Probing the Higgs with Angular Observables at Future $e^+e^-$ Colliders

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I summarize our recent works on using differential observables to explore the physics potential of future $e^+e^-$ colliders in the framework of Higgs effective field theory. We study angular observables in the $e^+e^- \rightarrow ZH\ell^+\ell^-b\bar{b}$ channel at future circular $e^+e^-$ colliders such as CEPC and FCC-ee. Taking into account the impact of realistic cut acceptance and detector effects, we forecast the precision of six angular asymmetries at CEPC (FCC-ee) with center-of-mass energy $\sqrt{s} = 240$ GeV and 5 (30) ab$^{-1}$ integrated luminosity. We then determine the projected sensitivity to a range of operators relevant for the Higgs-strahlung process in the dimension-6 Higgs EFT. Our results show that angular observables provide complementary sensitivity to rate measurements when constraining various tensor structures arising from new physics. We further find that angular asymmetries provide a novel means of constraining the “blind spot” in indirect limits on supersymmetric scalar top partners. We also discuss the possibility of using ZZ-fusion at $e^+e^-$ machines at different energies to probe new operators.

1. Introduction

Following the discovery of a Standard Model-like Higgs at the LHC$^{3,4}$ the study of Higgs properties has become one of the highest priorities for current and future colliders. High-luminosity electron-positron colliders are particularly well suited to this end, promising a large sample of relatively clean Higgs production events and the ability to directly probe Higgs properties in a model-independent fashion. Such precision tests of Higgs couplings will provide a window into physics beyond the Standard Model (BSM) well above the weak scale.

Thus far much attention has focused on the potential of future $e^+e^-$ colliders to probe deviations in Higgs properties in terms of a re-scaling of Standard Model couplings$^{5-7}$ with sensitivity exceeding the percent level in some channels. However, in general deviations in Higgs properties may encode additional information, for example in the form of operators with different tensor structure in the Higgs Effective Field Theory (EFT). Disentangling contributions from these different operators provides a further handle on BSM physics by both increasing the effective reach of $e^+e^-$ colliders and distinguishing different BSM scenarios in the event of deviations from the Standard Model.

$^a$This proceeding is based upon Ref. 1 and Ref. 2.
2. Angular Observables at CEPC and FCC-ee

Given the apparent parametric separation of scales between the Higgs boson and new physics, the Higgs EFT provides a useful framework for characterizing deviations in Higgs properties from their Standard Model (SM) predictions.

Here we will work in terms of a minimal operator basis given in Ref. [8]; for a comparable choice of basis, see Ref. [9]. The relevant operators defining our operator basis are given in Table 1 of Ref. [1]. After electroweak symmetry breaking, these dimension-6 operators give rise to a variety of interaction terms relevant for $e^+e^\rightarrow ZH$ of the form

$$\mathcal{L}_{\text{eff}} \supset c^{(1)}_{ZZ} h Z_\mu Z^\mu + c^{(2)}_{ZZ} h Z_{\mu\nu}Z^{\mu\nu} + c_{h} h Z_{\mu\nu} A^{\mu\nu} + c_{A h} h Z_{\mu\nu} \tilde{A}^{\mu\nu} + h Z_{\mu} \tilde{\epsilon}^{\mu}(c_V + c_A \gamma_5) \ell + Z_{\mu} \tilde{\epsilon}^{\mu}(g_V - g_A \gamma_5) \ell - g_{\text{em}} Q_{\ell} A_{\mu} \tilde{\epsilon}^{\mu} \ell,$$

where $h$ is the real CP-even Higgs scalar, $Z_{\mu\nu}$ and $A_{\mu\nu}$ are the $Z$ boson and photon gauge field strengths, and $\tilde{V}^{\mu\nu} = \epsilon^{\mu\nu\alpha\beta} V_{\alpha\beta}$. Here we again use the notation of Ref. [8] for clarity. The couplings in this broken-phase effective Lagrangian may be straightforwardly expressed in terms of coefficients in the dimension-6 Higgs EFT and is outlined in details in Ref. [1].

2.1. Angular observables in $ZH$ production

In general, these effective operators contribute to a shift in the cross section for $e^+e^- \rightarrow ZH$, so that a linear combination of Wilson coefficients can be constrained to high precision by future $e^+e^-$ colliders. However, there is additional information available in Higgsstrahlung events that allows us to constrain independent linear combinations of Wilson coefficients. This independent information can be effectively parameterized in terms of angular observables. In this paper we will work in terms of the parameterization in Ref. [8], although other definitions of angular observables are possible and in principle may prove more efficient in isolating specific Wilson coefficients.

