Exotic doubly charmed $D_{s0}^*(2171)D$ and $D_{s1}^*(2460)D^*$ molecules

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The $D_{s0}^*(2171)D$ and $D_{s1}^*(2460)D^*$ heavy meson systems can exchange a kaon that is emitted in the S-wave due to the opposite intrinsic parity of the $D_{s0}^*(2171)$ and $D(D^*)$ mesons. As a consequence of the mass difference of the $D_{s0}^*(2171)$ and $D(D^*)$ mesons, the range of the kaon exchange potential will be significantly longer than expected, corresponding to an effective mass of about 200 MeV. The potential will be very strong: the strength of the interaction is proportional to $(m_{D} - m_{D^*})^2/f_k^2$ and $(m_{D}, - m_{D^*})^2/f_k^2$. This combination of range and strength almost guarantees the existence of S-wave due to the opposite intrinsic parity of the mass difference of the $D_{s0}^*(2171)$ and $D_{s1}^*(2460)D^*$ bound states with $J^P = 0^-$ and $J^P = 0^+, 2^-$, respectively. Concrete calculations indicate a binding energy of 5–15 MeV independently of $J^P$. The $D_{s0}^*(2171)D$ and $D_{s1}^*(2460)D^*$ molecules have manifestly exotic flavor quantum numbers: $C = 2$, $S = 1$, and $I = 1/2$. We expect the existence of bottom counterparts composed of the $BB_{s0}$ and $B^*B^*_{s1}$ mesons, which will be more bound and have a richer spectrum that might include a shallow P-wave and an excited S-wave state.

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The existence of hadronic molecules was conjectured long ago [1,2] on the basis of a very simple idea: the exchange of light mesons between two hadrons generates a potential that might be able to bind them. The discovery of the $X(3872)$ by Belle [3] more than a decade ago provided a very strong candidate for a molecular state, due to its small width and to its closeness to the $D^0\bar{D}^{0*}$ threshold. Subsequently, other molecular candidates have been discovered, among them the $Z_c$’s [4,5] (which are conjectured to be $DD^*$ and $D^*\bar{D}^*$ molecules [6,7]), the $Z_b$’s [8,9] ($BB^*$ and $B^*B^*$ molecules [10,11]) and the $P_c$ (4450) pentaquarklike state [12] (a $\Sigma_c\bar{D}^*$ [13] or a $\Sigma_c\bar{D}^*$ molecule [14–17], in the latter case probably with a sizable $\Lambda_c(2590)\bar{D}$ component [18,19]).

Though it is easy to conjecture the existence of hadronic molecules from theoretical principles, making concrete predictions is considerably harder. The reason is that in most cases hadronic molecules are generated as a consequence of unknown short-range physics. This is manifest from the necessity of cutoffs/form factors. If we consider the one pion exchange (OPE) potential, which is expected to be the longest range piece of the interaction between two hadrons (provided they contain at least one light quark), we will quickly realize that it requires regularization: the OPE potential contains a tensor piece that is singular at short distances. The tensor force, if attractive, will be able to hold an infinite number of bound states. This situation is circumvented by the introduction of a form factor, cutoff or other regulator that renders predictions possible at the price of the introduction of an unknown new parameter [20–25]. Educated guesses are possible by making a judicious choice of the cutoff, the work by Törnqvist on heavy meson-antimeson bound states being an astonishing example [20], but there always remains a large degree of arbitrariness.

Yet the tensor force is not present in every hadron molecule. The richness of the hadron spectrum gives rise to other possibilities even if we only consider the exchange of a pseudo–Nambu–Goldstone boson. If a pion or a kaon is exchanged in a vertex involving hadrons with different parities a series of interesting situations can arise. If in addition a vertex involves hadrons with different masses, this can lead to interactions with an unusual long range for strong interactions. A recent example is a Coulomb-like force in the $\Lambda_c(2590)\Sigma_c$ and $\Lambda_c(2590)\bar{\Sigma}_c$.
In this case we have a $1/r$ potential that is regular at short distances: we can make predictions that do not crucially depend on a cutoff, though we still expect the unknown details of the short-range physics to have a certain impact.

