3}$ would correspond to a $Z'_{\eta}$ boson originating from the direct breaking of $E_6$ to a rank-5 group in superstring inspired models.

Another characteristic of extended models, apart from the $Z'$, is the existence of new matter, new heavy leptons and quarks. In $E_6$ models the fermion sector is enlarged, since the matter multiplets are in larger representations (the $27$ fundamental representation), that contains, in particular, a vector doublet of leptons. From the phenomenological point of view it is convenient to classify the fermions present in $E_6$ in terms of their transformation properties under $SU(2)$. We denote the particles with unconventional isospin assignments (right-handed doublets) as exotic fermions. We here consider two heavy left- and right-handed $SU(2)$ exotic lepton doublets

$$\begin{pmatrix} N \\ E^- \end{pmatrix}_L, \quad \begin{pmatrix} N \\ E^- \end{pmatrix}_R,$$

and one $Z'$ boson, with masses larger than $M_Z$ and coupling constants that may be different from those of the SM. These leptons are called vector leptons. We also assume that the new, “exotic” fermions only mix with the
standard ones within the same family (the electron and its neutrino being the ones relevant to process (2)), which assures the absence of tree-level generation-changing neutral currents [10]. In this paper, we also study the indirect effects induced by heavy lepton exchange in $W^+$ pair production (2) at the ILC, with a center of mass energy $\sqrt{s} = 0.5 - 1$ TeV and time-integrated luminosity of $\mathcal{L}_{\text{int}} = 0.5 - 1$ ab$^{-1}$.

The paper is organized as follows. In Section 2 we evaluate discovery and identification reaches on sneutrinos in process (1). In Section 3 we study the heavy neutral lepton and boson mixings in the process (2) and determine the discovery and identification reaches on the $N\nu e$ coupling constants. Concluding remarks are contained in Section 4.

2 Observables and NP parametrization in $\mu^+\mu^-$ production

For a sneutrino in an $R$-parity-violating theory, we take the basic couplings to leptons and quarks to be given by

$$\lambda_{ijk} L_i L_j \tilde{E}_k + \lambda'_{ijk} L_i Q_j \tilde{D}_k.$$  \hspace{1cm} (5)

Here, $L$ ($Q$) are the left-handed lepton (quark) doublet superfields, and $\tilde{E}$ ($\tilde{D}$) are the corresponding left-handed singlet fields. If just the $R$-parity violating $\lambda LL\tilde{E}$ terms of the superpotential are present it is clear that observables associated with leptonic process (1) will be affected by the exchange of $\tilde{\nu}$'s in the $t$- or $s$-channels [3]. For instance, in the case only one nonzero Yukawa coupling is present, $\tilde{\nu}$'s may contribute to, e.g. $e^+e^- \rightarrow \mu^+\mu^-$ via $t$-channel exchange. In particular, if $\lambda_{121}$, $\lambda_{122}$, $\lambda_{132}$, or $\lambda_{231}$ are nonzero, the $\mu^+\mu^-$ pair production proceeds via additional $t$-channel sneutrino exchange mechanism. However, if only the product of Yukawa, e.g. $\lambda_{131}\lambda_{232}$, is nonzero the $s$-channel $\tilde{\nu}_\tau$ exchange would contribute to the $\mu^+\mu^-$ pair final state. Below we denote by $\lambda$ the relevant Yukawa coupling from the superpotential (5) omitting the subscripts.

With $P^-$ and $P^+$ denoting the longitudinal polarizations of the electrons and positrons, respectively, and $\theta$ the angle between the incoming electron and the outgoing muon in the c.m. frame, the differential cross section of process (1) in the presence of contact interactions can be expressed
as \( z \equiv \cos \theta \):

\[
\frac{d\sigma^{\text{CI}}}{dz} = \frac{3}{8} [(1 + z)^2 \sigma^\text{CI}_+ + (1 - z)^2 \sigma^\text{CI}_-].
\] (6)