We define the angles $\cos \theta_1$, $\cos \theta_2$ and $\phi$ as follows: the $z$ direction is defined by the momentum of the on-shell $Z$ boson in the rest frame of the incoming $e^+e^-$ pair. The $xz$ plane is the plane defined by the momentum of the outgoing $Z$ boson and its $\ell^+$ decay product. Then $\theta_1$ is the angle between the momentum of the outgoing $\ell^+$ and the $z$ axis. $\theta_2$ is the angle between the momentum of the incoming $e^+$ and the momentum of the outgoing $h$ along the $z$ axis. Finally, the angle $\phi$ corresponds to the angle in the $xy$ plane between the planes defined by the incoming $e^+e^-$ and the outgoing $\ell^+\ell^-$, respectively. These angles are illustrated in Fig. [1].

In terms of these angles, we parameterize the triple differential cross section for $e^+e^- \rightarrow Z(\rightarrow \ell^+\ell^-)h$ as

$$
\frac{d\sigma}{d\cos \theta_1 d\cos \theta_2 d\phi} = \frac{1}{2^{10}(2\pi)^3} \frac{1}{\sqrt{s} \gamma_Z} \frac{1}{s^2} \frac{1}{m_h^2} \mathcal{J}(q^2, \theta_1, \theta_2, \phi),
$$

(2)
where \( r = m_Z^2/m_h^2 \approx 0.53, \gamma_Z = \Gamma_Z/m_h \approx 0.020, \) \( s = q^2/m_h^2, \lambda(a,b,c) = a^2 + b^2 + c^2 - 2ab - 2ac - 2bc, \) and the function \( \mathcal{J} \) contains nine independent angular structures with coefficients \( J_1, \ldots, J_9 \) decomposed as

\[
\mathcal{J}(q^2, \theta_1, \theta_2, \phi) = J_1(1 + \cos^2 \theta_1 \cos^2 \theta_2 + \cos^2 \theta_1 + \cos^2 \theta_2) + J_2 \sin^2 \theta_1 \sin^2 \theta_2 \\
+ J_3 \cos \theta_1 \cos \theta_2 + J_4 \sin \theta_1 \sin \theta_2 \sin \phi + J_5 \sin 2\theta_1 \sin 2\theta_2 \sin \phi.
\] (3)

The explicit form of the \( J_i \) in terms of the EFT coefficients and Standard Model parameters was computed by Ref. 8 and for convenience is given in Appendix of Ref. 1. The total integrated cross section for \( e^+e^- \to ZH \) is given in terms of the \( J_i \) simply by

\[
\sigma(s) = \frac{32\pi}{9} \frac{1}{2^{10}(2\pi)^3} \frac{1}{\sqrt{s}} \frac{\sqrt{\lambda(1,s,r)}}{s^2} \frac{1}{m_h^2} (4J_1 + J_2). \tag{4}
\]

It is useful to isolate various combinations of terms in the differential cross section through the following angular observables \( A_i \), normalized to \( \sigma \):

\[
A_{\theta_1} = \frac{1}{\sigma} \int_{-1}^{1} d\cos \theta_1 \sgn(\cos(2\theta_1)) \frac{d\sigma}{d\cos \theta_1} = 1 - \frac{5}{2\sqrt{2}} + \frac{3J_1}{\sqrt{2}(4J_1 + J_2)}.
\] (5)

\[
A_{\phi}^{(1)} = \frac{1}{\sigma} \int_{0}^{2\pi} d\phi \sgn(\sin \phi) \frac{d\sigma}{d\phi} = \frac{9\pi}{32} \frac{J_4}{4J_1 + J_2}.
\] (6)

\[
A_{\phi}^{(2)} = \frac{1}{\sigma} \int_{0}^{2\pi} d\phi \sgn(\sin(2\phi)) \frac{d\sigma}{d\phi} = \frac{2}{\pi} \frac{J_6}{4J_1 + J_2}.
\] (7)

\[
A_{\phi}^{(3)} = \frac{1}{\sigma} \int_{0}^{2\pi} d\phi \sgn(\cos \phi) \frac{d\sigma}{d\phi} = \frac{9\pi}{32} \frac{J_4}{4J_1 + J_2}.
\] (8)

\[
A_{\phi}^{(4)} = \frac{1}{\sigma} \int_{0}^{2\pi} d\phi \sgn(\cos(2\phi)) \frac{d\sigma}{d\phi} = \frac{2}{\pi} \frac{J_6}{4J_1 + J_2}.
\] (9)

