Gauged Supergravities and Spontaneous Supersymmetry Breaking from the Double Copy Construction

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Gauged supergravities—with gauged $R$ symmetry and Minkowski vacua allow for spontaneous supersymmetry breaking and, as such, provide a framework for building supergravity models of phenomenological relevance. In this Letter, we initiate the study of double copy constructions for these supergravities. We argue that, on general grounds, they expect their scattering amplitudes to be described by a double copy of the type (spontaneously broken gauge theory)⊗(gauge theory with broken supersymmetry). We present a simple realization in which the resulting supergravity has $U(1)_R$ gauge symmetry, spontaneously broken $\mathcal{N} = 2$ supersymmetry, and massive gravitini. This is the first instance of a double copy construction of a gauged supergravity and of a theory with spontaneously broken supersymmetry. The construction extends in a straightforward manner to a large family of gauged Yang-Mills-Einstein supergravity theories with or without spontaneous gauge-symmetry breaking.

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Recent advances in scattering amplitude calculations have been playing a key role in revealing hidden properties of gravity. Amplitudes in many supergravities admit a simpler formulation in terms of gauge-theory building blocks. A systematic framework for finding this description is provided by the double copy construction introduced in Refs. [10,11]. The double copy applies to tree- and loop-level amplitudes [12–14], as well as classical solutions [15–17], and extends earlier string-theory results [18]. Recent success in reformulating large families of Maxwell-Einstein (ME) [19–23] and Yang-Mills-Einstein (YME) supergravities [24–26] in the double copy language has prompted the proposal that all theories of gravity could be regarded as double copies of some sort [22] (see also [27]). Generalizing these constructions to gauged ME and YME supergravities constitutes a major step toward establishing this proposal and has the potential for incorporating a large body of supergravity literature into the rapidly developing field of amplitude calculations.

In this Letter, we propose a general strategy for expressing gauged supergravities as double copies. The main result is that amplitudes with the correct properties can be obtained from those of a theory with spontaneously broken gauge symmetry and a gauge theory with broken supersymmetry. We present an explicit example in which we gauge a $U(1)_R$ subgroup of the SU(2)$_R$ $R$ symmetry in theories belonging to the so-called generic Jordan family of $\mathcal{N} = 2$ ME supergravities.

Gauged supergravity always involves a minimal coupling between (some of the) gravitini and one or more vector fields. Consequently, for Minkowski vacua, there exist nonvanishing gravitini-vector amplitudes

$$\mathcal{M}_3(1\bar{\psi}_i, 2\psi_j, 3A^a) = ig_R r_{ij}^a v_3^{\mu} v_{2\mu} + O(g_R^0),$$

where $g_R$ is the gauge coupling constant, $v_{3\mu}(l = 1, 2)$ are the gravitini’s polarizations, and $r_{ij}^a$ are the representation...
matrices of the gauged $R$ symmetry subgroup, acting on the
two gravitini. We omitted terms involving field strengths
that do not explicitly depend on $g_\kappa$; these are unrelated to
the gauging. While seemingly innocuous, the amplitude (1)
is not invariant under a linearized supersymmetry trans-
formation, $v_{i\mu} \to v_{i\mu} + k_{i\mu} c$ (the spinor $c$ obeys $\nabla c = 0$ to
preserve the gamma tracelessness of $v_{i\mu}$). Hence, assuming
that the gauging procedure preserves the supersymmetry
of the Lagrangian, the amplitude above must belong to a
theory with spontaneously broken supersymmetry (pos-
sibly partially). Since local supersymmetry can no longer
be used to reduce the gravitino’s physical polarizations
down to two, a gravitino now has four distinct polarization
states corresponding to a massive spin-3/2 particle. Thus,
we need to consider a double copy construction valid for
massive gravitini.