In terms of the helicity cross sections \( \sigma^{\text{CI}}_{\alpha\beta} \) (with \( \alpha, \beta = \text{L}, \text{R} \)), directly related to the individual CI couplings \( \Delta_{\alpha\beta} \) (see Eq. (10)):

\[
\sigma^{\text{CI}}_+ = \frac{1}{4} [(1 - P^-)(1 + P^+) \sigma^{\text{CI}}_{\text{LL}} + (1 + P^-)(1 - P^+) \sigma^{\text{CI}}_{\text{RR}}]
\] (7)

\[
\sigma^{\text{CI}}_- = \frac{1}{4} [(1 - P^-)(1 + P^+) \sigma^{\text{CI}}_{\text{LR}} + (1 + P^-)(1 - P^+) \sigma^{\text{CI}}_{\text{RL}}]
\] (8)

where the first (second) subscript refers to the chirality of the electron (muon) current. Moreover, in Eqs. (7) and (8):

\[
\sigma^{\text{CI}}_{\alpha\beta} = \sigma_{\text{pt}} |\mathcal{M}_{\alpha\beta}^{\text{CI}}|^2,
\] (9)

where \( \sigma_{\text{pt}} \equiv \sigma(e^+e^- \rightarrow \gamma^* \rightarrow \mu^+\mu^-) = (4\pi \alpha^2_{\text{em}})/(3s) \). The helicity amplitudes \( \mathcal{M}_{\alpha\beta}^{\text{CI}} \) can be written as

\[
\mathcal{M}_{\alpha\beta}^{\text{CI}} = \mathcal{M}_{\alpha\beta}^{\text{SM}} + \Delta_{\alpha\beta} = Q_e Q_\mu + g_\alpha^e g_\beta^\mu \chi_Z + \Delta_{\alpha\beta},
\] (10)

where \( \chi_Z = s/(s - M_Z^2 + i M_Z \Gamma_Z) \) represents the Z propagator, \( g_\text{L}^l = (I_{3\text{L}} - Q_l s_W^2)/s_W c_W \) and \( g_\text{R}^l = -Q_l s_W^2/s_W c_W \) are the SM left- and right-handed lepton \( (l = e, \mu) \) couplings of the Z with \( s_W^2 = 1 - c_W^2 \equiv \sin^2 \theta_W \) and \( Q_l \) the leptonic electric charge. The \( \Delta_{\alpha\beta} \) functions represent the contact interaction contributions coming from TeV-scale physics.

The structure of the differential cross section (6) is particularly interesting in that it is equally valid for a wide variety of NP models such as composite fermions, extra gauge boson \( Z' \), AGC, TeV-scale extra dimensions and ADD model. Parametrization of the \( \Delta_{\alpha\beta} \) functions in different NP models \( (\alpha, \beta = \text{L}, \text{R}) \) can be found in [8].

The doubly polarized total cross section can be obtained from Eq. (6) after integration over \( z \) within the interval \(-1 \leq z \leq 1\). In the limit of \( s, t \) small compared to the CI mass scales, the result takes the form

\[
\sigma^{\text{CI}} = \sigma^\text{CI}_+ + \sigma^\text{CI}_- = \frac{1}{4} ((1 - P^-)(1 + P^+) \sigma^{\text{CI}}_{\text{LL}} + \sigma^{\text{CI}}_{\text{LR}}) + (1 + P^-)(1 - P^+) \sigma^{\text{CI}}_{\text{RR}} + \sigma^{\text{CI}}_{\text{RL}}).
\] (11)
It is clear that the formula in the SM has the same form where one should replace the superscript CI → SM in Eq. (11).

Since the $\tilde{\nu}$ exchanged in the s-channel does not interfere with the s-channel SM $\gamma$ and $Z$ exchanges, the differential cross section with both electron and positron beams polarized can be written as [11]

$$\frac{d\sigma^{\tilde{\nu}}}{dz} = \frac{3}{8} \left[ (1 + z)^2 \sigma_-^{\text{SM}} + (1 - z)^2 \sigma_+^{\text{SM}} + 2 \frac{1 + P^- P^+}{2} (\sigma_{\text{RL}}^{\tilde{\nu}} + \sigma_{\text{LR}}^{\tilde{\nu}}) \right]. \quad (12)$$

