An Experiment to Search for $\nu_\mu + \nu_\tau$ Neutrino Oscillations
Using an Enriched ($\nu_e/\bar{\nu}_e$) Beam

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ABSTRACT

We propose to construct an enriched $\nu_e/\bar{\nu}_e$ beam using $K^+$ decays
and the sign selected bare target beam elements. The ($\nu_e/\bar{\nu}_e$) beam
and the 15' bubble chamber filled with heavy neon will be used to
search for $\nu_\tau$ interactions arising from $\nu_e + \nu_\tau$ or $\nu_\mu + \nu_\tau$ neutrino
oscillations. Using the present analysis of neutrino oscillations
from Barger, et al., we find that $\lesssim 250$ $\nu_\tau$ interactions could be
observed in a 500,000 picture exposure depending on the neutrino
mixing parameters. If oscillations are observed, this experiment
would also establish the existence of the $\nu_\tau$ neutrino. The energy
of the primary proton beam is 400 GeV, although as an energy saving
alternative this experiment could also be operated at a reduced
machine energy of 200 GeV with an increased repetition rate.

*Università di Padova's participation is subject to Italian
authorities' approval.
Contents

1. Introduction
2. Search for $\nu_{\tau} \rightarrow \tau^-$ in the bubble chamber
3. The enriched $\nu_e/\bar{\nu}_e$ beam using the BTSS beam
4. Event Rates and Detection Efficiency
5. Appendices
   A. Possible Indications of Neutrino Oscillations, V. Barger, et al
   B. Mass and Mixing Scales of Neutrino Oscillations, V. Barger, et al.
1. Introduction

A recent analysis of existing neutrino oscillation data suggested the possibility that neutrino oscillations may have been observed, but no conclusive evidence exists at present (Appendix A)\(^1\). The most interesting possibility suggested by this analysis is that \(\nu_e + \nu_\tau\) oscillations may exist with a fairly large mixing angle and \(\delta m^2\)\(^1,2\). All previous experiments were insensitive to this kind of oscillation because of low \(\nu_e\) flux or very large \(\nu_\mu\) flux that produces a large background. We propose to construct an enriched \(\nu_e, \bar{\nu}_e\) beam with a large \(\nu_e/\nu_\mu\) ratio, compared to ordinary beams, in order to carry out a conclusive search for \(\nu_e + \nu_\tau\) oscillations. Figure 1 shows the range of sensitivity required to deserve \(\nu_e + \nu_\tau\) for the two solutions suggested in the analysis of Barger, et al\(^1,2\). We emphasize that 10 - 30 GeV (\(\nu_e, \bar{\nu}_e\)) beams with long flight paths (> 1 KM) and considerable purity from large \(\nu_\mu, \bar{\nu}_\mu\) flux are necessary to establish the signal of \(\nu_e + \nu_\tau\). Also the present data allow \(P(\nu_e + \nu_\tau)\) between the lower and upper kinks given by beam dump experiments (Fig. 1).

We propose to construct a \(K^0_L\) beam in the normal neutrino decay channel. Using the Bare Target Sign Selected train load, a modest, inexpensive beam defining system is available\(^3,4,5\). This system is shown schematically in Figure 2. The most suitable detector to perform this search is the 15' bubble chamber with a neon-hydrogen filling.
The use of the heavy liquid bubble chamber has the following well-known advantages:

1) excellent identification of electrons with both sign and momentum determination.

2) good efficiency in observing and measuring neutrino interactions down to very low energies \(\sim 500\) MeV/c.

3) excellent visibility of the vertex. It might be possible to see the \(\tau\) lepton decay vertex depending on \(\tau\) momentum spectra and lifetime. The addition of the high resolution camera to the stereo triad would considerably enhance this possibility.