**Gauged supergravities as double copies.**—The double
copy construction of [11] starts from gauge-theory ampli-
tudes organized in terms of cubic graphs whose edges are
labeled by representations of the gauge group. The color
factor $c_i$ of each graph is obtained by dressing each vertex
with the corresponding group-invariant symbol; the kin-
ematic numerator $n_i$ of each graph includes the dependence
on external polarizations as well as loop and internal
momenta. If (a) two gauge theories have common mass
spectra and conjugate gauge-group representations (so that
gravity states and interactions, reproducing the absence of
massive gravitini, this implies that the fermions of the other
gauge theory must be massive. We therefore propose that
gauged supergravities around Minkowski vacua can be
described as double copies of a spontaneously broken
gauge theory and a gauge theory whose supersymmetry is
explicitly broken by fermion masses. Schematically, the
double copy is

$$\text{(gauged supergravity)} = \text{(Higgs YM)} \otimes \text{(SYM)}. \quad (3)$$

A simple realization.—To illustrate the proposed con-
struction, we take as the left gauge theory (GT$_L$) a scalar-
coupled SU($N + M$) Yang-Mills (YM) theory with 4D
Lagrangian

$$L_0 = \frac{1}{g^2} \text{Tr} \left[ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} D_\mu \phi^a \bar{D}^\mu \phi^a + \frac{1}{4} [\phi^a, \phi^b]^2 \right], \quad (4)$$

with $a, b = 1, \ldots, n$. As discussed above, the gauge sym-
metry is spontaneously broken; we choose a scalar vacuum
expectation value (VEV) $\phi^a \to \phi^a + \langle \phi^a \rangle$ of the form

$$\langle \phi^a \rangle = V^a \times \text{diag} \left( \frac{1}{N} \mathbb{1}_N, -\frac{1}{M} \mathbb{1}_M \right),$$

where $V^a$ is constant. The subgroup $G = \text{SU}(M) \times \text{SU}(N) \times \text{U}(1)$
remains unbroken and the spectrum is

$$\text{GT}_L: \{A_{\pm}, \phi^a\}_G \oplus \{W_{\mu}, \psi^a\}_R \oplus \{\bar{W}_{\mu}, \bar{\psi}^a\}_{\bar{R}},$$

where $G$ denotes the adjoint representations of $G$, and $R$
and $\bar{R}$ are the bifundamental $(N, \bar{M})$ and $(\bar{N}, M)$ represen-
tations. All fields transforming in the $R$, $\bar{R}$ representa-
tions have the same mass $m$. The index $s = 2, \ldots, n$ runs
over the massive scalars, while $\mu$ runs over the three
physical polarizations of the massive $W$’s. It was shown
in Ref. [25] that this theory obeys color-kinematics duality.

The right gauge theory (GT$_R$) has explicitly broken
supersymmetry and Lagrangian

$$L_{\Delta = 2} = \frac{1}{g^2} \text{Tr} \left( -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} D_\mu \phi^a \bar{D}^\mu \phi^a + \frac{1}{4} [\phi^a, \phi^b]^2 \right.
\left. + i \tilde{\gamma}^\mu D_\mu \chi + \frac{1}{2} \tilde{\gamma}^\mu \bar{\phi}^a + \bar{\phi}^a \chi \right), \quad (7)$$

where $\chi$ is a six-dimensional Weyl fermion and $\alpha, \beta = 5, 6$
(this compact notation reflects the six-dimensional origin of
the theory). This theory preserves color-kinematics duality
because it can be realized as the orbifold of a spontane-
ously broken pure $\mathcal{N} = 2$ supersymmetric Yang-Mills
(SYM) theory. Indeed, we begin with the SU$(N + M)$
$\mathcal{N} = 2$ SYM theory and spontaneously break the gauge
group to $G = \text{SU}(N) \times \text{SU}(M) \times \text{U}(1)$ by introducing a VEV

$$\langle \phi^a \rangle = \bar{V}_a \times \text{diag} \left( \frac{1}{N} \mathbb{1}_N, -\frac{1}{M} \mathbb{1}_M \right),$$

which is chosen to have the same magnitude as the one in
the left gauge theory, $(V^a)^2 = (\bar{V}_a)^2$, so that the two
theories have common mass spectra. Conjugation by the matrix \( \chi = \text{diag}(\gamma_5, -\gamma_5) \) is a symmetry of the Lagrangian and so is the sign flip of the fermion fields. We may therefore orbifold by their composition

\[ A_\mu \mapsto \gamma A_\mu \gamma^{-1}, \quad \chi \mapsto -\gamma \chi \gamma^{-1}, \quad \varphi \mapsto \gamma \varphi \gamma^{-1}. \]  

Since, as shown in [20,25], each of these operations preserves color-kinematics duality, so must the resulting theory. Its Lagrangian is that of Eq. (7) and its spectrum is

\[ \text{GT}_R: \{ A_\pm, \varphi_a \}_G \oplus \{ \chi \}_R \oplus \{ \bar{\chi} \}_R. \]  