Here we consider the $DD^*_1(2317)$ and $D^*D^*_1(2460)$ systems, which share interesting similarities with the $\Lambda_c(2590)\Sigma_c$ molecule theorized in Ref. [26]. The $D$ and $D^*$ mesons have negative parity, while for the $D^*_1(2460)$ mesons ($D^*_0$ and $D^*_1$ from now on) the parity is positive. That is, they can exchange a kaon in $S$-wave. In addition the mass difference $m(D^*_0) - m(D)$ and $m(D^*_1) - m(D^*)$ is similar to the kaon mass, which means that the exchanged kaon will be near the mass shell and hence the range of the interaction will be unusually large. If this were not enough, chiral symmetry implies that the strength of the $D^*_0DK$ and $D^*_1D^*K$ vertices are proportional to the mass difference, which translates into an exceptional strength for the resulting Yukawa potential. This mechanism is also present in the $\Lambda(1405)/N$ and $\Xi(1690)/\Sigma$ via antikaon exchange and in the $\Lambda(1520)/\Sigma$ via pion exchange.

This type of kaon exchange leads to a different spectrum than the one obtained from standard OPE [20]. The strength of the former is independent of spin and isospin, while the later is proportional to $S_1 \cdot T_1 \cdot T_2$, with $S_1 \cdot T_1$ and $T_1 \cdot T_2$ the spin and isospin of hadrons 1 and 2 and the $+/-$ sign for hadron-hadron/hadron-antihadron [14]. For standard pion exchange flavor exotic states are suppressed as they require a symmetric/antisymmetric wave function. This limits the choices of total spin and isospin for which attraction is strong. In turn $CP$ exotic states are suppressed as they usually require $P$-wave, for which binding is less likely. The type of kaon exchange discussed here is independent of spin (the kaon is emitted in $S$-wave) and of isospin (the $D^*_0$ and $D^*_1$ are isoscalars). We nonetheless stress that the production of flavor exotic states is experimentally difficult and has only been achieved recently [27].

The $DD^*_0(2317)$ and $D^*D^*_1(2460)$ molecules are also interesting for another reason: their quark content is $cc\bar{s}\bar{q}$ with $\bar{q} = \bar{u}, \bar{d}$. This configuration is unlikely to form compact tetraquarks but narrow molecules instead, as argued by Manohar and Wise [28]. Lattice QCD [29,30] and quark model calculations [31–37] seem to indicate that compact $QQ\bar{q}\bar{q}$ structures exist in the bottom sector, but not in the charm one (maybe with the exception of an isoscalar $cc\bar{u}\bar{d}$ state with $J^P = 1^+$). As a consequence the potential discovery of a structure with $cc\bar{s}\bar{q}$ quark-content and negative parity will unmistakably point to a molecule.

Now we calculate the one kaon exchange (OKE) potential in the $DD^*_0$ and $D^*D^*_1$ molecules. We begin with the $D, D^*$ $S$-wave heavy mesons, which can be written as the heavy quark symmetric superfield:

$$H_a = \frac{1 + \gamma_5}{2} [D^{\mu}\gamma_\mu - D_a \gamma_5] , \quad (1)$$

where $a$ is an SU(3)-flavor index such that

$$D_a = \begin{pmatrix} D^0 \\ D^+ \\ D_s \end{pmatrix} , \quad D_a^* = \begin{pmatrix} D_0^* \\ D^{*+} \\ D_s^* \end{pmatrix} . \quad (2)$$

If we consider now the $D_0$ and $D_1$ P-wave heavy mesons (to which the $D^*_0, D^*_1$ belong), they can be arranged in the superfield

$$S^a = \frac{1 + \gamma_5}{2} [D^{\mu}_{1a}\gamma_\mu - D^{\mu}_0] , \quad (3)$$

with the SU(3)-flavor structure

$$D^{\mu}_0 = \begin{pmatrix} D^0_0 \\ D^0_1 \\ D^{*0}_1 \end{pmatrix} , \quad D^{\mu}_1 = \begin{pmatrix} D^0_0 \\ D^1_1 \\ D^{*0}_1 \end{pmatrix} . \quad (4)$$