4) unbiased data taking. The detection of events is independent of the kinematic characteristics and/or the event energy.
$\nu_\alpha \rightarrow \nu_\beta$ In The Small $L/E$ Region

CERN Beam Dump

$\nu_e \rightarrow \nu_\tau (C)$ (Mean)

$\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ Limit

$\nu_\mu \rightarrow \nu_e$ Limit

$20 \text{ GeV } \nu_e / 1000 \text{ m path}$

$10 \text{ GeV } \nu_e / 1000 \text{ m path}$

$6 \text{ GeV } \nu_e / 1200 \text{ m path}$

$\nu_\mu \rightarrow \nu_e (C)$

$\nu_\mu \rightarrow \nu_e (A)$

$P(\nu_\alpha \rightarrow \nu_\beta)$

$10^{-1}$

$10^{-2}$

$10^{-3}$

$10^{-4}$

$L/E$ Meter/MeV

FIGURE 1
Special $\nu_e, \bar{\nu}_e$ Beam
For Neutrino Oscillation Search

FIGURE 2
2. The Search for $\nu_{\tau} + \tau$ in the Bubble Chamber

We propose to detect the presence of $\nu_{\tau}$, $\bar{\nu}_{\tau}$ in the $\nu_e$, $\nu_\mu$ beam through the signature

$$\nu_{\tau} + N \rightarrow \tau + x$$

$\rightarrow$ lepton + missing transverse momentum

$$+ \tau + x$$

$\rightarrow$ hadrons + missing transverse momentum

This is schematically shown in Figure 3. Albright et al.\textsuperscript{6} and Barger et al.\textsuperscript{7} have made extensive calculations for the backgrounds to these signatures from charmed particle production (i.e.)

$$\nu_\mu + N \rightarrow \mu + \text{charm} + x$$

$\rightarrow$ lepton

$$+ \nu_\mu + \text{charm pair} + x$$

In normal neutrino beams the ratio of $\nu_\mu$ flux to $\nu_e$ flux is about 100 to 1. This is about the same ratio as the ratio of lepton pair production by $\nu_\mu$ through charm. About one order of magnitude smaller there appears to be evidence for events with same sign leptons which may be due to charm pair production. At some level all of these processes may contribute to fake $\nu_{\tau} + \tau$ events. Thus it is essential to reduce the large $\nu_\mu$ flux in a $\nu_e$ beam to reduce these backgrounds. The missing transverse momentum distributions for $\tau^\pm$ production and decay are shown in Figure 4 for leptonic and $\nu_{\tau} + \text{hadronic}$ decays.

We have estimated the expected background contribution for

$$\nu_e + N \rightarrow e^\pm + \text{mismeasured hadrons}$$
from existing data and find that less than $3 \times 10^{-3}$ of $\nu_e$, $\bar{\nu}_e$ interactions should have a "fake" missing $P_L$. Thus it appears possible to identify $\tau^\pm$ to this level in the bubble chamber.

The expected cross section and $y$ distribution for production at these low energies are shown in Figure 5 and Figure 6 respectively\(^8\). The characteristic $y$ distribution might provide additional evidence that $\tau^\pm$ have been observed. The relevant variable in neutrino oscillations is $L/E_\nu$ (the $\nu$ path length/neutrino energy). Since the path length is fixed, the important measurements are the rates as a function of neutrino energy. No other detector combines the sensitivity and precision at low neutrino energies as does the heavy liquid bubble chamber.
\( \tau \) Signatures

(a)

(b)

FIGURE 3
\[ \tau^- \rightarrow e^- + \bar{\nu}_e + \nu_{\tau} \]

\[ \tau^- \rightarrow \nu_{\tau} + \text{hadrons} \]

**FIGURE 4**
3. The Enriched $\nu_e/\bar{\nu}_e$ Beam

We propose to use $K^0_L$ decay to provide an enriched $\nu_e/\bar{\nu}_e$ beam. Practical beams were discussed some time ago. We follow closely the description by Mori, et al$^4,5$:

The Sign Selected Bare Target (SSBT) train used for the HPWF neutrino experiment can easily be modified for an electron neutrino beam. Figure 7 gives a schematic layout of the arrangement$^4$. A modification required for the SSBT Beam is to move the dump a few inches to allow the $K^0_L$ beam to enter the decay pipe.