Gauging U(1)\(_R\) in \( N = 2 \) supergravities.—General ME supergravity theories with \( N = 2 \) supersymmetry in five dimensions were constructed in Refs. [30,31]. Their gaugings were studied in Refs. [32,33]; gaugings that require dualization of some of the vector fields to tensor fields were constructed later [34,35]. Four-dimensional ME supergravities and their gaugings were studied in Refs. [36–40] (see [2] for further references). The fields of 5D ME supergravity (MESG) with \( n \) vector multiplets are

\[ \text{MESG:} \{ e^m_\mu, \Psi_i^a, A_\mu^I, \lambda^a, \varphi^s \}. \]  

where \( I = 0, \ldots, n; a, x = 1, \ldots, n, \) and \( i, j = 1, 2 \) are \( R \) symmetry indices [30]; ME theories are completely specified by the cubic polynomial \( \mathcal{V}(\xi) = (2/3)^{3/2} C_{IJK} \xi^I \xi^J \xi^K \), where \( \xi^I \) are coordinates of a \( (n + 1) \)-dimensional ambient space and \( C_{IJK} \) is a constant symmetric tensor. The scalar fields parametrize the \( \mathcal{V}(\xi) = 1 \) hypersurfaces in this ambient space. The metric \( \bar{\alpha}_{IJ} \) of the kinetic energy term of the vector fields is given by the restriction of the ambient-space metric to this hypersurface; it is written in terms of the vielbeine \( (h_I, h^i) \) as \( \bar{\alpha}_{IJ} = h_I h_J + h_I^i h_J^i \) (see Ref. [30] for explicit expressions). Thus, as relevant for the amplitude perspective, theories in the ME class are uniquely specified by their spectra and three-point interactions.

In this Letter, we will focus on the ME supergravities belonging to the generic Jordan family with symmetric target spaces in five and four spacetime dimensions. They have \( n > 1 \) vector multiplets and are defined by the cubic polynomial \( \mathcal{V}(\xi) = \sqrt{2} \xi^2 [(\xi^I)^2 - (\xi^J \xi^K)] \). Their double copy construction is given in Ref. [24].

As shown in Refs. [32,35], it is possible to gauge a U(1)\(_R\) subgroup of the \( R \) symmetry group SU(2)\(_R\) for all ME supergravity theories. The resulting actions admit Minkowski vacua with spontaneously broken supersymmetry. Thus, we expect them to admit a double copy construction as explained above. The relevant Lagrangians are obtained by covariantizing derivatives on the fermions with respect to the U(1)\(_R\) gauge field \( V_I A_\mu^I \) defined by an \( (n + 1) \)-dimensional constant vector field \( V_I \),

\[ D_\mu \Psi_i^a = \nabla_\mu \Psi_i^a + g_R V_I A_\mu^I \delta^a \psi_{ij}, \quad D_\mu \lambda^a = \nabla_\mu \lambda^a + g_R V_I A_\mu^I \delta^a \lambda^a, \]  

and adding the following terms to the 5D Lagrangian,

\[ \delta \mathcal{L} = -\frac{i}{8} g_R \bar{\Psi}^i \Gamma^a \Phi^i \delta_{ij} P_0 - \frac{1}{\sqrt{2}} g_R \delta^a \psi_{ij} P_0 - \frac{1}{2 \sqrt{2}} g_R \delta^a \lambda^a P_{ij} \]  

\[ + \frac{i}{2 \sqrt{6}} P_{ij} \phi^i \delta_{ij} P_0 - g_R^2 \phi^a \phi^a P_{ij}. \]

The coefficient functions \( P_0, P^a, \) and \( P_{ij} \) are given in terms of \( V_I \) as

\[ P_a(\varphi) = \sqrt{2} h_a V_I, \quad P_0(\varphi) = 2 h^i V_i, \]

\[ P_{ij} = \frac{1}{4} \delta_{ij} P_0 + 2 \sqrt{2} T_{abc} P^c, \]

with \( T_{abc} = C_{IJK} h_I^a h_J^b h_K^c \). The scalar potential \( P^{(R)}(\varphi) \) is given by [32,35]

\[ P^{(R)}(\varphi) = (P_0)^2 + P_a P^a = -4 C_{IJK} V_I V_J h_K. \]