Here, $\sigma_{\text{RL}}^{\tilde{\nu}}(\sigma_{\text{LR}}^{\tilde{\nu}}) = \sigma_{\text{pt}} |\mathcal{M}_{\text{RL}}^{\tilde{\nu}}|^2$, $\mathcal{M}_{\text{RL}}^{\tilde{\nu}} = \mathcal{M}_{\text{LR}}^{\tilde{\nu}} = \frac{1}{2} C_\chi^\tilde{\nu} C_\chi^\tilde{\nu}$, and $C_\chi^\tilde{\nu}$ and $\chi^\tilde{\nu}$ denote the product of the $R$-parity violating couplings and the propagator of the exchanged sneutrino. For the $s$-channel $\tilde{\nu}_r$ sneutrino exchange they read

$$C_\chi^\tilde{\nu} \chi^\tilde{\nu} = \frac{\lambda_{131} \lambda_{232}}{4\pi \alpha_{\text{em}}} \frac{s}{s - M_{\tilde{\nu}_r}^2 + i M_{\tilde{\nu}_r} \Gamma_{\tilde{\nu}_r}}. \quad (13)$$

Below we will use the abbreviation $\lambda^2 = \lambda_{131} \lambda_{232}.$

As seen from Eq. (12) the polarized differential cross section picks up a $z$-independent term in addition to the SM part. The corresponding total cross section can be written as

$$\sigma^{\tilde{\nu}} = \frac{1}{4} (1 - P^-)(1 + P^+) (\sigma_{\text{LL}}^{\text{SM}} + \sigma_{\text{LR}}^{\text{SM}}) + \frac{1}{4} (1 + P^-)(1 - P^+) \times$$

$$\times (\sigma_{\text{RR}}^{\text{SM}} + \sigma_{\text{RL}}^{\text{SM}}) + \frac{3}{2} \frac{1 + P^- P^+}{2} (\sigma_{\text{RL}}^{\tilde{\nu}} + \sigma_{\text{LR}}^{\tilde{\nu}}). \quad (14)$$

It is possible to uniquely identify the effect of the $s$-channel sneutrino exchange exploiting the double beam polarization asymmetry defined as [11]

$$A_{\text{double}} = \frac{\sigma(P_1, -P_2) + \sigma(-P_1, P_2) - \sigma(P_1, P_2) - \sigma(-P_1, -P_2)}{\sigma(P_1, -P_2) + \sigma(-P_1, P_2) + \sigma(P_1, P_2) + \sigma(-P_1, -P_2)}, \quad (15)$$

where $P_1 = |P^-|$, $P_2 = |P^+|$. From (11) and (15) one finds

$$A_{\text{double}}^{\text{SM}} = A_{\text{double}}^{\text{CI}} = P_1 P_2 = 0.48, \quad (16)$$

where the numerical value corresponds to electron and positron degrees of polarization: $P_1 = 0.8$, $P_2 = 0.6$. This is because these contact interactions contribute to the same amplitudes as shown in (10). Eq. (16) demonstrates
that $A_{\text{double}}^{\text{SM}}$ and $A_{\text{double}}^{\text{CI}}$ are indistinguishable for any values of the contact interaction parameters, $\Delta_{\alpha\beta}$, i.e. $\Delta A_{\text{double}} = A_{\text{double}}^{\text{CI}} - A_{\text{double}}^{\text{SM}} = 0$.

On the contrary, the $\tilde{\nu}$ exchange in the $s$-channel will force this observable to a smaller value, $\Delta A_{\text{double}} = A_{\text{double}}^{\tilde{\nu}} - A_{\text{double}}^{\text{SM}} \propto -P_1 P_2 |C_{\tilde{\nu}}^{\tilde{\nu}}|^2 < 0$. The value of $A_{\text{double}}$ below $P_1 P_2$ can provide a signature of scalar exchange in the $s$-channel. All those features in the $A_{\text{double}}$ behavior are shown in Fig. 1.

Figure 1: Double beam polarization asymmetry $A_{\text{double}}^{\tilde{\nu}}$ as a function of sneutrino mass $M_{\tilde{\nu}}$ for different choices of $\lambda$ (dashed lines) at the ILC with $\sqrt{s} = 0.5$ TeV (left panel) and $\sqrt{s} = 1.0$ TeV (right panel), $L_{\text{int}} = 0.5$ ab$^{-1}$. From left to right, $\lambda$ varies from 0.2 to 1.0 in steps of 0.2. The solid horizontal line corresponds to $A_{\text{double}}^{\text{SM}} = A_{\text{double}}^{\text{CI}}$. The yellow bands indicate the expected uncertainty in the SM case.