Figure 8 shows a calculated electron neutrino or antineutrino flux from the $K^0_L \rightarrow \pi^- e^+ \nu_e$ (or $\pi^+ e^- \bar{\nu}_e$) decay by a Monte Carlo program. The incident proton energy is 400 GeV. The $K^0_L$ production is assumed to be the average $K^+$ and $K^-$ production. Stefanski-White parameterization was used. Figure 9, 10 shows the spectra for 200 GeV protons. The muon neutrino background from the $\pi^+$ and $K^+$ decays is also shown in Figure 9, 10. This background is relatively independent of the incident proton energy. The muon antineutrino background from the $\pi^-$ and $K^-$ decays is substantially smaller than the muon neutrino background. The muon neutrino or antineutrino flux from the $K^0_L \rightarrow \pi^- \mu^+ \nu_\mu$ (or $\pi^+ \mu^- \bar{\nu}_\mu$) is 70% of the electron neutrino or antineutrino flux from the $K^0_L$ decay. The muon neutrino or antineutrino backgrounds from pion decays of the $K^0_S \rightarrow \pi^+ \pi^-$ are estimated to be relatively small in the present arrangement.

In summary, computed electron neutrino fluxes for the present electron neutrino beams are shown in Figures 8, 9, 10 for the incident proton energies of 400 and 200 GeV.
BTSS Beam Set for $K^0_L$

C1: Magnet (M1) Protection Collimator
C2, C3, C4: ± 2 Mrad XY Collimators (No Cooling)
Dumps: 3 Meter Long Aluminum Blocks (Water Cooled)

FIGURE 7
$K^0 \rightarrow \pi^0 e \nu_e$

$E_p = 400$ GeV

$\nu_e, \bar{\nu}_e$

Neutrinos/BeV/M$^2$/10$^{13}$ Interaction Protons

$E_\nu$(GeV)

FIGURE 8
$\nu_e, \bar{\nu}_e$ flux with $\nu_\mu$ background

$E_P = 200$ GeV

$\nu_\mu$ Background

$10^5$

$10^4$

$10^3$

Neutrinos/GeV/m$^2$/10$^{13}$ Interacting Protons

$E_\nu$ (GeV)

$K_S^0$

$K^+$

$\nu_e, \bar{\nu}_e$

FIGURE 9
\( \nu_e, \bar{\nu}_e \) flux with \( \bar{\nu}_\mu \) background

\[ E_P = 200 \text{ GeV} \]

FIGURE 10
4. Event Rates and Detection Efficiency

We propose to fill the bubble chamber with a heavy neon mixture, about 15 tons of neon in the fiducial volume, we also assume $2 \times 10^{13}$ protons per pulse. In a 500,000 picture exposure we expect 2000 $\nu_e$ interactions and 1000 $\bar{\nu}_e$ interactions for 400 GeV operation using the spectra shown in Figure 8. We expect comparable numbers for 200 GeV operation.

However, the operation of the machine in a dedicated experiment at 200 GeV primary beam energy could have several important advantages.

i) About 3-3.5 $\times 10^{13}$ proton could reliably be delivered on target with a $\approx 3$ second cycle time with no flat-top.

ii) The accelerator power requirements are greatly reduced.

iii) Such a run of about six weeks would result in $\approx 3 \times 10^{19}$ protons on target or more than 5000 $\nu_e$ events. The short duration of the run will minimize the bubble chamber operational costs and enable a prompt analysis of the data.

The threshold cross section behavior of $\tau^{\pm}$ production will suppress the low energy $\nu_\tau$, $\bar{\nu}_\tau$ rates as shown in Figure 5.