Here \( \sgn(\pm|x|) = \pm 1. \) In addition to these five angular observables, it is also useful to define the forward-backward asymmetry
\[ A_{c\theta_1, c\theta_2} = \frac{1}{\sigma} \int_{-1}^{1} d\cos\theta_1 \text{sgn}(\cos\theta_1) \int_{-1}^{1} d\cos\theta_2 \text{sgn}(\cos\theta_2) \frac{d^2\sigma}{d\cos\theta_1 d\cos\theta_2} \]

\[ = \frac{9}{16} \frac{J_3}{4J_1 + J_2}. \tag{10} \]

Although there are nine \( J_i \), only six are independent, leading to six independent angular observables corresponding to the six independent form factors in the \( e^+ e^- \rightarrow ZH \) amplitude. Each of the angular observables is sensitive to a different linear combination of coefficients in the dimension-6 Higgs EFT.

The numerical values of the input parameters are chosen carefully and discussed in detail in Ref. 1. Working only to linear order in the Wilson coefficients, at \( \sqrt{s} = 240 \text{ GeV} \) the dependence of the total cross section on the hatted coefficients \( \tilde{\alpha}_i \) in the unbroken-phase HEFT and similarly, the dependence of the angular observables on the hatted coefficients \( \tilde{\alpha}_i \) in the unbroken-phase HEFT are both numerically expressed in Ref. 1. The inclusive cross section \( \sigma \) is unsurprisingly sensitive to all CP-even operators, with particular sensitivity to operators that shift the couplings between gauge bosons and leptons, as well as those that generate new \( hZ\ell\ell \) contact terms. The asymmetry variables \( A_{\phi} \) and \( A_{\phi}^{(4)} \) provide independent sensitivity to the operators \( O_{\Phi W}, O_{\Phi B}, O_{WB} \), which are also the operators in this basis that are constrained by measurements of \( h \rightarrow \gamma\gamma \). The forward-backward asymmetry \( A_{c\theta_1, c\theta_2} \) and the angular asymmetry \( A_{\phi}^{(3)} \) are sensitive to independent linear combinations of the CP-even operators (excepting \( O_{\Phi \Box} \), whose contribution has been eliminated by construction in the angular asymmetries). Finally, the asymmetries \( A_{\phi}^{(1)} \) and \( A_{\phi}^{(2)} \) are sensitive to independent linear combinations of CP-odd operators.

2.2. Expected precision and statistical uncertainty

Having defined the set of angular variables relevant for probing the Higgs EFT in \( e^+ e^- \rightarrow ZH \), we now develop projections for the sensitivity attainable at various proposed Higgs factories. In particular, we study the reach in angular observables at two proposed future \( e^+ e^- \) colliders: the Circular Electron-Positron Collider (CEPC) and the \( e^+ e^- \) mode of the CERN Future Circular Collider (FCC-ee). Both of these colliders are designed to produce large numbers of \( e^+ e^- \rightarrow ZH \) events at the center-of-mass energy \( \sqrt{s} = 240 \text{ GeV} \). With a proposed luminosity of \( 2 \times 10^{34} \text{ cm}^{-2}\text{s}^{-1} \) per Interaction Point (IP), the integrated luminosity at CEPC will be \( 5 \text{ ab}^{-1} \) over a running time of 10 years with 2 IPs.\(^{11}\) The machine parameters of FCC-ee\(^{14}\) project that its luminosity can reach \( 6 \times 10^{34} \text{ cm}^{-2}\text{s}^{-1} \) at \( \sqrt{s} = 240 \text{ GeV} \), which is three times that of CEPC. In addition, there is a factor of 2 increase in luminosity on account of the projected 4 IPs at FCC-ee, bringing the total FCC-ee luminosity to six times that of CEPC. Considering the same running time of 10 years, we therefore take the integrated luminosity at FCC-ee to be \( 30 \text{ ab}^{-1} \) for the purpose of our projections.
Table 1. The SM expectation at $\sqrt{s} = 240$ GeV for the asymmetry observables and the standard deviation ($\sigma_A$) for different sample sizes. We consider the process with $Z \rightarrow \mu^+\mu^-/e^+e^-$ and $H \rightarrow b\bar{b}$, which is almost entirely background-free. According to Section 3.3.3.1 in the CEPC pre-CDR the number of events after basic cuts is 22100 for 5 ab$^{-1}$. We use this number here and also scale it up with luminosity for 30 ab$^{-1}$ and the full statistics scenario detailed in text.