In the numerical analysis, cross sections are evaluated including initial- and final-state radiation by means of the program ZFITTER, together with ZEFIT, with $m_{\text{top}} = 175$ GeV and $m_{H} = 125$ GeV.

As numerical inputs, we shall assume the identification efficiencies of $\epsilon = 95\%$ for $\mu^+\mu^-$ final states, integrated luminosity of $L_{\text{int}} = 0.5$ ab$^{-1}$ with uncertainty $\delta L_{\text{int}}/L_{\text{int}} = 0.5\%$, and a fiducial experimental angular range $|\cos \theta| \leq 0.99$. Also, regarding electron and positron degrees of polarization, we shall consider the following values: $P^- = \pm 0.8$; $P^+ = \pm 0.6$, with $\delta P^-/P^- = \delta P^+/P^+ = 0.5\%$.

Discovery and identification reaches on the sneutrino mass $M_{\tilde{\nu}}$ (95\% C.L.) plotted in Fig. 2 are obtained from conventional $\chi^2$ analysis.

For comparison, current limits from low-energy data are also shown. From Fig. 2 one can see that identification of sneutrino exchange effects in
Figure 2: Discovery and identification reaches on sneutrino mass $M_\tilde{\nu}$ (95% C.L.) as a function of $\lambda$ for the process $e^+e^- \rightarrow \mu^+\mu^-$ at the ILC with $\sqrt{s} = 0.5$ TeV (left panel) and $\sqrt{s} = 1.0$ TeV (right panel), $\mathcal{L}_{\text{int}} = 0.5$ ab$^{-1}$. For comparison, current limits from low energy data are also displayed.

the $s$-channel with $A_{\text{double}}$ is feasible in the region of parameter and mass space far beyond the current limits.

### 3 Lepton and $Z - Z'$ mixing in $W^+W^-$ production

In process (2) fermionic coupling constants modified by leptonic mixing can be written as [9]:

$$g_a^e = g_a^0 c_{1a} + g_a^{E^0} s_{1a}, \quad g_a^{\mu} = g_a^0 c_{1a} + g_a^{E^0} s_{1a};$$  

(17)

$$G_L^\nu = c_{1L} c_{2L} - 2 T_{3L}^E s_{1L} s_{2L}, \quad G_R^\nu = -2 T_{3R}^E s_{1R} s_{2R};$$  

(18)

$$G_L^{Ne} = -s_{2L} c_{1L} - 2 T_{3L}^E c_{2L} s_{1L}, \quad G_R^{Ne} = -2 T_{3R}^E c_{2R} s_{1R}.$$  

(19)

Here, $(\varepsilon^0 = e^0, \ E^0)$, $g_a^0 = (T_{3a}^0 - Q_{\text{em,a}} s_W^2) g_Z$ and $T_{3a}^0$ is the third isospin component, $g_Z = 1/s_W c_W$, with $c_W = \cos \theta_W$, $s_{1a} = \sin \psi_{1a}$ and $s_{2a} = \sin \psi_{2a}$. $\psi_{1a}$ and $\psi_{2a}$ being the mixing angles in charged and neutral lepton sectors, respectively.
$Z-Z'$ mixing can be parametrized as

$$
\begin{pmatrix}
    Z_1 \\
    Z_2
\end{pmatrix} = \begin{pmatrix}
    \cos \phi & \sin \phi \\
    -\sin \phi & \cos \phi
\end{pmatrix} \begin{pmatrix}
    Z \\
    Z'
\end{pmatrix},
$$

(20)

where $Z, Z'$ are weak eigenstates, $Z_1, Z_2$ are mass eigenstates and $\phi$ is the $Z-Z'$ mixing angle. Taking Eq. (20) into account, the lepton neutral current couplings to $Z_1$ and $Z_2$ are, respectively:

$$
g^e_{1a} = g^e_a \cos \phi + g^e_a \sin \phi; \quad g^e_{2a} = -g^e_a \sin \phi + g^e_a \cos \phi.
$$

(21)

In the Born approximation the process (2) is described by the set of five diagrams shown in Fig. 3 and corresponding to mass-eigenstate exchanges (i.e. $\gamma, \nu, N, Z_1$ and $Z_2$), with couplings given by Eqs. (17)-(19) and (21).

![Feynman diagrams](image)

Figure 3: Feynman diagrams.