While the $D^*_0$ and $D^*_1$ are narrow and thus good candidates for being part of a molecule, the $D^0_0, D^0_1, D^{*0}_1$ and $D^{*0}_1$ are broad ($\Gamma \approx 200–300$ MeV) and as a consequence unlikely to form bound state, except with kaons [38]. Besides the $D^*_0$ and $D^*_1$ are expected to contain a non-negligible $DK$ and $D^*K$ molecular component [38–40] (about 50%–70% according to Refs. [41,42]) plus a $D_s\eta$ and $D_s'\eta$ component [43]. The binding momentum is about 200 MeV for the $DK$ and $D^*K$ and about 400 MeV for the $D_s\eta$ and $D_s'\eta$. If the binding momentum of a $DD^*_0/D^*D^*_1$ molecule is smaller than these figures, it will be safe to ignore the possible compound structure of the $D^*_0$ and $D^*_1$ mesons.

The heavy meson chiral lagrangian for the interaction between the $S$- and $P$-wave heavy mesons is [44]

$$\mathcal{L} = \frac{\hbar}{2} \text{Tr} [\bar{H}_a S_b A_{ab} \gamma_5] + \text{H.c.} , \quad (5)$$

with $a, b$ SU(3)-indices, $A_{ab}$ the axial current of the pseudo-Nambu-Goldstone field and where H.c. indicates the Hermitian conjugate. We have $A^\mu = -\frac{1}{f_\pi} \partial_\mu M$ with $f_\pi \approx 130$ MeV, where

$$M = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & -\frac{K^0}{\sqrt{2}} & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}} \eta \end{pmatrix} . \quad (6)$$

We determine the coupling $\hbar$ from two different assumptions of the structure of $D^*_0$ and $D^*_1$ states. In the first scenario, assuming that the $D^*_0$ and $D^*_1$ are $c\bar{s}$ states, we can infer $\hbar$ from the decays of $D_0$ and $D_1$ mesons.
\[\Gamma(D_0 \to D\pi) = \Gamma(D_0 \to D^0\pi) + \Gamma(D_0 \to D^{\mp}\pi)\]
\[= \frac{3}{2} \Gamma(D_0 \to D^{\pm}\pi)\]
\[= \frac{3}{2} m_D \frac{q_x}{m_{D_0}} \frac{h^2}{f_\pi} (m_{D_0} - m_D)^2. \quad (7)\]

plus the analogous formula for the \(D_1 \to D^*\pi\) decay, where
\[q_x = \sqrt{(m_{D_0} - m_D)^2 - m_{\pi}^2}\]
is the momentum of the pion.

If the widths of the \(D_0\) and \(D_1\) heavy mesons are saturated by the pion decays above, we obtain \(h \approx 0.5-0.9\) where the large spread comes from the uncertainties in the masses and widths of the \(D_0\) and \(D_1\) mesons and also because it depends on whether we use the \(D_0^0\) or the \(D_1^+\) decay width (notice that the \(D_1^{+}\) has not been detected yet). For instance Ref. [45] obtains the values \(h = 0.61 \pm 0.07, 0.50 \pm 0.06\) and \(0.8 \pm 0.2\) for the three previous cases.

Determinations of this coupling from QCD sum rules [46,47] and lattice QCD [48] lie in the previous range. That is, if the \(D_{s0}^*\) and \(D_{s1}^*\) are compact \(\pi\delta\) states the uncertainty in \(h\) is likely to be large, for instance \(h = 0.7 \pm 0.2\).

In the second scenario we deduce \(h\) from the molecular hypothesis, where the \(D_{s0}^* \to DK (D_{s1}^* \to D^* K)\) coupling \(g\) is extracted from the residues of the scattering amplitude at the pole [41,49]. We have the relation
\[g = \sqrt{2m_{D_{s0}} m_{D_{s1}} \omega_K \frac{h}{f_\pi}}, \quad (8)\]
from which a typical \(g \sim 10-12\) GeV [41,49] translates into \(h \sim 0.7-0.8\), where the higher value comes from analyzing lattice QCD data [50]. In this scenario we can use \(h = 0.7 \pm 0.1\) for \(f_\pi = 130\) MeV. Choosing \(f_K = 160\) MeV instead of \(f_\pi\) only amounts to the change \(h = 0.9 \pm 0.1\). This makes no difference because the potential is proportional to \(g^2\).

