I. INTRODUCTION

The standard SU(3)$_c \times$ SU(2)$_L \times$ U(1) model of strong and electroweak interactions contains 18 (or more$^1$) independent parameters. Three are the gauge couplings $g_3$, $g_2$, and $g_1$, associated with SU(3)$_c$, SU(2)$_L$ and U(1) symmetries, respectively, while the other 15 are elementary particle masses and quark mixing angles. By embedding the standard model in a grand unified theory (GUT)$^2$ such as SU(5), SO(10), E(6), O(18), etc. (i.e., a compact simple gauge group), the bare couplings are naturally rendered equal, $g_{30} = g_{20} = g_{10}$. This leads to calculable relationships$^3$ among the renormalized couplings as well as testable predictions. The high degree of symmetry in GUTs also promotes $\theta_W^0$, the bare weak mixing angle, from an infinite counterterm to a finite number, usually $\sin^2 \theta_W^0 = 3/8$, and relates electric charge and color charge quantization. Unfortunately, GUTs add little to our understanding of the other 15 parameters. In fact, rather than explaining their origin in a more fundamental manner, GUTs significantly increase the number of arbitrary mass and mixing parameters that must be accommodated by the so-called Higgs mechanism. Therefore, even though GUTs represent a significant theoretical advancement, they certainly are not the final word.

Two of the most spectacular predictions of GUTs are that the proton decays$^2-^3$ (baryon number is not conserved) and superheavy...
magnetic monopoles should exist.\textsuperscript{4} Even more incredible, Rubakov and Callan have convincingly argued that the GUT magnetic monopoles would catalyze proton decay,\textsuperscript{5} $p + M \rightarrow e^+ + M + X$, with strong cross sections $\approx 10^{-26} \text{cm}^2$. So, if monopoles were detected, baryon number violation should tag along as an added bonus.

Of course, the idea that the proton may be unstable predates GUTs. In the period 1929–1949 Weyl, Stueckelberg and Wigner formulated what we now refer to as the law of baryon number conservation\textsuperscript{6} and in 1954 Reines, Cowan and Goldhaber\textsuperscript{7} carried out the first experimental search for baryon number violation. Then in 1967, Sakharov\textsuperscript{8} noted that CP and baryon number violations might together explain the observed baryon asymmetry of the universe (excess of baryons over antibaryons). A new incentive to search for proton decay and a theoretical framework for estimating decay rates and branching ratios was later provided by the unified models of Pati and Salam\textsuperscript{9} and the SU(5) GUT of Georgi and Glashow.\textsuperscript{2} In recent years, GUTs have become the natural interface between elementary particle physics and early universe cosmology.

Interest in magnetic monopoles dates back to 1931 when Dirac\textsuperscript{10} postulated their existence in order to explain electric charge quantization through his famous relationship

$$Q_eQ_m = n/2, \quad n = \text{integer}. \quad (1.1)$$

Unfortunately, little could be said about the properties of Dirac’s monopoles other than that they should carry a large magnetic charge $\approx 137 ne/2$ ($e =$ electron’s charge). Because they were pointlike, they had infinite self-energy (just like a classical point electric charge); hence their mass was not predicted. In contrast, GUT magnetic monopoles exhibit structure at very short distances $\approx 10^{-30}$ cm. That core structure cuts off the $1/r$ singularity in their potential and thereby leads to a finite, albeit superheavy mass $\approx 10^{16}$ GeV. It also provides a region of strong baryon number violation that can induce proton decay catalysis. So it seems that Dirac’s magnetic monopole conjecture finds a natural setting in GUTs, and within that framework it exhibits more fantastic properties than one could have possibly imagined in 1931.

Our plan in this Comment is to survey the predictions of GUTs
and compare them with experiment. In Section II we outline the basic features of the SU(5) model and scrutinize its predictions for $\sin^2 \theta_W$ and $m_X$, the unification mass. New particle “appendages” to the minimal SU(5) model and their effect on $m_X$ and $\sin^2 \theta_W(m_W)$ are also described. General aspects of proton decay are the subject of Section III. There we compare the decay rate predicted by minimal SU(5) with existing experimental bounds. That model, as well as any GUT which employs a “great desert hypothesis,” now appears to be ruled out by experiment; however, there are various sources of uncertainty in the theory on which we comment. Higgs scalar mediated proton decay and effects due to supersymmetry are discussed. Instanton and quantum gravity induced proton decay are also briefly touched upon. In Section IV we review the expected properties of GUT magnetic monopoles and examine the consequences of strong proton decay catalysis. Finally, in Section V we present some conclusions and an outlook for the future.

II. GUT PREDICTIONS—$\sin^2 \theta_W$ and $m_X$

The SU(5) model proposed by Georgi and Glashow\(^2\) is the most economical of any GUT and the most definite in its predictions. It incorporates the entire SU(3)$_c \times$ SU(2)$_L \times$ U(1) standard model along with its three sequential fermion generations without requiring any additional fermion fields. Each generation forms a $5^* + 10$ representation of SU(5), e.g., the first generation assignment is

\[
\begin{pmatrix}
\bar{d}_1 \\
\bar{d}_2 \\
\bar{d}_3 \\
v_e \\
e^-
\end{pmatrix}
= \frac{1}{\sqrt{2}}
\begin{pmatrix}
0 & -\bar{u}_3 & -\bar{u}_2 & -u_1 & -d_1 \\
-\bar{u}_3 & 0 & \bar{u}_1 & -u_2 & -d_2 \\
\bar{u}_2 & -\bar{u}_1 & 0 & -u_3 & -d_3 \\
u_1 & u_2 & u_3 & 0 & -e^+ \\
d_1 & d_2 & d_3 & e^+ & 0
\end{pmatrix}
\]

(2.1)

where the bar denotes antiparticle. Similar assignments hold for the $\mu$, $\nu_\mu$, $s$, $c$ and $\tau$, $\nu_\tau$, $b$, $t$ generations.
There are 24 gauge bosons in the SU(5) model. Twelve are the usual eight gluons, $W^\pm$, $Z$ and $\gamma$ of the standard model, while the additional 12 superheavy bosons are more exotic in that they carry color and have fractional electric charges $\pm 4/3$ and $\pm 1/3$. These latter $X^{\pm 4/3}$ and $Y^{\pm 1/3}$ bosons belong to SU(3)$_c$ triplets and together form an SU(2)$_L$ isodoublet; hence they are degenerate $m_X = m_Y$, up to small SU(2)$_L$ breaking effects of $O(\alpha m_W)$. It is the $X^{\pm 4/3}$ and $Y^{\pm 1/3}$ bosons that are responsible for proton decay and the unusual properties of magnetic monopoles. The gauge boson spectrum and pattern of symmetry breaking in the SU(5) model are illustrated in Fig. 1. Larger GUTs such as SO(10), E(6), O(18), etc. have many more gauge bosons (45, 78 and 153, respectively) and various possible patterns of symmetry breaking.\footnote{\textsuperscript{11}} We will not give detailed discussions on any specific models other than SU(5); but we do want to point out that they often exhibit features such as left–right symmetry, irreducible fermion generation representations and generation unification which make them attractive alternatives.