The polarized cross section for the process (2) can be written as

$$
\frac{d\sigma(P^-_L, P^+_L)}{d \cos \theta} = \frac{1}{4} \left[ (1 + P^-_L)(1 - P^+_L) \frac{d\sigma^{RL}}{d \cos \theta} + \\
+ (1 - P^-_L)(1 + P^+_L) \frac{d\sigma^{LR}}{d \cos \theta} + (1 + P^-_L)(1 + P^+_L) \frac{d\sigma^{RR}}{d \cos \theta} + \\
+ (1 - P^-_L)(1 - P^+_L) \frac{d\sigma^{LL}}{d \cos \theta} \right],
$$

(22)

where $P^-_L$ ($P^+_L$) are degrees of longitudinal polarization of $e^-$ ($e^+$), $\theta$ the scattering angle of the $W^-$ with respect to the $e^-$ direction. The superscript "RL" refers to a right-handed electron and a left-handed positron, and similarly for the other terms. The relevant polarized differential cross sections for $e_a^- e_b^+ \rightarrow W_{\alpha}^- W_{\beta}^+$ contained in Eq. (22) can be expressed as [12]

$$
\frac{d\sigma^{ab}_{\alpha\beta}}{d \cos \theta} = C \sum_{k=0}^{k=2} F^{ab}_k \mathcal{O}_{k\alpha\beta},
$$

(23)
where \( C = \pi \alpha_{e,m}^2 \beta_W/2s \), \( \beta_W = (1 - 4M_W^2/s)^{1/2} \) the \( W \) velocity in the CM frame, and the helicities of the initial \( e^- e^+ \) and final \( W^- W^+ \) states are labeled as \( ab = (RL, LR, LL, RR) \) and \( \alpha\beta = (LL, TT, TL) \), respectively. The \( O_k \) are functions of the kinematical variables dependent on energy \( \sqrt{s} \), the scattering angle \( \theta \) and the \( W \) mass, \( M_W \), which characterize the various possibilities for the final \( W^+ W^- \) polarizations (\( TT, LL, TL + LT \) or the sum over all \( W^+ W^- \) polarization states for unpolarized \( W \)'s).

The \( F_k \) are combinations of lepton and trilinear gauge boson couplings, \( g_{WWZ_1} \) and \( g_{WWZ_2} \), including lepton and \( Z^-Z' \) mixing as well as propagators of the intermediate states. For instance, for the \( LR \) case one finds

\[
F_0^{LR} = \frac{1}{16s_W^4} \left[(G_L^\nu)^2 + r_N \left(G_L^{Ne}\right)^2\right]^2, \\
F_1^{LR} = 2 \left[1 - g_{WWZ_1}g_{1L}^e \chi_1 - g_{WWZ_2}g_{2L}^e \chi_2\right]^2, \\
F_2^{LR} = -\frac{1}{2s_W^2} \left[(G_L^\nu)^2 + r_N \left(G_L^{Ne}\right)^2\right] \times \\
\quad \quad \left[1 - g_{WWZ_1}g_{1L}^e \chi_1 - g_{WWZ_2}g_{2L}^e \chi_2\right], \tag{24}
\]

where the \( \chi_j \) (\( j = 1, 2 \)) are the \( Z_1 \) and \( Z_2 \) propagators, i.e. \( \chi_j = s/(s - M_j^2 + iM_j\Gamma_j) \), \( r_N = t/(t - m_N^2) \), with \( t = M_W^2 - s/2 + s \cos\theta \beta_W/2 \), and \( m_N \) is the neutral heavy lepton mass. Also, in Eq. (24), \( g_{WWZ_1} = g_{WWZ} \cos\phi \) and \( g_{WWZ_2} = -g_{WWZ} \sin\phi \) where \( g_{WWZ} = \cot\beta_W. \) Note that Eq. (24) is obtained in the approximation where the imaginary parts of the \( Z_1 \) and \( Z_2 \) boson propagators are neglected, which is fully appropriate far away from the poles.

The first term \( F_0^{LR} \) describes the contributions to the cross section caused by neutrino \( \nu \) and heavy neutral lepton \( N \) exchanges in the \( t \)-channel while the second one, \( F_1^{LR}, \) is responsible for \( s \)-channel exchange of the photon \( \gamma \) and the gauge bosons \( Z_1 \) and \( Z_2 \). The interference between \( s \)- and \( t \)-channel amplitudes is contained in the term \( F_2^{LR} \). The \( RL \) case is simply obtained from Eq. (24) by exchanging \( L \to R \).