The leading order (LO) potential for the \(DD_{s0}^*\) and \(D^*D_{s1}^*\) system is generated from kaon exchange and is not diagonal. If we consider the bases \(\{DD_{s0}^*, D_{s0}^*D\}\) and \(\{D^*D_{s1}^*, D_{s1}^*D^*\}\), the momentum space potential reads
\[V_{OKE}(\vec{q}) = -\hbar \omega_K^2 \frac{a_K \hbar}{f_\pi} \frac{1}{m_K^2 - \omega_K^2 + \vec{q}^2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad (9)\]
with \(\omega_K = m_{D_{s0}} - m_{D}\) or \(m_{D_{s1}} - m_{D^*}\) depending on the case. We have used \(f_\pi\) instead of \(f_K\) as they are only different at next-leading order in the chiral expansion. The interesting point is that the range of the potential is set by the effective kaon mass \(\mu_K\)
\[\mu_K^2 = m_K^2 - \omega_K^2, \quad (10)\]
which is about 200 MeV, moderately long-ranged. This enhanced range also happens in \(\Lambda(1405)/N\) [51]. For \(S = 0, 2\) we have the linear combinations \(|DD_{s0}^* + D_{s0}^*D|/\sqrt{2}\) and \(|D^*D_{s1}^* + D_{s1}^*D^*|/\sqrt{2}\) for which the potential is attractive\(^1\) and reads as
\[V_{OKE}(r) = -\hbar \omega_K^2 \frac{a_K \hbar}{f_\pi} \frac{1}{4\pi r^2} e^{-\mu_K r}, \quad (11)\]
in configuration space, which has bound states if
\[\lambda_B = \frac{2\mu_K}{\mu_K^2} \frac{\hbar^2}{4\pi f_\pi^2} \geq 1.68, \quad (12)\]
with \(\mu_K\) the reduced mass of the system. This condition is probably satisfied: the evaluation of the expression above yields \(9.16\hbar^2\) and \(10.70\hbar^2\) for the \(DD_{s0}^*\) and \(DD_{s1}^*\) cases, respectively, and a bound state exists for \(|h| > 0.43\) and 0.40. For \(\lambda_B \geq 6.45\) there will be two bound states, a condition that requires \(|h| > 0.84\) and 0.78, which makes the existence of the second state less probable but still possible.

Concrete calculations of the binding will be divided in two scenarios: a compact and a molecular \(D_{s0}^*/D_{s1}^*\). In the first case, the predictions will be subjected to large errors due to the poor knowledge of the coupling \(h\). In the second the coupling \(h\) is well determined, but the finite size of the \(DK\) and \(D^*K\) molecule has to be considered. The OKE potential is regular but its short-range behavior is not necessarily physical. We regularize it in order to obtain more realistic results. For that, we apply a nonlocal gaussian regulator to the momentum space OKE potential with a cutoff of the order of the hard scale (\(\Lambda = 0.5-1.0\) GeV), which we plug into the Lippmann-Schwinger equation [52,53]. This choice is not the easiest one—-it generates a nonlocal potential—but it is more convenient for a prospective three body \(DDK\) calculation. Choices such as a monopolar or a gaussian form factor depending on \(\vec{q}\) will lead to a local potential that can be used in the Schrödinger equation.

In the first scenario—\(D_{s0}^*/D_{s1}^*\) as a compact meson—a \(DD_{s0}^* (D^*D_{s1}^*)\) bound state is very likely but the uncertainties are large. For \(h = 0.7\) the binding energy is \(E_B = -(4-13)\) MeV, with the spread reflecting the cutoff range. This figure decreases to \(E_B = -(1-5)\) MeV if we choose \(f_K\) instead of \(f_\pi\) in the OKE potential. The system binds for most choices of the parameters except for \(h = 0.5\) with \(f_K\) (though there is still a virtual state at \(E_V = -0.7\) MeV). The resilience against short-range physics can be illustrated by changing the regulator to