The minimum Higgs scheme required to break the gauge symmetry and produce a realistic fermion mass spectrum consists of a real $24$-plet, complex $5$-plet and complex $45$-plet\textsuperscript{2} ($124$ scalars!). Fifteen of these fields become longitudinal components of gauge bosons and endow them with mass while $109$ remain as massive physical scalar particles. For simplicity, one generally assumes that the single neutral scalar of the standard model has mass $\approx m_W$.

\begin{figure}
\centering
\begin{tikzpicture}
  \node (SU5) {SU(5)};
  \node (SU3xSU2xU1) at (0,0) {SU(3)$_c$ $\times$ SU(2)$_L$ $\times$ U(1)};
  \node (SU3xU1em) at (0,-1) {SU(3)$_c$ $\times$ U(1)$_{em}$};
  \draw[->] (SU5) -- (SU3xSU2xU1);
  \draw[->] (SU3xSU2xU1) -- (SU3xU1em);
\end{tikzpicture}
\caption{Pattern of symmetry breaking and resulting mass scales of gauge bosons in the SU(5) model.}
\end{figure}

$X^{\pm 4/3}$, $Y^{\pm 1/3}$ acquire mass

\begin{align*}
m_X &= m_Y = 10^{14} \sim 10^{15} \text{ GeV}
W^\pm$, $Z^0$ acquire mass

\begin{align*}
m_W &= 83 \text{ GeV}, m_Z = 94 \text{ GeV}
8$ gluons and photon massless

[$\text{FIGURE 1}$ Pattern of symmetry breaking and resulting mass scales of gauge bosons in the SU(5) model.]

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while the 108 other scalars have mass $= m_X$. This scenario is called the “great desert hypothesis” (no new physics below $m_X$). It simplifies the analysis and leads to rather definite predictions which can be tested by experiment. Consequences of relaxing this constraint will be subsequently examined.

As mentioned in the Introduction, an appealing feature of grand unification is that it forces all bare gauge couplings to be equal

$$g_{3_0} = g_{2_0} = g_{1_0} = g_{G_0}$$

(2.2)

where $g_{G_0}$ is the universal gauge coupling of the simple covering group. Furthermore, in theories (such as SU(5)) where each fermion generation forms a complete representation, one finds\(^2\)\(^1\)\(^2\)

$$\sin^2 \theta_w^0 = 3/8$$

(2.3a)

$$e_0^2/g_0^2 = 3/8, \ i = 1, 2, 3$$

(2.3b)

These relationships are renormalized by finite vacuum polarization effects which enhance or diminish coupling strengths as we change the energy (distance) scale probed. The observed unequal strengths of strong, weak and electromagnetic interactions is merely a consequence of performing laboratory experiments at relatively low energies. Equality is realized only at energies greater than the unification scale $m_X$.

Corrections to the relationships in Eq. (2.3) are easily analyzed by renormalization group techniques.\(^3\) Defining the renormalized running couplings $\alpha_i(\mu) = g_i^2(\mu)/4\pi, i = 1, 2, 3$ of the standard model by $\overline{\text{MS}}$ (modified minimal subtraction) and employing the relations\(^2\)\(^1\)

$$\alpha^{-1}(\mu) = \frac{5}{3} \alpha_1^{-1}(\mu) + \alpha_2^{-1}(\mu)$$

(2.4a)

$$\sin^2 \theta_w(\mu) = \frac{\alpha(\mu)}{\alpha_2(\mu)}$$

(2.4b)

$$m_W = 38.53 \text{ GeV}/\sin \theta_w(m_W)$$

(2.5a)

$$m_Z = 77.13 \text{ GeV}/\sin 2\theta_w(m_W)$$

(2.5b)
one finds for three fermion generations (assuming the "great desert hypothesis")

\[
\frac{\alpha(m_w)}{\alpha_3(m_w)} = \frac{3}{8} \left[ 1 - \frac{67}{6} \frac{\alpha(m_w)}{\pi} \ln \left( \frac{m_X}{m_w} \right) + \frac{\alpha(m_w)}{2\pi} \right.
\]
\[
+ \frac{\alpha(m_w)}{4\pi} \left\{ \frac{96}{7} \ln \left( \frac{\alpha_3(m_X)}{\alpha_3(m_w)} \right)
\]
\[
- \frac{10}{19} \ln \left( \frac{\alpha_2(m_X)}{\alpha_2(m_w)} \right)
\]
\[
- \frac{46}{41} \ln \left( \frac{\alpha_1(m_X)}{\alpha_1(m_w)} \right) \right\] (2.6)
\]

\[
\sin^2\theta_w(m_w) = \frac{3}{8} \left[ 1 - \frac{109}{18} \frac{\alpha(m_w)}{\pi} \ln \left( \frac{m_X}{m_w} \right) + \frac{5\alpha(m_w)}{18\pi} \right.
\]
\[
+ \frac{\alpha(m_w)}{4\pi} \left\{ \frac{-16}{21} \ln \left( \frac{\alpha_3(m_X)}{\alpha_3(m_w)} \right)
\]
\[
- \frac{94}{57} \ln \left( \frac{\alpha_2(m_X)}{\alpha_2(m_w)} \right)
\]
\[
- \frac{154}{123} \ln \left( \frac{\alpha_1(m_X)}{\alpha_1(m_w)} \right) \right\] (2.7)
\]

These formulas include all leading and next-to-leading logarithmic corrections as well as all ordinary $O(\alpha)$ corrections. They are valid for any grand unified theory $G$ which breaks down to the standard model at mass scale $m_X$, i.e., has a desert below $m_X$; however, because this scenario fits most easily into the SU(5) model, we refer to Eqs. (2.6) and (2.7) as minimal SU(5) model predictions.

To obtain predictions for $m_X$ and $\sin^2\theta_w(m_w)$, one solves the above equations iteratively using $\alpha(m_w)$ and $\alpha_3(m_w)$ as input with $m_w = 83$ GeV. Knowing $\alpha^{-1} = 137.036$, one finds from fermion vacuum polarization effects (for $m_t = 35$ GeV, see Section III)\textsuperscript{13}

\[
\alpha^{-1}(m_w) = 127.70 \pm 0.30 \quad (2.8)
\]
Given a value for $\Lambda_{\overline{\text{MS}}}^{(4)}$, the effective four flavor QCD mass scale, $\alpha_3(m_w)$ is also determined. For example, radiative upsilon decay gives

$$\Lambda_{\overline{\text{MS}}}^{(4)} = 100^{+100}_{-50} \text{ MeV} \rightarrow \alpha_3(m_w) = 0.101^{+0.011}_{-0.009} \quad (2.9)$$

In that way the predictions in Table I are obtained.