The deformation breaks the \( R \) symmetry down to a U(1)\(_R\) subgroup. Minkowski vacua correspond to gauge with vanishing potentials, \( P^{(R)}(\varphi) = 0 \); they break supersymmetry spontaneously [32,35]. Up to rotations and overall rescaling, the simplest choice of \( V_I \) leading to theories with this property is \( V_I^{(0)} = (0, 1, \pm 1, 0, \ldots, 0) \) [41]. This choice breaks the global symmetry down to the Euclidean group \( E(n-2) \) for \( (n-2) \) internal dimensions. To study the spectrum of the theory, it is convenient to redefine the massive gravitini as

\[ \bar{z}^i = \Psi^a - \frac{i}{\sqrt{12}} \Gamma^a \lambda^a P_0 P_0 + \frac{\sqrt{2}}{g_R P_0} D_\mu \left( \frac{P_a\lambda^a}{P_0} \right). \]

After this operation, the Goldstino field \( \eta^i = \lambda^a P_a / P_0 \) no longer appears in the Lagrangian, having become the longitudinal component of the gravitino which is now massive (this is analogous to the unitary gauge in spontaneously broken YM theories). Mass matrices for gravitini and remaining spin-1/2 fields can be written as

\[ M_{ij} = \frac{\sqrt{6}}{4} g_R P_0 \delta_{ij}, \quad M_{ab} = \frac{g_R}{\sqrt{6}} \left( P_{ab} - \frac{5 P_a P_b}{2} P_0 \right) \delta_{ij}. \]

Taking into account that the nonvanishing coefficient matrices at the scalar base point are \( P_0|_{c_i} = 2 \sqrt{2}/3, \)
The combination on the first line is manifestly transverse and gamma traceless. $U$ is a unitary matrix diagonalizing the spin-1/2 mass terms and the index $s = 2, 3, \ldots, n$ runs over all spin-1/2 fields except the Goldstino. Since the $U(1)_R$ gauging affects only the fermionic terms in the Lagrangian, the double copy origin of the vector fields will be the same as for the ungauged construction [24]

\begin{align*}
A^{\pm}_+ &= A_+ \otimes \phi^0 \pm \phi^2 \otimes A_+.
\end{align*}

(22)

In order to match the amplitudes from the double copy with the ones from the supergravity Lagrangian, we employ the massive spinor-helicity formalism, writing massive momenta as $p_i = p_i^\pm - (i/2)q \cdot g_R q$. Here $q$ is a reference momentum and $p_i^\pm, q$ are both massless. Polarizations for massive spinors are written as $v_i^\pm = (\pm 1, m\{q\} / (i^2 q \cdot i^2 q))$ and $v_i^\pm = (m^2 q / (i^2 q \cdot i^2 q))$. Explicit expressions for the massive-vector polarizations can be found in [42] (see also [43]). We consider massive gravitini with ± polarizations and rewrite selected gravitini-vector amplitudes as $(I = 0, \ldots, n)$

\begin{align*}
M_3(1\xi^+, 2\xi^-, 3A^+) &= -\sqrt{2}i m\Omega V^{(+)} (\xi^+ (2^+ q / (1^+ q)),
\end{align*}

(23)

We note that, aside from the gravitino minimal coupling, the first amplitude has a contribution coming from a cubic interaction of the form $2i H^{-1} \equiv P \chi$, where $P \equiv \chi$ denotes the chirality projector and $A_R^{-1}$ is the gravitopho. The overall factor of $\Omega = 3^{-1/2}((1^+ 2^+ 3^+))$ is equal to the gauge-theory amplitude between two massive vectors and one massless vector. The $(n + 2)$-dimensional vector $V_A$ defines the choice of $U(1)_R$ gauge vector and is given in (19). This result matches the one from the double copy, provided that the gauge-theory VEVs are

\begin{align*}
V_a &= (0, m), \quad V^{(\pm)}(a) = (0, \pm m, 0, \ldots, 0).
\end{align*}

(24)

The magnitude of the VEVs in the two gauge theories determines the supergravity parameter $g_R$ or, alternatively, the masses of gravitini and spin-1/2 fields. Similarly, the direction of the gauge-theory VEVs is identified with the supergravity vector $V_A$ which defines the $U(1)_R$ gauge field. From the point of view of the underlying gauge theories, the vanishing of the first entry in each VEV arises because the scalar fields $\phi^1$ and $\phi_5$ descend from the 5D

\[ P_1 |_{c_1} = -\sqrt{2/3} = P_{11} |_{c_1}, \quad P_2 |_{c_2} = \pm \sqrt{2}, \quad P_{12} |_{c_1} = \pm 2 \sqrt{2}, \]

\[ P_{22} |_{c_2} = \sqrt{6}, \quad P_{31} |_{c_1} = -1 \quad (s = 3, \ldots, n) \]

immediately follows that the masses of the two gravitini and one pair of spin-1/2 fermions are $m = g_R$. The remaining nonzero fermion masses are equal to $-g_R$.