<table>
<thead>
<tr>
<th>observable</th>
<th>SM expectation</th>
<th>$\sigma_A$ (5 ab$^{-1}$)</th>
<th>$\sigma_A$ (30 ab$^{-1}$)</th>
<th>$\sigma_A$ (Full Stat.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mathcal{A}_\theta^{(1)}$</td>
<td>-0.448</td>
<td>0.0066</td>
<td>0.0025</td>
<td>0.00078</td>
</tr>
<tr>
<td>$\mathcal{A}_\phi^{(1)}$</td>
<td>0</td>
<td>0.0067</td>
<td>0.0027</td>
<td>0.00087</td>
</tr>
<tr>
<td>$\mathcal{A}_\phi^{(2)}$</td>
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<td>0.0067</td>
<td>0.0027</td>
<td>0.00087</td>
</tr>
<tr>
<td>$\mathcal{A}_\phi^{(3)}$</td>
<td>0.0136</td>
<td>0.0067</td>
<td>0.0027</td>
<td>0.00087</td>
</tr>
<tr>
<td>$\mathcal{A}_\phi^{(4)}$</td>
<td>0.0959</td>
<td>0.0067</td>
<td>0.0027</td>
<td>0.00086</td>
</tr>
<tr>
<td>$\mathcal{A}_{c\theta_1 c\theta_2}$</td>
<td>-0.0075</td>
<td>0.0067</td>
<td>0.0027</td>
<td>0.00087</td>
</tr>
</tbody>
</table>

In Table 1 we list the theoretical expectations for all the relevant asymmetry observables assuming only Standard Model contributions, as well as the 1σ errors ($\sigma_A$) for various integrated luminosity benchmarks. We consider only the process with $Z \rightarrow \mu^+\mu^-/e^+e^-$ and $H \rightarrow b\bar{b}$, which is almost entirely background-free. According to Section 3.3.3.1 in the pre-CDR of CEPC the number of events after basic cuts in both $\mu^+\mu^-$ and $e^+e^-$ channels is 11067 + 11033 = 22100 for 5 ab$^{-1}$. We assume for simplicity that FCC-ee will conduct a very similar study on this channel, and consequently scale the statistics up directly to 30 ab$^{-1}$ for FCC-ee.

For the purposes of forecasting, we assume the experimental results are SM-like and obtain the expected constraints on new physics using a simple $\chi^2$ fit. For the sake of concreteness, we focus on the channel $e^+e^- \rightarrow ZH \rightarrow \ell^+\ell^- b\bar{b}$ at CEPC with $\sqrt{s} = 240$ GeV and 5 ab$^{-1}$ integrated luminosity, although we also forecast sensitivity for several scenarios at FCC-ee. To compensate the omission of systematics, we judiciously apply a universal 5% penalty factor for the sensitivities of asymmetry observables to Wilson coefficients, as mentioned in Section 2. For the uncertainty in the cross section, we adopt the values in the preCDR[7] which are 0.9% for the $\mu^+\mu^- b\bar{b}$ channel and 1.1% for the $e^+e^- b\bar{b}$ channel. The combined precision for the $e^+e^- \rightarrow ZH \rightarrow \ell^+\ell^- b\bar{b}$ channel is therefore 0.7%, assuming statistical uncertainties dominate.