These results apply to any GUT in which the "great desert" is assumed. Notice that $m_X$ is approximately proportional to $\Lambda_{\overline{\text{MS}}}^{(4)}$

$$m_X = 1.3 \Lambda_{\overline{\text{MS}}}^{(4)} \times 10^{15} \quad (2.10)$$

and $\sin^2 \theta_w(m_w)$ is tightly constrained

$$\sin^2 \theta_w(m_w) = 0.216 + 0.006 \ln(0.1 \text{ GeV}/\Lambda_{\overline{\text{MS}}}^{(4)}) \quad (2.11)$$

How do these predictions compare with experiment? For $\Lambda_{\overline{\text{MS}}}^{(4)} = 100^{+100}_{-50} \text{ MeV}$, minimal SU(5) predicts

$$\sin^2 \theta_w(m_w) = 0.216 \pm 0.004 \quad (2.12)$$

which is to be compared with the experimental world average

$$\sin^2 \theta_w(m_w)_{\exp} = 0.219 \pm 0.006 \quad (2.13)$$

obtained from $\nu_\mu N$ and $eD$ scattering results combined with recent $m_w$ and $m_z$ determinations.\textsuperscript{15}

The agreement between theory and experiment in Eqs. (2.12) and (2.13) is very impressive. It represents the single successful

**TABLE I**

<table>
<thead>
<tr>
<th>$\Lambda_{\overline{\text{MS}}}^{(4)}$ (MeV)</th>
<th>$m_X$ (GeV)</th>
<th>$\sin^2 \theta_w(m_w)$</th>
<th>$m_w$ (GeV)</th>
<th>$m_z$ (GeV)</th>
<th>$\alpha(m_X)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>$3 \times 10^{13}$</td>
<td>0.224</td>
<td>81.3</td>
<td>92.3</td>
<td>0.024</td>
</tr>
<tr>
<td>50</td>
<td>$6.2 \times 10^{13}$</td>
<td>0.220</td>
<td>82.0</td>
<td>92.8</td>
<td>0.024</td>
</tr>
<tr>
<td>100</td>
<td>$1.3 \times 10^{14}$</td>
<td>0.216</td>
<td>82.8</td>
<td>93.6</td>
<td>0.024</td>
</tr>
<tr>
<td>200</td>
<td>$2.7 \times 10^{14}$</td>
<td>0.212</td>
<td>83.5</td>
<td>94.3</td>
<td>0.024</td>
</tr>
<tr>
<td>400</td>
<td>$5.5 \times 10^{14}$</td>
<td>0.208</td>
<td>84.3</td>
<td>94.9</td>
<td>0.024</td>
</tr>
</tbody>
</table>
quantitative prediction of GUTs with a "great desert hypothesis." Unfortunately, the prediction regarding $m_X$ in Eq. (2.10) has not fared so well. For $\Lambda_{\text{MS}}^{(d)} = 100^{+100}_{-50}$ MeV, the predicted range of $m_X$ leads to a partial lifetime for the decay $p \rightarrow e^+ \pi^0$ ($\tau_p$ scales as $m_X^4$, see Section III)

$$\tau(p \rightarrow e^+ \pi^0) = (0.06-240) \times 10^{29} \text{yr} \quad (2.14)$$

This prediction is to be compared with the present IMB bound$^{16}$

$$1/\Gamma(p \rightarrow e^+ \pi^0) > 2.5 \times 10^{32} \text{yr} \quad (2.15)$$

which appears to rule out the minimal SU(5) model or any simple GUT with a great desert between the standard model and $m_X$. Of course, there may be additional uncertainties in the theory. We will examine those uncertainties in Section III where a more complete discussion of proton decay will be given.

If one takes the experimental bound in Eq. (2.15) and the theoretical prediction in Eq. (2.14) both very seriously, one concludes that the minimal SU(5) model is ruled out. New physics at a mass scale $< m_X$ must modify the predictions in Eqs. (2.6) and (2.7) such that (for $\Lambda_{\text{MS}}^{(d)} = 100$ MeV)

$$m_X \geq 4 \times 10^{14} \text{GeV} \quad (2.16)$$

(the proton lifetime is proportional to $m_X^4$) and at the same time not upset the successful $\sin^2\theta_w(m_w)$ prediction. There are many ways in which this could happen. New physics at a mass scale $< m_X$ in the form of gauge bosons, fermions and/or scalars could modify Eqs. (2.6) and (2.7) and push up $m_X$. We will briefly discuss the gauge boson and fermion cases and then go into some detail in the case of light new scalars. We also consider the effect of supersymmetry which adds a large number of new fermion and scalar particles to the low energy spectrum.

Additional Gauge Bosons: GUTs based on symmetry groups larger than SU(5) (such as the SO(10) model$^{17}$) naturally include additional gauge bosons. If some of these populate the region between $m_W$ and $m_X$, they could increase the value of $m_X$ and
lengthen \( \tau_p \). For example, a popular symmetry breaking scheme\(^\text{18}\)

\[
SO(10) \xrightarrow{m_X} SU(4) \times SU(2)_R \times SU(2)_L
\]

\[
\xrightarrow{m_e} SU(3)_c \times U(1) \times SU(2)_R \times SU(2)_L
\]

\[
\xrightarrow{m_R} SU(3)_c \times U(1) \times U(1)_R \times SU(2)_L
\]

\[
\xrightarrow{m'} SU(3)_c \times SU(2)_L \times U(1)
\]

\[
\xrightarrow{m_W} SU(3)_c \times U(1)_{\text{em}}
\]

introduces arbitrary gauge boson mass scales \( m_e, m_R \) and \( m' \) between \( m_W \) and \( m_X \). These enter into new coupling constant relationships analogous to Eqs. (2.6) and (2.7). The added freedom of these mass scales loosens but does not completely unconstrain \( m_X \). For example, an analysis by Tosa, Branco and Marshak\(^\text{18}\) implies (neglecting Higgs scalar effects)

\[
\exp[55\Delta] \leq m_X/(1.3 \Lambda_{\text{MS}}^{(4)} \times 10^{15}) \leq \exp[219\Delta] \quad (2.17a)
\]

where

\[
\Delta = \sin^2 \theta_W(m_W)_{\text{exp}} - 0.216 - 0.006 \ln(0.1 \text{ GeV}/\Lambda_{\text{MS}}^{(4)}) \quad (2.17b)
\]

Employing \( \sin^2 \theta_W(m_X)_{\text{exp}} = 0.219 \pm 0.006 \) and \( \Lambda_{\text{MS}}^{(4)} = 0.1^{+0.1}_{-0.05} \) GeV, these constraints give

\[
0.5 \times 10^{14} \text{ GeV} \leq m_X \leq 5 \times 10^{15} \text{ GeV} \quad (2.18)
\]

which implies

\[
\tau_p(p \rightarrow \pi^0 e^+) \leq 1.6 \times 10^{35} \text{ yr} \quad (2.19)
\]

This example illustrates the need to push \( \tau_p \) bounds as far as possible and the importance of determining \( \Lambda_{\text{MS}}^{(4)} \) and \( \sin^2 \theta_W(m_W) \) with
the highest experimental precision possible. With regard to the latter, we note that measurements of $m_w$ and $m_z$ will provide a value for $\sin^2 \theta_w(m_w)$ to better than $\pm 0.001$.

*New Fermion Representations:* Merely adding a few more light fermion generations with mass $\approx m_w$ to the SU(5) model does not significantly modify its predictions. A fourth generation increases the predicted proton lifetime by about 30%, while a fifth and sixth could potentially increase it only by about a factor of 2 each. However, we note that for the case of eight relatively light generations (as suggested by O(18) GUTs), Bagger, Dimopoulos and Masso find that the predicted proton lifetime is several hundred times longer while $\sin^2 \theta_w(m_w)$ is almost unchanged. Such a scenario would therefore reconcile theory and present experimental bounds. It also suggests that proton decay should be observed in the near future. A more radical suggestion by P. Frampton and S. Glashow is to introduce split SU(5) fermion representations in which some members have mass $\approx m_w$ while the others have mass $m_x$. This leads to changes in the coefficients of the $\ln(m_x/m_w)$ terms in Eqs. (2.6) and (2.7) such that $m_x$ may be increased (or decreased). For some of the examples they considered, one can accommodate proton lifetimes consistent with the experimental bound in Eq. (2.15). Unfortunately, in such scenarios predictability is lost.