\(^1\)This is a consequence of extended Bose-Einstein statistics. The potential exchanges the \(D\) by the \(D_{s0}^*\) and vice versa, which means that it is convenient to consider the \(D\) and \(D_{s0}^*\) as identical particles. Alternatively, we can notice that the potential is defined in the \(DD_{s0}^* \to DD_{s0}^*\) channel, which leads to an overall \((-1)^S\) factor (see for instance Ref. [51]).
\[ V(r; R_c) = V(r)\theta(r - R_c). \]  

where \( R_c \) is a cutoff radius. With this regulator OKE binds for \( R_c \leq 0.8-1.3 \text{ fm} (0.9-1.4 \text{ fm}) \) depending on whether we use \( f_\pi \) or \( f_K \). This is larger than the typical range of short distance physics, which as we will see below also happen to be suppressed. If we consider the exchange of other light mesons, we notice that SU(3) flavor symmetry and the OZI rule imply that the coupling of the \( D_{s0}^*/D_{s1}^* \) to the sigma and omega mesons vanishes. The only nonsuppressed light meson exchange is that of the \( K^* \), which generates a spin-spin interaction that vanishes for \( D_{s0}D \) while it is repulsive (attractive) for \( S = 0 \) \( (S = 2) \) \( D_{s1}^*D^* \). That is, OKE dominates the low energy physics of this system.

The importance of OKE can also be understood by reinterpreting the previous predictions as the leading order (LO) calculation in an effective field theory (EFT) with the heavy mesons and the pseudo–Nambu-Golstone bosons as the low energy degrees of freedom. Within this framework, the longest range correction to the OKE potential comes from two pion exchange (TPE),\(^2\) in particular the football diagram.\(^3\) For a sharp cutoff \( \Lambda \) the cutoff is chosen to be \( \Lambda \sim m_\rho \) (analogous to those in the \( D^0\bar{D}^0 \) and \( D^{*0}\bar{D}^{*0} \) decays of the \( X(3872) [62-64] \)). While the width of the \( D^* \) is of the order of 100 keV [65,66], the widths of the \( D, D_{s0} \) and \( D_{s1}^* \) are not that well known experimentally (except for upper bounds). Theory suggests that they are narrow: \( D \) only decays weakly while the \( D_{s0} \) and \( D_{s1}^* \) decays require isospin violation, where estimates of the width of the former range from a few keV [39,40,67] to about a pair of hundred of keV at most [68]. Besides the binding energy of the \( D^*\bar{D}_{s1}^* \) molecule precludes the possibility of the \( D^* \to D\pi \) decay, as chances are that this molecule is below the \( D_{s1}^*\pi \) threshold. From this we can conclude that the width of these two molecular states is really narrow, well below 1 MeV, in agreement with the original expectations about \( QQ\bar{q} \bar{q} \) states [28].

Yet the binding momenta of the \( D_{s0}^*D/D_{s1}^*D \) molecules is about 100–200 MeV, comparable to that of a \( DK/D^*K \) molecule. This points to corrections from the underlying \( DDK \) structure. The interactions in the \( DDK \) system are of a short-range nature, which makes a Faddeev calculation simple in this system (details will be provided in a future publication). If we fix the \( DK \) contact-range interaction to reproduce the \( D_{s0}^* \) pole and assume that there is no \( DD \) interaction, for \( \Lambda = 0.5-1.0 \text{ GeV} \) we find a bound state at \( E_B = -(44-57) \text{ MeV} \) below the \( D_{s0}^*K \) threshold. From rho and omega exchange we expect the \( DD \) potential to be repulsive at short distances. This is taken into account by saturating the \( DD \) contact-range coupling by the exchange of these two mesons as in [58]. In this case we find \( E_B = -(15-28) \text{ MeV} \). Finally the inclusion of relativistic kaon kinematics (in the formalism of Refs. [59–61]) and the correct energy dependence of the Weinberg-Tomozawa term for the \( DK \) interaction give modest corrections to the numbers above, about \( \Delta E_B = +5 \text{ MeV} \) for a non-interacting \( DD \) pair and \( \Delta E_B = +1 \text{ MeV} \) if we include \( DD \) repulsion.