2.3. Constraining Wilson coefficients

In this section we present the model-independent constraints on the Wilson coefficients in the Higgs effective Lagrangian, Eq. (1), parameterized the 9 Wilson coefficients,

$$\hat{\alpha}_{ZZ}, \hat{\alpha}_{ZZ}^{(1)}, \hat{\alpha}_{V}^{V}, \hat{\alpha}_{A\ell}, \hat{\alpha}_{AZ}, \hat{\delta}_V, \hat{\delta}_A, \hat{\alpha}_{Z\tilde{Z}}, \hat{\alpha}_{A\tilde{Z}}.$$
Treating the 9 coefficients as independent parameters, there are totally 7 constraints from the rate and the six asymmetry observables, less than the number of unknowns. Therefore, one cannot obtain independent constraints on the Wilson coefficients without making further assumptions. However, with a reduced set of coefficients the angular observables can break the degeneracy of the rate measurement, which by itself could only constrain one linear combination of the Wilson coefficients. To illustrate this point, we focus on two coefficients at a time while setting the rest to zero. One of the coefficients is always chosen to be $\hat{\alpha}_{ZZ}^{(1)}$, which parameterizes a modification of the SM $HZ\mu\mu$ interaction and is most strongly constrained by the rate measurement. The angular observables, being normalized to the total cross section, are independent of $\hat{\alpha}_{ZZ}^{(1)}$ by construction. We provide in Table 2 the constraints on individual Wilson coefficients with the assumption that all other coefficients are zero. Table 2 shows the 1σ uncertainties for each Wilson coefficient (setting others to zero) from the rate measurements only, the angular observables measurements only, and the combination of the two. We use "$\infty$" to denote coefficients for which no constraint can be derived within our procedure. In particular, the angular observables are insensitive to $\hat{\alpha}_{ZZ}^{(1)}$ by construction, while the rate measurements are independent of the CP-odd operators at leading order in the Wilson coefficients.

As discussed in earlier, with the same running time FCC-ee is able to deliver a sample size 6 times larger than that of CEPC. It is also reasonable to expect that statistical uncertainties dominate for the $e^+e^- \rightarrow ZH \rightarrow \ell^+\ell^- b\bar{b}$ process at FCC-ee as they do at CEPC. Furthermore, the inclusion of additional decay modes of $H$ and $Z$ would increase the statistics and could potentially significantly increase the constraining power. While the reaches of other channels would require further study, to illustrate their potential usefulness we perform a naive scaling of statistics from the FCC-ee $e^+e^- \rightarrow ZH \rightarrow \ell^+\ell^- b\bar{b}$ process by another factor of 10, and denote this scenario as FCC-ee FS (full statistics).

### 2.4. Constraining Stops

As a final example of the discriminating power of angular observables, we consider a concrete weakly-coupled model that may be constrained with precision measurements at $e^+e^-$ colliders: scalar top partners (stops) in supersymmetric extensions.
of the Standard Model. For simplicity, we will consider stops with degenerate stop soft masses $m_{\tilde{t}}^2 = m_{\tilde{Q}}^2 = m_{\tilde{t}R}^2$ plus mixing terms of the form $X_t = A_t - \mu \cot \beta$. The mass scale of the effective operators is $\Lambda = m_{\tilde{t}}$. Wilson coefficients for this scenario were computed in [12] while the constraint on the stop parameter space due to rate measurements at $e^+e^-$ colliders was determined in [9]. Here we include the additional sensitivity contributed by angular observables by translating the results of [9,12] into our preferred basis of Wilson coefficients and applying the results of the previous section.

In Fig. 2 we show the sensitivity provided by rate measurements and the inclusion of angular observables in the plane of the two stop mass eigenvalues $M_1$ and $M_2$, which are functions of $m_{\tilde{t}}^2$ and $X_t$ given by the stop mass mixing matrix. For definiteness we have set $\tan \beta = 10$, while the results are insensitive to $\tan \beta$ as long as $\tan \beta \gtrsim$ few.

The blue contours show the constraints from the rate measurements only and the red contours show the total combined constraints from the measurements of rate and the angular observables. The solid (dotted) lines corresponds to 68%(95%) CL. The region in the upper-right part of each plot is allowed by projected coupling measurements.

The features of the exclusion provided by rate measurements were discussed extensively in Ref. [9]. The most noteworthy feature of the rate measurements is the so-called “blind spot” along the line $M_2 = M_1 + \sqrt{2}m_t$ where the shift in the $hZZ$ rate is zero. Such blind spots arise more generally in stop corrections to various Higgs properties such as $hgg$, $h\gamma\gamma$ couplings and precision electroweak observables. Each blind spot corresponds to a zero in physical linear combinations of Wilson coefficients where the contributions from two stops cancel. While the exact zeroes in observables arise in different places in the $M_1 - M_2$ plane, they are collected around the line in which the coupling of the lightest stop mass eigenstate to the Higgs goes to zero.