*Higgs Scalars:* In the minimal SU(5) model analysis, it was assumed that all physical scalars have mass $m_x$, except the one Higgs scalar of the standard model which has mass $\approx m_w$. One way to increase $m_x$ and the proton lifetime is to eliminate that constraint. Let us see what happens to $\sin^2 \theta_w(m_w)$ and $m_x$ if we do not assume scalar mass degeneracy. For generality we consider complex 5, 10, 15, 45, and 50 SU(5) scalar multiplets. They are the only scalars that can couple to the known fermions (they all occur in the 126 of SO(10)). In addition we include the real 24 that breaks SU(5) in our analysis.

Decomposing the SU(5) scalar multiplets according to their SU(3)$_c \times$ SU(2)$_L \times$ U(1) components, one finds

$$5 = \left(\frac{1,2,1}{m_1}\right) + \left(\frac{3,1,-2/3}{m_2}\right)$$
Each submultiplet is labeled by \((d_3,d_2,Y)\) where \(d_3\) and \(d_2\) are its \(SU(3)_c\) and \(SU(2)_L\) dimensions and \(Y\) is the \(U(1)\) hypercharge. Two submultiplets of the 24 become longitudinal components of the \(X^{\pm 4/3}\), \(Y^{\pm 1/3}\); so they have effective mass \(m_X\). Similarly a linear combination of the \((1,2,1)\) doublets in the 5 and 45 have an effective mass \(m_W\). For simplicity we take \(m_1 = m_W\). All other masses \(m_i\), \(i = 2\ldots 24\) will be left arbitrary in the following discussion.

First consider the 24-plet. If \(m_{22} = m_{23} = m_{24} = m_X\), minimal \(SU(5)\) predictions are left unchanged. On the other hand, if \(m_{22} = m_{23} = m_{24} = m_W\), one finds that \(\sin^2\theta_W(m_W)\) is unchanged while \(m_X\) increases by about 50\% (a factor of \(m_X/m_W)^{1/22}\) \(\tau_p\) increases by a factor of 5).

Next, we consider the 5, 10, 15, 45 and 50 scalars. Some of their components, \((3,1,-2/3), (3,3,-2/3), (\overline{3},1,8/3)\), can mediate proton decay at the tree level (see Section III); hence they must be
quite massive $>10^{10}$ GeV. The other mass parameters are more
or less arbitrary (they generally should not be $<m_w$). Allowing
arbitrary masses, one finds (in a leading-log approximation)\(^{22}\)

$$
\sin^2 \theta_w(m_w) = 0.212 + 0.006 \ln(0.1 \text{ GeV}/\Lambda_{\text{MS}}^{(4)}) - 7 \times 10^{-5}
$$

\begin{equation}
\times \ln \left[ \frac{m_1^2 m_4^4 m_6^7 m_8^2 m_9^2 m_{11}^{34} m_{20}^{33}}{m_2^2 m_3^4 m_{18}^{11} m_{10}^2 m_{12}^7 m_{13}^4 m_{14}^9 m_{15}^4 m_{16}^4 m_{17}^4 m_{18}^{19} m_{21}^{15}} \right]
\end{equation}

(2.20)

$$
m_X = 2 \times 10^{15} \Lambda_{\text{MS}}^{(4)}
$$

\begin{equation}
\times \left( \frac{m_1 m_3 m_5^3 m_9 m_{11}^3 m_{12}^7 m_{13}^4 m_{16}^4 m_{17}^4 m_{19}^4}{m_2^2 m_5 m_7 m_8 m_{10}^6 m_{14}^6 m_{15}^6 m_{17}^6 m_{20}^6 m_{21}^6} \right)^{1/66}
\end{equation}

(2.21)

It is clear from these formulas that predictability is lost. By selectively making some scalars heavier than others, one can increase
\(m_X\) thereby rendering \(\tau_p\) acceptably long while keeping \(\sin^2 \theta_w(m_w)\)
close to \(\sin^2 \theta_w(m_w)_{\exp} = 0.219 \pm 0.006\). Let us, however, point
out some general observations.\(^{22}\) (1) Making color singlets relatively
light, i.e., \(m_1, m_3, m_6, m_9, m_{16}\), decreases \(m_X\) and \(\tau_p\). (2) Only
relatively light \(m_5, m_7,\) or \(m_{20}\) lead to increases in both \(\sin^2 \theta_w(m_w)\)
and \(m_X\). (3) Assuming a hierarchy condition \(0.1 \leq m_i/m_j \leq 10, i,j
= 2,3, \ldots, 21\) leads to at most a factor of 3.5 increase in \(m_X\) (or
a factor of 150 increase in \(\tau_p\)) while leaving \(\sin^2 \theta_w(m_w)\) practically
unchanged. The last scenario again points out why it is important
to push the search for \(p \to e^+ \pi^0\) as far as possible.

**Supersymmetry:** The basic idea of supersymmetry is that each
known boson (fermion) has a fermion (boson) partner. Assuming
that all partners of standard model particles are light \(= m_w\) while
partners of superheavies have mass \(m_X\), one obtains the predictions\(^{24}\)
in Table II. Note that the results depend sensitively on the number
of light Higgs doublets \(N_H\) \((N_H = 4\) seems to be clearly ruled out
by \(\sin^2 \theta_w(m_w)_{\exp}\)). The value of \(m_X\) is significantly increased for
\(N_H = 2\), rendering \(\tau_p(p \to e^+ \pi^0) = 10^{35}\) yr unobservably long with
presently existing detectors. However, \(\sin^2 \theta_w(m_w)\) is larger than
the experimental value in Eq. (2.13). Of course there are ways to
lower \(\sin^2 \theta_w(m_w)\), e.g., nondegenerate scalars.\(^{25}\) In so doing, how-
TABLE II
Supersymmetric SU(5) predictions for $\Lambda_{\overline{\text{MS}}}^{(4)} = 100$ GeV

<table>
<thead>
<tr>
<th>$N_H$</th>
<th>$m_x$ (GeV)</th>
<th>$\sin^2 \theta_w (m_w)$</th>
<th>$m_w$ (GeV)</th>
<th>$m_z$ (GeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>$5 \times 10^{15}$</td>
<td>0.239</td>
<td>78.8</td>
<td>90.3</td>
</tr>
<tr>
<td>4</td>
<td>$3 \times 10^{14}$</td>
<td>0.260</td>
<td>75.5</td>
<td>87.8</td>
</tr>
</tbody>
</table>

However, it is possible that $\tau_p$ may also be reduced (or further increased). Supersymmetric GUTs are especially interesting because they naturally arise in unified theories which incorporate quantum gravity, such as superstring models. Constraints from $\sin^2 \theta_w (m_w)$ and $\tau_p$ severely restrict the allowed class of such theories.