The expected detection efficiency for $\tau^+$ identification depends on the backgrounds in the bubble chamber. In principle all the decay modes of the $\tau^\pm$ could be detected either through

$$\nu_\tau + N + \tau^- + x$$

\[\text{charged lepton + missing } P_L\]

or

$$+ \tau^- + x$$

\[\text{hadron jet + missing } P_L\] double jet events
The latter process is separated from the ordinary neutral current processes
\[ \nu_{e,\mu} + N \rightarrow \nu_{e,\mu} + x \]
through the two jet signature.

For solution C we expect \( \lesssim 250 \) detected \( \tau \) events and for solution A \( \lesssim (5-10) \) events. We believe that the background rates are sufficiently low to be able to clearly separate the signal from the background in either case.
References


POSSIBLE INDICATIONS OF NEUTRINO OSCILLATIONS

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ABSTRACT

We analyze neutrino oscillations of the $\nu_e$, $\nu_\mu$, $\nu_\tau$ system. Presently available reactor antineutrino data contain indications of oscillations, that have hitherto escaped attention, corresponding to an eigenmass squared difference of $\delta m^2 = 1 \text{ eV}^2$. Two other classes of oscillation solutions are contrasted and further experimental tests are indicated. All the $\delta m^2$ must be greater than $10^{-3} \text{ eV}^2$ to explain solar and deep mine observations.
The interesting possibility of neutrino oscillations has long been recognized but no clear signal has yet been established. In this Letter we observe, however, that the reactor antineutrino data of Reines et al. are consistent with an oscillation effect of shorter wavelength than hitherto considered. Solar neutrino observations and deep mine neutrino data reinforce the indication that neutrino oscillations occur, and constrain their parameters.

Neutrino oscillations depend on differences in mass \( m_1 \) between the neutrino mass eigenstates \( \nu_i \). The latter are related to the weak charged current eigenstates \( \nu_\alpha \) (distinguished by Greek suffices) through a unitary transformation \( |\nu_\alpha > = U_{\alpha i} |\nu_i > \). Starting with an initial neutrino \( \nu_\alpha \) of energy \( E \), the probability for finding a neutrino \( \nu_\beta \) after a path length \( L \) can be compactly written (for \( E^2 >> m_1^2 \)):

\[
P(\nu_\alpha \rightarrow \nu_\beta) = \delta_{\alpha,\beta} + \sum_{i<j} 2 |U_{\alpha i}U_{\beta i}U_{\alpha j}U_{\beta j}| \left[ \cos(\Delta_{ij} - \phi_{\alpha i j}) - \cos \phi_{\alpha i j} \right] \tag{1}
\]

where \( \phi_{\alpha i j} = \arg(U_{\alpha i}^* U_{\beta i} U_{\alpha j}^* U_{\beta j}) \) and \( \Delta_{ij} = \frac{1}{2}(m_i^2 - m_j^2) L/E \). For a diagonal transition or an off-diagonal transition with CP conservation (\( U \) real), we obtain the simple formula

\[
P(\nu_\alpha \rightarrow \nu_\beta) = \delta_{\alpha,\beta} - \sum_{i<j} 4 |U_{\alpha i} U_{\beta i}^* U_{\alpha j} U_{\beta j}| \sin^2 \left( \frac{\Delta_{ij}}{2} \right) . \tag{2}\]

With \( L/E \) in \( m/\text{MeV} \) and \( m_1 \) in eV units, the oscillation argument in radians is

\[
\frac{\Delta_{ij}}{2} = 1.27 \frac{\delta m_{ij}^2}{m_1} L/E \tag{3}
\]

where \( \delta m_{ij}^2 \equiv m_i^2 - m_j^2 \). For antineutrinos replace \( U \) by \( U^* \) above.