A direct comparison of double copy amplitudes with supergravity calculations requires that we properly identify the mass dependence (i.e., the dependence on $g_R$ in supergravity). Apart from its explicit appearance in the Lagrangian, in both gauge theory and supergravity, the mass is also hidden in the massive particle wave functions. To expose it, we shall use spinor-helicity notation and reduce the supergravity Lagrangian to four dimensions. For the 5D spinors rewritten as Dirac spinors, the reduction is straightforward. The reduction of a massive gravitino yields the 4D gravitino $\xi_\mu$ and a further spin-1/2 field $\xi$. The precise decomposition of the 5D gravitino is chosen such that the 4D quadratic terms are canonically normalized.

To obtain diagonal kinetic terms for the bosons in the 4D Lagrangian, we dualize the vector $A^{-1}_\mu$ from dimensional reduction of the graviton, and redefine fields as

\[ \begin{pmatrix} A^{-1}_\mu \\ A^0_\mu \\ A^+_\mu \end{pmatrix} = -\frac{1}{4} \begin{pmatrix} -1 & 1 & 2 \sqrt{2} \\ 2 & -2 & 2 \sqrt{2} \\ 2 \sqrt{2} & 2 \sqrt{2} & 0 \end{pmatrix} \begin{pmatrix} A^{-1}_\mu \\ A^0_\mu \\ A^-_\mu \end{pmatrix}. \]

(18)

After this operation, $A^{-1}_\mu$ is the 4D graviphoton and the vector identifying the $U(1)_R$ gauge boson is expressed as

\[ V_A^{(\pm)} = (V_-, V_+^{(\pm)}) = \left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0, \pm 1, 0, \ldots, 0\right). \]

(19)

Supergravity amplitudes can now be straightforwardly computed from the Lagrangian and matched with the ones from the double copy method. We focus in particular on the amplitudes involving the $U(1)_R$ gauge field and two gravitini, which have the form in Eq. (1), where $t^\pm_{ij}$ is replaced by the identity (note that the polarization vector-spinors $v_{ij}$ need to be transverse and gamma traceless). Such amplitudes can be reproduced with the following double copy field map for the fermions

\[ \xi_\mu = W_\mu \otimes \chi - W_\mu \otimes \left(\frac{\gamma_\mu}{3} - \frac{i p_\mu}{3m}\right) \gamma^\perp \chi, \]

\[ \xi = W_\mu \otimes \gamma^\perp \chi. \]

(20)

The combination on the first line is manifestly transverse and gamma traceless. $U$ is a unitary matrix diagonalizing the spin-1/2 mass terms and the index $s = 2, 3, \ldots, n$ runs over all spin-1/2 fields except the Goldstino. Since the $U(1)_R$ gauging affects only the fermionic terms in the Lagrangian, the double copy origin of the vector fields will be the same as for the ungauged construction [24]

\[ A^{\pm}_+ = A_+ \otimes \phi^0 \pm \phi^2 \otimes A_+. \]

(21)

The factor of $i$ arises because the double copy is most naturally formulated in a symplectic frame with $SO(n)$ compact isometry, which differs from the one singled out by dimensional reduction by the dualization of one vector field. The gauge boson defined by (19) has the following simple double copy realization

\[ A^{V(\pm)} = -A_+ \otimes \phi^0 \pm \phi^2 \otimes A_+. \]

(22)
Breaking, as well as extensions to including hypermultiplets and partial supersymmetry may be generalized in several other directions, such as

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References


[28] One may consider more general double copies in which the gauge theories have different gauge groups, provided that the pairing of the representations obeys certain consistency requirements.

[41] In addition to $V^{(c)}_I$, the choice $V_I = (\sqrt{2}, 0, \ldots, 0)$ leads to flat gauging and will be considered in forthcoming work.

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