The above scenarios illustrate a few ways in which a new physics at mass $< m_x$ can change the SU(5) model predictions. It is obvious that $m_x$ and $\tau_p$ can be increased and brought into agreement with experiment without ruining the successful $\sin^2 \theta_w (m_w)$ prediction. The implication is clear: $\Lambda_{\overline{\text{MS}}}^{(4)}$ and $\sin^2 \theta_w (m_w)$ should be more precisely measured, the search for proton decay should be pushed as far as possible, and we should continue to anticipate and to look for new physics at scales $> m_w$.

III. PROTON DECAY

In this section we focus on theoretical predictions for proton decay and their comparison with present experimental bounds. Gauge boson mediated decays will be our main topic; however, scalar mediated decays as well as instanton and quantum gravity induced decays will also be briefly surveyed.

Gauge Boson Mediated Proton Decay

The superheavy $X^{\pm 4/3}$ and $Y^{\pm 1/3}$ gauge bosons of the SU(5) model can mediate B and L violating processes such as proton decay (B-L is conserved). Some graphical illustrations of proton decay are given in Fig. 2. Higher rank models such as SO(10) contain, in addition, a second color triplet, SU(2)$_L$ doublet pair $X''^{\pm 2/3}$, $Y''^{\pm 1/3}$ which can also mediate proton decay. These two pairs of
gauge bosons are the only ones that induce tree level proton decay via four Fermi interactions.\textsuperscript{26,27} Their exchange leads to decay rates proportional to $1/m_X^4$ and $1/m_Y^4$, which are naturally suppressed due to the enormous magnitude of these masses $>10^{14}$ GeV (see Section II).

Virtual exchange of $X$, $Y$, $X'$ and $Y'$ bosons between quarks and leptons gives rise to the following effective low energy Hamiltonian\textsuperscript{26,28}

$$H = \frac{g_G^2(m_X)}{2m_X^2} \left( \frac{m_X^2 + m_{X'}^2}{m_{X'}^2} \right)$$

$$\times A e_{ijk} \left[ \bar{u}_i^c \gamma_\mu u_j^L (\bar{e}_R^+ \gamma_\mu d_i^R + r_c (\bar{e}_L^+ \gamma_\mu d_i^L)) - \bar{u}_i^c \gamma_\mu d_i^L \bar{e}_R^c \gamma_\mu d_i^R \right]$$

$$+ \text{h.c.} + \text{heavier generation terms} \quad (3.1)$$

where $g_G(m_X)$ is the value of the gauge coupling at mass scale $m_X$, $A \approx 3$ is an enhancement factor due to virtual gluon, $W^\pm$, $Z$ and $\gamma$ radiated corrections and $r_c = 2m_X^2/(m_X^2 + m_{X'}^2)$. (In the SU(5) model one can set $m_{X'} \rightarrow \infty$, $r_c \approx 2$.) Heavier generation interactions have not been explicitly exhibited. The ones relevant for proton decay are obtained from Eq. (3.1) by setting $m_{X'} \rightarrow \infty$ and making the substitution

$$\bar{e}_R^+ \gamma_\mu d_i^R \rightarrow \bar{\mu}_R^+ \gamma_\mu s_i^R, \quad r_c \bar{e}_L^+ \gamma_\mu d_i^L \rightarrow \bar{\mu}_L^+ \gamma_\mu s_i^L,$$

$$\bar{\nu}_e^c \gamma_\mu d_i^R \rightarrow \bar{\nu}_\mu^c \gamma_\mu s_i^R.$$

FIGURE 2 Examples of proton decay amplitudes.
In addition, there can be generation mixing which in principle could render specific amplitudes arbitrary.\textsuperscript{29} We shall follow the "kinship" hypothesis introduced by Wilczek and Zee\textsuperscript{27} which assumes that generation mixing is small, i.e., on the order of the Cabibbo angle, and hence not very important. Such a situation naturally occurs in the SU(5) model if only the Higgs 5-plet (and not the 45) is used to generate fermion masses. Although the "kinship" hypothesis is quite reasonable, the reader should bear in mind that one could modify the subsequent decay rate predictions by relaxing this assumption and introducing significant mixing effects.\textsuperscript{29}

The predictions that follow from Eq. (3.1) depend on $g_G(m_X)$, $A$, $r_e$, and most sensitively on $m_X$ and $m_X'$. In the minimal SU(5) model described in Section II, $g_G(m_X)/4\pi \simeq 0.024$, $A \simeq 2.7$, $r_e \simeq 2$ and $m_X \simeq 1.3 \times 10^{14}$ GeV for $\Lambda^{(4)}_{\overline{\text{MS}}} = 100$ MeV; so $H$ is fully determined.\textsuperscript{30} (Allowing for a factor of 2 uncertainty in $\Lambda^{(4)}_{\overline{\text{MS}}}$ results in about a factor of 10 uncertainty in subsequent decay rates due to changes in $m_X$ and $A$.) Unfortunately, the calculation of hadronic matrix elements of $H$ which interpolate from an initial nucleon to final state decay products is dependent on the model of hadronic structure employed. This dependence implies a degree of uncertainty in decay rate predictions. Consider for example the decay mode $p \to e^+\pi^0$ discussed in Section II. It is extremely important, because this mode is quite easily identified in water Cerenkov detectors. The matrix element $\langle e^+\pi^0|H|p \rangle$ receives contributions from two quark annihilation (see Fig. 2) as well as three quark fusion in the proton. These distinct amplitudes are proportional to $1/R_p^{3/2}$ and $1/R_p^3$, respectively, where $R_p$ is the effective proton radius. Hence, the relative importance of each contribution as well as the overall rate depends sensitively on $R_p$. (Most calculations now find these two contributions to be about equal in magnitude and to constructively interfere.) Results for the partial lifetime $1/\Gamma(p \to e^+\pi^0)$ arrived at by a variety of different calculations are illustrated in Table III. It is difficult to assess the uncertainty in these various calculations. For the moment, we take $1/\Gamma(p \to e^+\pi^0) \simeq 1.2 \times 10^{29}$ yr as a representative central value and allow for a factor of 2 variance. This central prediction is more than three orders of magnitude below the IMB bound in Eq. (2.15) and therefore certainly ruled out. The factor of 2 uncertainty in $\Lambda^{(4)}_{\overline{\text{MS}}}$ (see Eq. (2.9)) gives
TABLE III

Results of different calculations for the partial lifetime $1/\Gamma(p \to e^+\pi^0)$ in the minimal SU(5) model with $\Lambda_{\text{MS}}^{(4)} = 100$ MeV

<table>
<thead>
<tr>
<th>Group</th>
<th>Method</th>
<th>$1/\Gamma(p \to e^+\pi^0)$ (yr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tomazawa$^{31}$</td>
<td>PCAC</td>
<td>$(4 \sim 25) \times 10^{28}$</td>
</tr>
<tr>
<td>Berezinsky, Joffe and Kogan$^{32}$</td>
<td>QCD Sum Rule</td>
<td>$2 \times 10^{28}$</td>
</tr>
<tr>
<td>Donoghue and Golowich$^{33}$</td>
<td>MIT BAG</td>
<td>$1.4 \times 10^{29}$</td>
</tr>
<tr>
<td>Lucha$^{34}$</td>
<td>B-S Equation</td>
<td>$1.8 \times 10^{29}$</td>
</tr>
<tr>
<td>Isgur and Wise$^{35}$</td>
<td>N.R.Q.M.</td>
<td>$1.1 \times 10^{29}$</td>
</tr>
<tr>
<td>Thomas and McKeller$^{36}$</td>
<td>Cloudy Bag</td>
<td>$1.5 \times 10^{28}$</td>
</tr>
</tbody>
</table>