For the \( LL \) and \( RR \) cases there is only \( N \)-exchange contribution,

\[
F_0^{LL} = F_0^{RR} = \frac{1}{16s_W^4} r_N^2 \left(G_L^{Ne}G_R^{Ne}\right)^2. \tag{25}
\]

Concerning the \( O_{k\alpha\beta} \) multiplying the expression in Eq. (25) (see Eq. (23)) their explicit expressions for polarized and unpolarized final states \( W^+ W^- \) can be found in, e.g.
Let us start the analysis with a case where there is only lepton mixing and no $Z$-$Z'$ mixing, i.e., $\phi = 0$. Since the mixing angles are bounded by $s_i^2$ at most of order $10^{-2}$, we can expect that retaining only the terms of order $s_1^2$, $s_2^2$ and $s_1 s_2$ in the cross section (22) should be an adequate approximation. To do that we expand the couplings of Eqs. (17)-(19) taking $g_{\alpha}^{\epsilon_0} = (T_{3\alpha} - Q_{\alpha}^{\epsilon_0} s_{\epsilon_0}^2 s_{\epsilon_0}^r) g_Z$ into account. We find for $E_6$ models, where $T_{3L}^E = T_{3R}^E = -1/2$:

\[
\begin{align*}
G_{L}^{N_\epsilon} &= s_{1L} - s_{2L}, & G_{R}^{N_\epsilon} &= s_{1R}, \\
G_{\epsilon_0}^L &= g_{\epsilon_0}^{\epsilon_0}, & G_{\epsilon_0}^R &= g_{\epsilon_0}^{\epsilon_0} - \frac{1}{2} (G_{R}^{N_\epsilon})^2 g_Z, \\
G_{\epsilon_0}^L &= G_{R}^{N_\epsilon} - \frac{1}{2} (G_{L}^{N_\epsilon})^2, & G_{\epsilon_0}^R &= s_{1R} s_{2R}.
\end{align*}
\]

From Eqs. (24)-(26) one can see that in the adopted approximation the cross section (22) allows to constrain basically the pair of heavy lepton couplings squared, $((G_{L}^{N_\epsilon})^2, (G_{R}^{N_\epsilon})^2)$, it is not possible to constrain $s_{2R}^2$, which represents mixing in the right-handed neutral-lepton sector.

The sensitivity of the polarized differential cross section (22) to the couplings $G_{L}^{N_\epsilon}$ and $G_{R}^{N_\epsilon}$ is evaluated numerically by dividing the angular range $|\cos \theta| \leq 0.98$ into 10 equal bins, and defining a $\chi^2$ function in terms of the expected number of events $N(i)$ in each bin for a given combination of beam polarizations [13]:

\[
\chi^2 = \sum_{\{P_{L}, P_{L}^\perp\}} \sum_{i} \left[ \frac{N_{SM+NP}(i) - N_{SM}(i)}{\delta N_{SM}(i)} \right]^2,
\]

where $N(i) = \mathcal{L}_{int} \sigma_i \varepsilon_W$ with $\mathcal{L}_{int}$ the time-integrated luminosity. Furthermore,

\[
\sigma_i = \sigma(z, z_{i+1}) = \int_{z_i}^{z_{i+1}} \left( \frac{d\sigma}{dz} \right) dz,
\]

where $z = \cos \theta$ and polarization indices have been suppressed. Also, $\varepsilon_W$ is the efficiency for $W^+ W^-$ reconstruction, for which we take the channel of lepton pairs ($e\nu + \mu \nu$) plus two hadronic jets, giving $\varepsilon_W \simeq 0.3$ basically from the relevant branching ratios. The procedure outlined above is followed to evaluate both $N_{SM}(i)$ and $N_{SM+NP}(i)$.

Also, in our numerical analysis to evaluate the sensitivity of the differential distribution to model parameters we include initial-state QED
corrections to on-shell $W^\pm$ pair production in the flux function approach that assures a good approximation within the expected accuracy of the data.