In general, the addition of angular observables does not lead to immense improvements over the rate measurement in generic regions of parameter space. This is not surprising, since the relevant Wilson coefficients well-constrained by angular
observables are generated at one loop and thus are small for all values of the stop masses. However, it is apparent in Fig. 2 that the addition of angular observables provides significantly improved sensitivity in the blind spot of the $hZZ$ rate measurement. This is simply because the Wilson coefficients contributing to angular observables are suppressed but nonzero along the line where the $ZH$ cross section shift is zero, and so provide complementary sensitivity at small $M_1, M_2$ provided sufficient statistics. This demonstrates the value of angular observables even in the case of BSM scenarios that are generally well-constrained by rate measurements.

3. Physics potential from measuring Higgs inclusive rate at different energies

BSM physics could give rise to modifications of the Higgs couplings. The proper framework to describe such possibilities in a model-independent manner is the effective field theory approach. With respect to the SM gauge symmetry, such effects are expressed by dimension-six Higgs operators after integrating out heavy particles or loop functions.\[13,14\] The operators modifying Higgs to $ZZ$ couplings are naturally of particular interest in our case. This is partly because it will be one of the most precisely determined quantities through a recoil-mass measurement and partly because it is one of the key couplings that could help reveal the underlying dynamics of electroweak symmetry breaking. Certain operators may have different momentum dependence and thus measurements of differential cross sections may be more sensitive to the new effects.\[c\]

To demonstrate this important feature, we consider the following two representative operators

\[ O_H = \partial^\mu (\phi^\dagger \phi) \partial_\mu (\phi^\dagger \phi), \quad O_{HB} = g' D^\mu \phi^\dagger D^\nu \phi B_{\mu\nu}, \]

with

\[ \mathcal{L}_{\text{dim-6}} \supset \frac{c_H}{2\Lambda^2} O_H + \frac{c_{HB}}{\Lambda^2} O_{HB}, \]

where $\phi$ is the SM SU(2)$_L$ doublet and $\Lambda$ is the new physics scale. The coefficients $c_H$ and $c_{HB}$ are generically of order unity. We adopt the scaled coefficients $\tilde{c}_H = \frac{\Lambda^2}{\Lambda^2} c_H$ and $\tilde{c}_{HB} = \frac{m_W^2}{\Lambda^2} c_{HB}$. This translates to generic values of $\tilde{c}_H \approx 0.06$ and $\tilde{c}_{HB} \approx 0.006$ for $\Lambda = 1$ TeV.

The operator $O_H$ modifies the Higgs-$ZZ$ coupling in a momentum-independent way at lowest order. This operator renormalizes the Higgs kinetic term and thus

\[\text{For recent reviews of these operators, see e.g., Refs. \[17\]-\[20\]. Many of these operators not only contribute to Higgs physics, but also modify electroweak precision tests simultaneously. \[21,22\]}\]

\[\text{For discussions of the effects on Higgs decays due to these operators, see Ref. \[8\].}\]

\[\text{Assuming existence of a single operator at a time, limits can be derived, see, e.g., Ref. \[9\].}\]
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Fig. 3. Left panel: total cross section (in fb) for $e^-e^+ \rightarrow e^-e^++h$ at ILC versus $\sqrt{s}$. The dashed curve is for Higgsstrahlung mode only. Right panel: constraints on coefficients of dimension-six operators $c_H$ and $c_{HB}$ with and without the inclusion of the ZZ fusion channel. The dashed and dot-dashed lines represent $2\sigma$ deviations from zero in the Zh channel at 250 and 500 GeV (blue lines), respectively. The solid (red) lines indicates the constraint from ZZ fusion for 500 GeV plus 1 TeV. The outer (black-dashed) contour shows the constraint from combined Zh measurements and the middle (yellow) and inner (green) contours show the combined $2\sigma$ and $1\sigma$ results with ZZ fusion included.

modifies the Higgs coupling to any particles universally.\textsuperscript{25,26} Equivalently, one may think of rescaling the standard model coupling constant. In contrast, the operator $O_{HB}$ generates a momentum-dependent Higgs-ZZ coupling. This leads to a larger variation of the production rate versus c.m. energy for the Zh process than the ZZ fusion because of the energy difference in the intermediate Z bosons.