A factor of 10 uncertainty in $\Gamma(p \to e^+\pi^0)$; but this is not enough to bring theory and experiment into accord. There could, however, be an additional source of suppression missed by all the calculations in Table III. Indeed, Goldhaber, Goldman and Nussinov (GGN)$^{37}$ have argued that proton decay requires quantum barrier penetration which may suppress its rate by as much as an extra order of magnitude. Allowing for such a possibility as well as the factor of 2 uncertainty in $\Lambda_{\text{MS}}^{(4)}$ implies

$$1/\Gamma(p \to e^+\pi^0) = 6 \sim 240 \times 10^{28 \pm 1} \text{ yr} \quad (3.2)$$

for the range of predictions in the minimal SU(5) model. The upper extreme in this prediction range is still almost a factor of 10 below the IMB bound. To increase the prediction (including the GGN suppression factor) so as to be in accord with the IMB bound requires

$$\Lambda_{\text{MS}}^{(4)} > 370 \text{ MeV} \quad (\text{corresponds to } m_x > 5 \times 10^{14} \text{ GeV}) \quad (3.3)$$

$$\sin^2\theta_w(m_W) < 0.208 \quad (3.4)$$

These values are somewhat in conflict with the experimental constraints given in Eqs. (2.9) and (2.12). However, before the minimal SU(5) model can be discarded, the bound on $1/\Gamma(p \to e^+\pi^0)$ should be pushed further and $\Lambda_{\text{MS}}^{(4)}$ or $\sin^2\theta_w(m_W)$ should be more precisely determined. Re-examination of the $\langle e^+\pi^0|H|p \rangle$ matrix element should also be undertaken.
Aside from uncertainties in $\Lambda^{(4)}_{\overline{MS}}$ and the hadronic matrix elements, what other effects might modify the $1/\Gamma(p \rightarrow e^+ \pi^0)$ prediction? Nuclear physics effects in oxygen (the main source of protons in the IMB water detector) are expected to give less than a factor of 2 suppression\(^{38}\) and such corrections have already been applied to the data arriving at the bound in Eq. (2.15). Other uncertainties such as quark mass values used in the renormalization group analysis determination of $\alpha_s(m_w)$ and $\alpha(m_w)$ (such as $m_t$) have a fairly insignificant effect on the proton decay rate.\(^{23}\)

Of course, as we explained in Section II, the value of $m_X$ and predicted proton lifetime can be easily increased by populating the "great desert" between $m_W$ and $m_X$ with new physics. That new physics could be in the form of additional gauge bosons, fermions or scalars which modify the renormalization group equations of Eqs. (2.6) and (2.7) such that $m_X$ is increased. Keeping $\Lambda^{(4)}_{\overline{MS}} = 100$ MeV, one only needs to increase $m_X$ to $> 4 \times 10^{14}$ GeV (assuming $\alpha_s(m_X)$ is unchanged) in order to satisfy the IMB bound ($> 7 \times 10^{14}$ GeV if the GGN suppression factor is not included). Unfortunately, once $m_X$ becomes unconstrained, we lose the prediction for $\sin^2 \theta_W(m_w)$ and the proton decay rate. It is therefore very important to have an idea what dominant decay modes and branching ratios should be expected, independent of $m_X$. We now examine this issue.

From the form of $H$ is Eq. (3.1), we learn that $\Delta B = \Delta L$ (in addition $\Delta S/\Delta B = 0$ or $-1$). Next, using isospin symmetry and several model dependent estimates, one finds the following (approximate) two body proton decay branching ratios (for $m_X > m_W$)\(^{26}\)

\[
\begin{align*}
\text{p} &\rightarrow e^+ \pi^0, \; e^+ \omega \text{ or } e^+ \rho, \; e^+ \eta, \; \bar{\nu}_e \pi^+, \; \bar{\nu}_e \rho^+, \; \mu^+ K^0, \; \bar{\nu}_\mu K^+ \\
&0.40 : 0.30 : 0.01 : 0.16 : 0.04 : 0.03 : 0.02 \\
\end{align*}
\tag{3.5}
\]

Note that $\text{p} \rightarrow e^+ \pi^0$ is expected to be the dominant two body decay mode. In the case of the neutron one obtains

\[
\begin{align*}
n &\rightarrow e^+ \pi^-, \; e^+ \rho^-, \; \bar{\nu}_e \pi^0, \; \bar{\nu}_\mu K^0 \\
&0.80 : 0.05 : 0.08 : 0.02 \\
\end{align*}
\tag{3.6}
\]
The baryon number violating total decay rates of the proton and neutron are predicted to be about equal.

Some of the above relative rates are firm predictions because they depend only on isospin and the value of \( r_e \). For example,

\[
\Gamma(p \to e^+\pi^0) = \frac{1}{2} \Gamma(n \to e^+p^-) = \frac{1 + r_e^2}{2} \Gamma(p \to \bar{\nu}_e\pi^+)
\]

\[
\approx (1 + r_e^2) \Gamma(n \to \bar{\nu}_e\pi^0)
\]  \hspace{1cm} (3.7)

where \( r_e = 2 \) in the SU(5) model. Measurements of ratios such as \( \Gamma(p \to e^+\pi^0)/\Gamma(p \to \bar{\nu}_e\pi^+) \) could therefore determine \( r_e \) and be used to indicate the presence or absence of \( m_{\chi'} \) effects. Another way of determining \( r_e \) would be to measure the \( e^+ \) polarization which is predicted in \( \Delta S = 0 \) proton decays to be \((1 - r_e^2)/(1 + r_e^2)\).

Before leaving the subject of gauge boson mediated proton and neutron decay, we list some additional IMB bounds for particularly relevant decay modes (all at 90% C.L.).\(^{16}\)

\[
\frac{1}{\Gamma(p \to e^+\rho)} > 2.5 \times 10^{31} \text{ yr} \quad (3.8)
\]

\[
\frac{1}{\Gamma(p \to e^+\omega)} > 4.0 \times 10^{31} \text{ yr} \quad (3.9)
\]

\[
\frac{1}{\Gamma(p \to e^+\eta)} > 1.2 \times 10^{32} \text{ yr} \quad (3.10)
\]

\[
\frac{1}{\Gamma(p \to \bar{\nu}\rho^+)} > 1.1 \times 10^{31} \text{ yr} \quad (3.11)
\]

\[
\frac{1}{\Gamma(p \to \mu^+K^0)} > 2.9 \times 10^{31} \text{ yr} \quad (3.12)
\]

\[
\frac{1}{\Gamma(p \to \bar{\nu}K^+)} > 0.7 \times 10^{31} \text{ yr} \quad (3.13)
\]

\[
\frac{1}{\Gamma(n \to e^+\pi^-)} > 4.3 \times 10^{31} \text{ yr} \quad (3.14)
\]

\[
\frac{1}{\Gamma(n \to e^+\rho^-)} > 1.2 \times 10^{31} \text{ yr} \quad (3.15)
\]

\[
\frac{1}{\Gamma(n \to \bar{\nu}\pi^0)} > 0.7 \times 10^{31} \text{ yr} \quad (3.16)
\]

\[
\frac{1}{\Gamma(n \to \bar{\nu}K^0)} > 1.0 \times 10^{31} \text{ yr} \quad (3.17)
\]
In some cases, the IMB collaboration actually has candidate events. Other experiments have also reported proton decay candidates with partial lifetimes of order $10^{31}$ yr. Improved statistics and reproducibility are necessary to determine whether those candidates are actual proton decays or merely neutrino background events.