Now we turn to the generic case where both lepton mixing and $Z-Z'$ mixing occur, so that the leptonic coupling constants are as in Eq. (21) and the $Z_1, Z_2$ couplings to $W^\pm$ are as in Eq. (24). In this case, in order to evaluate the influence of the $Z-Z'$ mixing on the allowed discovery region on the heavy lepton coupling plane ($G_L^{Ne}$, $G_R^{Ne}$) one should vary the mixing angle $\phi$ within its current constraints which depend on the specific $Z'$ model, namely $-0.0018 < \phi < 0.0009$ for the $\psi$ model and $-0.0016 < \phi < 0.0006$ for the $\chi$ model. Within a specific $Z'$ model and with fixed $m_N$, the $\chi^2$ function basically depends on three parameters: $\phi, G_L^{Ne}$ and $G_R^{Ne}$. In this case, $\chi^2 \leq \chi^2_{\min} + \chi^2_{CL}$ describes a tree-dimensional surface. Its projection on the $(G_L^{Ne}, G_R^{Ne})$ plane demonstrates the interplay between leptonic and $Z-Z'$ mixings. Fig. 4 shows, as a typical example, the results of this analysis for the $\chi$-model (left panel) and the $\psi$-model (right panel), respectively, with fixed $m_N = 0.3$ TeV. As one can see, the shapes of the allowed regions for the coupling constants $G_L^{Ne}$ and $G_R^{Ne}$ are quite dependent on the $Z'$ model and different for these two cases. From the explicit calculation it turns out that this is due to the different relative signs between the lepton and $Z-Z'$ mixing contributions to the deviations of the cross section $\Delta \sigma$.

Concerning Fig. 4 and the corresponding analysis for the $\chi$ and $\psi$ models, we should note that the bounds on the lepton couplings ($G_L^{Ne}$) and ($G_R^{Ne}$) are somewhat looser than in the case $\phi = 0$ discussed above (roughly, by a factor as large as two), but still numerically competitive with the current situation. Also, we can remark that the cross sections for longitudinal $W^+W^-$ production provide by themselves the most stringent constraints for this model.

By "identification" we shall here mean exclusion of a certain set of competitive models, including the SM, to a certain confidence level. For this purpose, we use the double beam polarization asymmetry, defined in (15). From Eqs. (22) and (15) one finds for the $A_{\text{double}}$ of the process (2)

$$A_{\text{double}} = P_1 P_2 \frac{(\sigma^{RL} + \sigma^{LR}) - (\sigma^{RR} + \sigma^{LL})}{(\sigma^{RL} + \sigma^{LR}) + (\sigma^{RR} + \sigma^{LL})}. \tag{29}$$

We note that this asymmetry is only available if both initial beams are polarized.
Figure 4: Discovery reach at 95% CL on the heavy neutral lepton coupling plane \(((G_L^N)^2, (G_R^N)^2)\) at \(m_N = 0.3\) TeV in the case where both lepton mixing and \(Z-Z'\) mixing are simultaneously allowed for the \(Z'_\chi\) model (left panel) and the \(Z'_\psi\) model (right panel), obtained from combined analysis of polarized differential cross sections \(d\sigma(W_L^+ W_L^-)/dz\) at different sets of polarization, \(P_L^- = \pm 0.8\), \(P_L^+ = \mp 0.6\), at the ILC with \(\sqrt{s} = 0.5\) TeV and \(\mathcal{L}_{\text{int}} = 1\) ab\(^{-1}\). The dashed curves labelled \(\phi = 0\) refer to the case of no \(Z-Z'\) mixing.

It is important to also note that the SM gives rise only to \(\sigma^{LR}\) and \(\sigma^{RL}\) such that the structure of the integrated cross section has the form

\[
\sigma_{\text{SM}} = \frac{1}{4} \left[ (1 + P_L^-) (1 - P_L^+) \sigma_{\text{SM}}^{RL} + (1 - P_L^-) (1 + P_L^+) \sigma_{\text{SM}}^{LR} \right].
\]