Such operators receive direct constraints from the LHC from similar production processes\textsuperscript{21,22} off-shell Higgs-to-ZZ measurement\textsuperscript{25} etc., all of which lack desirable sensitivities due to the challenging hadron collider environment. Based on an analysis of current data the coefficient $c_{HB}$ is excluded for values outside the window $(-0.045,0.075)$ and $c_H$ is far less constrained\textsuperscript{21,22}

We only list above the cross sections which can be precisely measured at different ILC stages, with corresponding polarizations taken into account. The distinction between ZZ fusion($e^-e^+h$) and Zh-associated production with $Z$ decaying to electron-positron pairs is easily made by applying a minimal $m_{ee}$ cut above $m_Z$.

In Fig. 3 we plot the expected constraints on the constants $c_H$ and $c_{HB}$ from the Zh and ZZ processes measured at the ILC, assuming only these two constants among the six-dimensional terms are nonzero. We show the 95% C.L. contours for different measurements. The dashed(dot-dashed) blue line represents the contour from Zh-associated measurement at ILC 250 GeV(500 GeV). The red line represents the contour from combined ZZ fusion measurements at ILC 500 GeV and

\textsuperscript{c}The window is $(-0.053,0.044)$ for single-operator analysis. This smallness of the difference between the marginalized analysis and single-operator analysis illustrates that this operator mainly affects Higgs physics and thus other electroweak precision observables do not provide much information.
1 TeV. One can see that at a given energy for a simple production mode only a linear combination of the two operators is constrained, resulting in a flat direction in the contours. However, measurements of $Zh$ at two different energies would allow us to measure both simultaneously, as shown in the gray contour. Moreover, the addition of the $ZZ$ information at 1 TeV would offer significant improvements as shown in the yellow contour. This allows us to measure $\tau_H$ and $\tau_{HB}$ at the level of 0.04 and 0.004 respectively. Much of the improvement comes from the fact that in $ZZ$ fusion, in contrast to $Zh$-associate production, the $O_{HB}$ operator contributes with the opposite sign of the $O_H$ operator.

4. Conclusions
Future $e^+e^-$ provide unprecedented opportunities to explore the Higgs sector. The large sample size of clean Higgs events may be used to constrain not only deviations in Higgs couplings, but also non-standard tensor structures arising from BSM physics. While the former are readily probed by rate measurements, the latter may be effectively probed using appropriately-constructed angular asymmetries. In this work we have initiated the study of angular observables at future $e^+e^-$ colliders such as CEPC and FCC-ee. We have taken particular care to account for the impact of realistic cut acceptance and detector effects on angular asymmetries.

Our primary result is a forecast of the precision with which angular asymmetries may be measured at future $e^+e^-$ colliders. We have translated this forecast into projected sensitivity to a range of operators in the dimension-6 EFT, where angular measurements provide complementary sensitivity to rate measurements. Among other things, we have found that angular asymmetries provide a novel means of probing BSM corrections to the $hZ\gamma$ coupling beyond direct measurement of $e^+e^- \rightarrow h\gamma$. We also apply our results to a complete model of BSM physics, namely scalar top partners in supersymmetric extensions of the Standard Model, where angular observables help to constrain the well-known “blind spot” in rate measurements.

There are a wide range of interesting future directions. In this work we have focused on $ZH$ events with $Z \rightarrow \ell^+\ell^-$ and $h \rightarrow b\bar{b}$ in order to obtain a relatively pure sample of signal events without significant background contamination. Of course, there will be far more events involving alternate decays of the $Z$ and Higgs which, while not background-free, could add considerable discriminating power. It would be useful to conduct a realistic study of these additional channels to determine the maximum possible sensitivity of angular asymmetries. Although we have taken care to account for the impact of cut acceptance and detector effects on angular asymmetries, our work has neglected the possible impact of theory uncertainties in the Standard Model prediction for angular asymmetries. A detailed estimate of current and projected theory uncertainties in the Standard Model prediction for Higgsstrahlung differential distributions would be broadly useful to future studies. More generally, this work serves as a starting point for investigating the full set of Higgs properties accessible at future $e^+e^-$ colliders.
We also show sensitivities on the inclusive cross section $\sigma^{inc}_Z$ at multiple energies offer the possibility to distinguish contributions from different higher-dimensional operators induced by BSM physics. We demonstrate the ability to simultaneously constrain two operators whose effects are difficult to observe at the LHC, as shown in Sec. 3. Including the ZZ fusion channel provides as large as 50% relative improvement for the constraint on the chosen operators compared to the $Zh$-associated production channel alone.

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References