**Higgs Scalar Mediated Proton Decay**

Proton decay can also be mediated by Higgs scalars. At the lowest order tree level, only the $\text{SU}(3)_c \times \text{SU}(2)_L \times \text{U}(1)$ scalar multiplets $(3,1,-2/3), (3,3,-2/3)$ and $(\overline{3},1,8/3)$ induce proton decay. The first of these multiplets occurs in the SU(5) 5-plet while all three are present in the 45 (see Section II). If we allow arbitrary Higgs–fermion couplings, then there is no prediction for Higgs induced decay rates. However, if we assume that those couplings are proportional to the fermion masses, then proton and neutron decays into muons and kaons should dominate. For example, Golowich found that if only the Higgs 5-plet were used to provide fermion masses, the amplitudes for $(3,1,-2/3)$ scalar induced proton decay would have the following ratios:

\[
p \rightarrow \overline{\nu}_\mu K^+, \quad \mu^+ K^0, \quad \mu^+ \pi^0, \quad e^+ K^0, \quad e^+ \pi^0
\]

where $\theta_c$ is the Cabibbo angle ($\sin^2 \theta_c \approx 0.05$). Supplementing Golowich’s result with phase space effects and three quark fusion enhancements, one expects two body branching ratios:

\[
p \rightarrow \overline{\nu}_\mu K^+, \quad \mu^+ K^0, \quad \mu^+ \pi^0, \quad e^+ K^0, \quad e^+ \pi^0
\]

\[
0.75 : 0.18 : 0.07 : 0.007 : 0.004
\]

We see that $\overline{\nu}_\mu K^+$ and $\mu^+ K^0$ decays dominate; however, the more easily observed mode $\mu^+ \pi^0$ is still quite appreciable. For the neutron, $n \rightarrow K^0 \overline{\nu}_\mu$ dominates. In this scenario, a Higgs mass of $m_H \approx 10^{10}$ GeV corresponds to a proton lifetime of order $10^{30}$ yr (it scales like $m_H^2$). Of course in the SU(5) model the mass of the $(3,1,-2/3)$ scalar is somewhat arbitrary. We generally take its mass
to be $m_x$, in which case it leads to proton lifetimes of $O(10^{44}\text{ yr})$ which are unobservably long. In some locally supersymmetric theories, scalars naturally acquire mass $\approx (m_\nu m_{p_1})^{1/2} \approx 10^{10}$ GeV ($m_{p_1} = \text{Planck mass} \approx 10^{19}$ GeV). Such models would suggest that observation of $p \rightarrow \bar{\nu}K^+$, $\mu^+K^0$ or $\mu^+\pi^0$ should have already occurred or may be close at hand (cf. Eqs. (3.8)–(3.17)).

Before leaving the subject of induced scalar effects, several other effects should be mentioned. Higgs scalars can induce dimension 9 six-fermion operators which lead to $\Delta B = 2$, $n-\bar{n}$ oscillations. Such a reaction in a heavy nucleus such as $^{16}$O would manifest itself as a 2 GeV outburst of pions. The lack of observation of such events in the IMB experiment allows one to deduce a free oscillation time

$$T_{n\bar{n}} \approx 8 \times 10^7 \text{ s}$$

(3.20)

Scalars can also induce dimension 12, eight-fermion operators which give rise to hydrogen–antihydrogen oscillations and double proton decay $pp \rightarrow e^+e^+$ or $\mu^+\mu^+$. The latter reaction would appear as a rather spectacular 2 GeV event with back-to-back leptons. So far, no such decays have been observed.

Dimension 5 Operators

The gauge boson and Higgs scalar mediated proton decays just described result from tree diagrams which lead directly to dimension 6 four-fermion operators. (Each fermion field has dimension 3/2.) The resulting decay rates are proportional to $1/m_X^4$ or $1/m_H^4$ and hence naturally suppressed by the large values of $m_X$ and $m_H$.

It was, however, pointed out by Weinberg and Sakai and Yanagida that B and L violating dimension 5 operators may be induced by higgsino mixing (higgsino is the spin-1/2 partner of the Higgs scalar) in some supersymmetric GUTs. (The dimension 5 operators entail fermion-fermion–scalar-scalar couplings. Scalars have dimension 1.) Through loop effects such operators can give rise to dimension 6 B and L violating four-fermion amplitudes suppressed only by $1/m_X$ and mixing factors. The resulting proton lifetime is of $O(10^{39}$ yr) with $p \rightarrow \bar{\nu}_rK^+$ and $n \rightarrow \bar{\nu}_rK^0$ being the dominant decay modes. The suggested rates for those decays appear to be in
conflict with the bounds in Eqs. (3.13) and (3.17) by about an order of magnitude. However, there is considerable uncertainty in the theoretical prediction. Furthermore, the dimension 5 operators can be exorcised by symmetries. Nevertheless, this possibility supports findings in the scalar mediated decay analysis that $p \rightarrow \bar{\nu}K^+$ and $n \rightarrow \bar{\nu}K^0$ appear to be the best modes for detecting scalar induced $B$ and $L$ violation.

Instanton Effects

't Hooft has shown that even in the standard SU(3)$_c \times$ SU(2)$_L \times$ U(1) model, baryon number is not exactly conserved due to instanton effects in the SU(2)$_L$ weak sector.\textsuperscript{47} Those effects are, however, extremely small due to a variety of suppressions. The amplitude for such a quantum tunneling phenomenon is proportional to $\exp[-8\pi^2/g^2(m_w)] \approx 10^{-77}$ and all fermion doublets must be involved in the effective interaction. For three generations the induced operators have dimension 18 (12 Fermi-operators) and must entail $\Delta B = \Delta L = 3$. Processes such as $ppp \rightarrow e^+\mu^+\tau^+$, $nnn \rightarrow \bar{\nu}_e\nu_\mu\bar{\nu}_\tau$, $pp \rightarrow \bar{\nu}e^+\mu^+\bar{\nu}_\tau$, etc. could in principle occur, but they would be suppressed by Cabibbo-like mixings between generations as well as the factor $10^{-77}$ and matrix element suppressions; hence their rates are completely negligible. Recently, Rubakov\textsuperscript{48} has made the observation that very massive baryons >2 TeV, if they exist, may violate baryon number in their decays via the 't Hooft mechanism without the $10^{-77}$ suppression factor. Such a scenario could lead to interesting signatures at a high energy collider, since all light fermion generations would be present in the decay products.