This is also the case for anomalous gauge couplings (AGC) [12], and \(Z'\)-boson exchange (including \(Z-Z'\) mixing and \(Z_2\) exchange) [13]. The corresponding expressions for those cross sections can be obtained from (30) by replacing the specification \(\text{SM} \rightarrow \text{AGC} \) and \(Z'\), respectively. Accordingly, the double beam polarization asymmetry has a common form for all those cases:

\[
A_{\text{double}}^{\text{SM}} = A_{\text{double}}^{\text{AGC}} = A_{\text{double}}^{Z'} = P_1 P_2 = 0.48,
\]

where the numerical value corresponds to the product of the electron and positron degrees of polarization: \(P_1 = 0.8\), \(P_2 = 0.6\). Eq. (31) demonstrates that \(A_{\text{double}}^{\text{SM}}\), \(A_{\text{double}}^{\text{AGC}}\) and \(A_{\text{double}}^{Z'}\) are indistinguishable for any values of NP parameters, AGC or \(Z'\) mass and strength of \(Z-Z'\) mixing, i.e. \(\Delta A_{\text{double}} = A_{\text{double}}^{\text{AGC}} - A_{\text{double}}^{\text{SM}} = A_{\text{double}}^{Z'} - A_{\text{double}}^{\text{SM}} = 0\).

On the contrary, the heavy neutral lepton \(N\)-exchange in the \(t\)-channel will induce non-vanishing contributions to \(\sigma^{LL}\) and \(\sigma^{RR}\), and thus force \(A_{\text{double}}\) to a smaller value, \(\Delta A_{\text{double}} = A_{\text{double}}^N - A_{\text{double}}^{\text{SM}}\) and \(\Delta A_{\text{double}} \propto \).
\[-P_L P_R \tau_N^2 \left( G_{L}^{N \tau e} G_{R}^{N \tau e} \right)^2 < 0 \] irreversibly of the simultaneous lepton and Z-Z' mixing contributions to \( \sigma^{RL} \) and \( \sigma^{LR} \). A value of \( A_{\text{double}} \) below \( P_L P_R \) can provide a signature of heavy neutral lepton \( N \)-exchange in the process (2).

The identification reach (ID) on the plane of heavy lepton coupling \( (G_{L}^{N \tau e})^2, (G_{R}^{N \tau e})^2 \) (at 95% C.L.) for various lepton masses \( m_N \) plotted in Fig. 5 is obtained from conventional \( \chi^2 \) analysis with \( A_{\text{double}} \). Note that discovery is possible in the green and yellow regions, whereas identification is only possible in the green region. The hyperbola-like limit of the identification reach is due to the appearance of a product of the squared couplings \( (G_{L}^{N \tau e})^2 \) and \( (G_{R}^{N \tau e})^2 \) in the deviation from the SM cross section, given by Eq. (25).

Figure 5: Left panel: discovery (DIS) and identification (ID) reaches at 95% CL on the heavy neutral lepton coupling plane \( (G_{L}^{N \tau e})^2, (G_{R}^{N \tau e})^2 \), obtained from a combined analysis of polarized differential cross sections \( d\sigma(W_L^+W_L^-)/dz \) at different sets of polarization, \( P_L^- = \pm 0.8, \ P_L^+ = \mp 0.6 \), and exploiting the double polarization asymmetry. Furthermore, \( m_N = 0.3 \) TeV, \( \sqrt{s} = 0.5 \) TeV and \( \mathcal{L}_{\text{int}} = 1 \) ab\(^{-1} \). Right panel: similar, with \( \sqrt{s} = 1.0 \) TeV and for \( m_N = 0.6 \) TeV. The dashed curves labelled "\( \phi = 0 \)" refer to the case of no Z-Z' mixing, whereas the outer contour labelled "DIS" refer to the minimum discovery reach in the presence of mixing.
4 Concluding remarks

In this note we have studied how uniquely identify the indirect (propagator) effects of spin-0 sneutrino predicted by supersymmetric theories with $R$-parity violation, against other new physics scenarios in high energy $e^+e^-$ annihilation into lepton-pairs at the ILC in process (1).

To evaluate the identification reach on the sneutrino exchange signature, we develop a technique based on a double polarization asymmetry formed by polarizing both beams in the initial state, that is particularly suitable to directly test for such $s$-channel sneutrino exchange effects in the data analysis.

We have also studied the process $e^+e^- \rightarrow W^+W^-$ and seen how to identify the propagator and exotic-lepton mixing effects of a heavy neutral lepton exchange in the $t$-channel. Discovery of new physics, meaning exclusion of the Standard Model, does not depend on having both initial beams polarized, but the sensitivity is improved with beam polarization. We shown that the availability of both beams being polarized, plays a crucial rôle in identifying those new physics scenarios.

References