Other interesting aspect of the \( D_{s0}^* \) and \( D_{s1}^* \) molecules is their decays, which are given by the decays of their components plus interference and binding effects (analogous to those in the \( D^0\bar{D}^0 \) and \( D^{*0}\bar{D}^{*0} \) decays of the \( X(3872) [62-64] \)). While the width of the \( D^* \) is of the order of 100 keV [65,66], the widths of the \( D, D_{s0} \) and \( D_{s1}^* \) are not that well known experimentally (except for upper bounds). Theory suggests that they are narrow: \( D \) only decays weakly while the \( D_{s0} \) and \( D_{s1}^* \) decays require isospin violation, where estimates of the width of the former range from a few keV [39,40,67] to about a pair of hundred of keV at most [68]. Besides the binding energy of the \( D^*\bar{D}_{s1}^* \) molecule precludes the possibility of the \( D^* \to D\pi \) decay, as chances are that this molecule is below the \( D_{s1}^*\pi \) threshold. From this we can conclude that the width of these two molecular states is really narrow, well below 1 MeV, in agreement with the original expectations about \( QQ\bar{q} \bar{q} \) states [28].

Probably the most effective way to produce the \( DD_{s0} \) and \( D_{s1}^* \) molecules in experiments involves heavy ion collisions, the reason being their double charm content. The production yields for the theoretical \( T_{cc} \) tetraquarks (\( cc\bar{q}\bar{q} \)) and other exotic hadrons have been estimated for electron-positron [69] and heavy ion collisions [70], where the predicted yields may be reachable by the LHCb in the future (double charm baryon production has been recently achieved [27]). Yet we note that the production of double charm molecules is probably different from the estimates above, which refer to the more compact \( T_{cc} \) tetraquarks.

The previous ideas also apply to the bottom sector, where the \( B_{s0}(5730) \) and \( B_{s1}(5776) \) bottom-strange mesons have been theorized to have a significant molecular component and a similar binding energy as the \( D_{s0}^* \) and \( D_{s1}^* \) mesons [39,40,43] (they also appear in lattice QCD...
calculations [71]). They are theoretical however and have not been experimentally discovered yet. If we consider the $B_{s0}$ and $B^*_{s1}$ molecules, the OKE potential is identical to the one for charm mesons but the spectrum will be more bound due to the heavier reduced mass. For $\Lambda = 0.5$ GeV/1 GeV we find a $B_{s0}$ bound state at $E_B = -14.7^{+6}_{-9}$ MeV/$-40^{+20}_{-20}$ MeV. The $B^*_{s1}$ predictions are almost identical because the reduced mass is nearly the same. Due to their wave number, a sizeable three-body component is possible. Because the reduced mass is nearly the same, due to their mass, a number of sizeable three-body component is possible. For $h = 0.8$ an excited shallow S-wave state is clearly visible. For the P-wave, there is a bound state with $E_B = -4.7^{+4}_{-7}$ MeV/$-14^{+7}_{-10}$ MeV. The previous uncertainties only take into account the error in the coupling $h$. The biggest uncertainty will come from the actual location of the $B_{s0}$ and $B^*_{s1}$ states: the closer they are to the $B K$ and $B^* K$ thresholds, the longer the range of the OKE potential and the more probable additional bound states will be.

To summarize, the $DD^*_{s0}$ and $D^* D^*_{s1}$ systems interact via a long-ranged kaon exchange Yukawa potential. This potential is a consequence of the different parities and masses of the $D(D^*)$ and $D^*_{s0}(D^*_{s1})$ heavy mesons. It also provides an excellent opportunity to predict the existence of bound states: due to the nonsingular character of the Yukawa potential, predictions do not crucially depend on arbitrary short-range physics, though there is still a moderate dependence on the cutoff. We find that there must be bound states with a binding energy of 5–15 MeV where the exact number should be fairly independent on whether we have a $0^- DD^*_{s0}$ or a $0^-/2^- D^* D^*_{s1}$ molecule. These predictions are robust against short-range dynamics, partly because the latter are suppressed phenomenologically. If the bound states become too deep their description probably requires the inclusion of a three-body component ($DDK$ and $D^*D^*K$, respectively). This does not affect the prediction of bound states, only their location. We expect the existence of similar bound states in the bottom sector, i.e., $B_{s0}$ and $B^*_{s1}$. They will be more bound and might have a richer spectrum than their charm counterparts (there is probably a P-wave state and an excited S-wave one), but we remind that the $B_{s0}$ and $B^*_{s1}$ heavy mesons have not been observed yet in experiments. The mechanism behind these molecules and the dual three body description probably extends to other hadron systems, for instance $\Lambda(1405)N$, $\Sigma(1690)\Sigma$ and $\Lambda(1520)\Sigma^*$ to name a few prominent examples.

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