In some preon-like theories where the $W^\pm$, $Z$, lepton, quarks, etc. are made from some more fundamental constituents, the non-Abelian couplings analogous to $g_2$ may be large and instanton contributions to proton decay could be sizeable. Unfortunately, without a realistic preon model it is impossible to be specific about what decay modes and rates one might expect.

Gravitational Theories

In quantum gravity theories, baryon number violation is also expected to occur.\textsuperscript{49} The basic idea is that although black holes must
conserves quantum numbers associated with long range gauge interactions (such as electric charge, color, etc.), they need not conserve global quantities such as baryon or lepton number. So, for example, given a proton, two of its quarks can fall into a virtual black hole and re-emerge as an antiquark + antilepton; hence the decay \( p \to e^+\pi^0 \). There are of course large uncertainties in the expected rate for this reaction. If the fundamental interaction is a four-fermion one, the rate should scale as \( 1/m_{\text{P1}}^4 \left( m_{\text{P1}} = G^{-1/2} = \text{Planck mass} \approx 10^{19} \text{GeV} \right) \) and the resulting lifetime \( \tau_p \) is expected to be of \( O(10^{50} \text{yr}) \). In some supersymmetric gravity theories, or Kaluza-Klein models, however, there could be dimension 5 operators which give \( 1/m_{\text{P1}}^2 \) rates which may be observable.\(^{50}\) It has been suggested that in such theories, the decays \( p \to \bar{\nu}K^+ \) and \( n \to \bar{\nu}K^0 \) dominate.

A final scenario for proton decay is through magnetic monopole catalysis in GUTs. This topic is the subject of the following section.

**IV. MAGNETIC MONOPOLES**

Grand unified theories which undergo spontaneous symmetry breaking \( G \to H \times \text{U}(1) \) at mass scale \( m_X \) predict the existence of superheavy magnetic monopoles.\(^4\) In the SU(5) model the lightest (stable) such monopole carries \( \pm 1/2e \) units of ordinary magnetic charge as well as (screened) color magnetic charge.\(^{51}\) (Colorless magnetic monopoles must carry \( 3n/2e \), \( n = \pm 1, \pm 2, \ldots \) units of ordinary magnetic charge.) Its ordinary magnetic field appears point-like down to distances of \( O(1/m_X) \), where \( X^{\pm 4/3} \) and \( Y^{\pm 1/3} \) structure effects are manifest due to the soliton-like nature of the monopole. The monopole mass, \( m_M \), is surprisingly well bounded\(^{52}\)

\[
\frac{3m_X}{8\alpha(m_X)} \leq m_M \leq \frac{5.36m_X}{8\alpha(m_X)} \quad (4.1)
\]

(In minimal SU(5), \( \alpha(m_X) \approx 1/110 \).) Such monopoles could have been produced in the early universe and may still be around. In fact, crude estimates in the framework of big bang cosmology suggest that magnetic monopoles should be as abundant as protons.
Of course this cannot be the case. Indeed, bounds on the gravitational contraction rate of the universe combined with the estimate $m_M \approx 10^{16}$ GeV imply a number density $n_M < 10^{-14} n_p$. This need to suppress the number of superheavy monopoles by 14 orders of magnitude is referred to as the "monopole problem" of GUTs. Attempts to find a solution have given rise to the "inflationary universe" scenario which completely eliminates the possibility of superheavy remnant monopoles. Of course the "inflationary universe" cosmology is still speculative, so it is worth contemplating ways to detect leftover monopoles. Although a candidate magnetic monopole event was reported several years ago, subsequent experiments have failed to confirm this finding. In addition, there are arguments based on the existence of galactic magnetic fields as well as searches for monopole ionizing effects which severely constrain the possible fluxes of such monopoles. Nevertheless, searches for magnetic monopoles are certainly worthwhile when one considers the payoff for a successful hunt. To that end, one should anticipate that such superheavy monopoles are most likely nonrelativistic $\beta = v/c \approx 10^{-3} \sim 10^{-4}$ and may undergo hadronic interactions with matter due to their screened color magnetic charge.

The most incredible property of GUT monopoles, uncovered by Rubakov and Callan, is their apparent ability to catalyze proton decay with strong interaction cross sections $O(10^{-26} \text{ cm}^2)$. The basic idea is that $J = 0$ fermion–monopole scattering must lead to helicity conserving reactions which involve electric and color charge exchange. At the quark level they give rise to $d_L + M \rightarrow e_T^+ + \bar{u}_R + \bar{u}_R + M$ amplitudes which are not suppressed by powers of $1/m_X$. At the nucleon level, one therefore expects

$$p + M \rightarrow e^+ + M + X \quad (X = \text{pions})$$

with cross sections $\approx \pi/k^2$. Of course, there may be long distance effects or weak interaction subtleties which actually reduce the rate for catalysis. A completely reliable estimate of proton decay catalysis cross sections does not as yet exist.

Accepting a large cross section for proton decay catalysis, then large underground proton decay detectors can also double as magnetic monopole detectors. Indeed, the signal that a monopole has
passed through the detector would be a spectacular trail of proton and neutron decays. Imagine discovering a magnetic monopole as well as proton decay with a single event. To date no such signal has been observed.58

V. CONCLUSION AND OUTLOOK

Just a few years ago, the possibility of proton decay and magnetic monopoles were considered far out speculations. Now they have become an integral part of theoretical lore. Indeed, mechanisms for baryon number nonconservation naturally arise in GUTs and quantum gravity. Furthermore, the observed matter–antimatter asymmetry of the universe (excess of $p$’s over $\bar{p}$’s) strongly suggests that baryon number violation must have been quite large in the very early universe. We now find ourselves asking not whether the proton decays, but whether we will be able to experimentally observe its decay. Unfortunately, lifetimes $>10^{34}$ yr would be swamped by backgrounds.

The minimal SU(5) model with a “great desert” scored a significant success by correctly predicting $\sin^2 \theta_W(m_W)$. Unfortunately, it now appears to be ruled out by experimental bounds on $\tau(p \rightarrow e^+\pi^0)$. However, as we have shown, one can always accommodate a longer proton lifetime by increasing $m_X$ through the addition of new particles at mass scales $<m_X$. Indeed, our lack of understanding of fermion mixing and mass parameters strongly suggests the existence of some new physics at or below $\sim 1$ TeV. If such new physics is found it will affect proton lifetime predictions. In the meantime experiments will continue to search not only for $p \rightarrow e^+\pi^0$ but also for any other experimentally accessible decay modes.

The fact that Dirac’s magnetic monopole finds its natural setting in GUTs and in that framework exhibits the ability to catalyze proton decay is remarkable. If we could ever trap such monopoles, they would allow a direct probe of $X^{\pm 4/3}, Y^{\pm 1/3}$ physics ($M + \bar{M} \rightarrow \approx 200$ $X$ and $Y$ bosons) and perhaps even provide an almost unlimited source of energy through their catalyzing reactions.

The last few years have been exciting times in theory and experiment. By their sheer elegance, GUTs have earned a perhaps permanent place in high energy physics and motivated several
impressive experiments. Stringent limits have been set on a variety of potential proton decay modes. The few proton decay candidates that have been reported deserve further investigation. During the coming years we can look forward to continued searches for proton decay, magnetic monopoles, etc. Perhaps during these quests we will find that nature has some even more amazing surprises just waiting to be found.

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