The oscillations are periodic in \( L/E \). Oscillations arising from a given \( \delta m_{ij}^2 \) can be most readily mapped out at \( L/E \) values of order \( 1/\delta m_{ij}^2 \). The
experimentally accessible ranges of $L/E$ in m/MeV are $\sim 10^{10}$ (solar), $\sim 10^{-5}$ (deep mine), 1-7 (low energy accelerators), 1-20 (reactors), 0.3-3. (meson factories), and 0.01-0.05 (high energy accelerators). After many cycles, detectors cannot measure $L$ or $E$ precisely enough to resolve individual oscillations and are sensitive only to average values. In the limit $L/E >> (m_i^2 - m_j^2)^{-1}$ for all $i \neq j$, the average asymptotic values are given by

$$<P(\nu_\alpha \rightarrow \nu_\beta)> = \sum_i |U_{\alpha i} U_{\beta i}^*|^2.$$  (4)

Since only $\nu_e$, $\nu_\mu$, and $\nu_\tau$ neutrino types are known, we specialize to a three neutrino world. The matrix $U$ can then be parameterized in the form introduced by Kobayashi and Maskawa, in terms of angles $\theta_1, \theta_2, \theta_3$ with ranges $(0, \pi/2)$ and phase $\delta$ with range $(-\pi, \pi)$. In our present analysis we neglect CP violation (thus $\delta = 0$ or $\pm \pi$). To limit the regions of the $\theta_i$, $\delta m_{1j}^2$ parameter space, we consider first the constraints placed by solar, deep mine and accelerator data.

**Solar neutrino observations and deep mine experiments:** The solar neutrino data suggest that $<P(\nu_e \rightarrow \nu_e)> \approx 0.3-0.5$ at $L/E \sim 10^{10}$ m/MeV. For three neutrinos, Eq. (4) gives

$$<P(\nu_e \rightarrow \nu_e)> = c_1^4 + s_1^4 c_3^4 + s_1^4 s_3^4$$  (5)

where $c_1 = \cos \theta_1$ and $s_1 = \sin \theta_1$. The minimum value of Eq. (5) is 1/3 and this requires $c_1 = 1/\sqrt{3}$, $c_3 = 1/\sqrt{2}$. For $<P(\nu_e \rightarrow \nu_e)>$ to be near its minimum, all mass differences must satisfy $\delta m^2 >> 10^{-10} \text{ eV}^2$. At this minimum all transition averages are specified, independent of $\theta_2$; in particular $<P(\nu_\mu \rightarrow \nu_\mu)> = 1/2$. In fact, there are indications from deep mine experiments that $<P(\nu_\mu \rightarrow \nu_\mu) > \sim 1/2$ (see footnote f in Table 1). Since the $\nu_\mu$
neutrinos detected in deep mines have traversed terrestrial distances, this
measurement suggests that all $\delta m^2 \gtrsim 10^{-3}$ eV$^2$. Based on these considerations
we may suppose that the true solution is not far from the above $\theta_1$, $\theta_3$ values.
If we only require $\langle P(\nu_e \rightarrow \nu_e) \rangle < 0.5$, then $\theta_1$ and $\theta_3$ are constrained to a re-
gion approximated by the triangle $\theta_1 > 35^\circ$, $\theta_3 > \theta_1 - 45^\circ$, and $\theta_3 < 135^\circ - \theta_1$.

$\nu_\mu$, $\nu_e$ oscillations: Stringent experimental limits exist on these trans-
itions$^{10,13}$ at L/E in the range 0.01 to 0.3 m/MeV (see Table 1). For
$\delta m^2 \ll 1$ eV$^2$, these oscillations do not appear until L/E $>> 1$ m/MeV. With a
single $\delta m^2 \gtrsim 1$ eV$^2$, these oscillations can be suppressed by choice of $\theta_2$
(if $\theta_1, \theta_3$ are taken as above).

Reactor $\overline{\nu}_e$-oscillations: The $\overline{\nu}_e$ flux at distances L = 6 m and 11.2 m from a
reactor core center was measured by Reines et al.$^{3,4}$ using the known cross
section for the inverse beta-decay reaction $\overline{\nu}_e p \rightarrow e^+ n$. The reactor $\overline{\nu}_e$ flux
at the core has been calculated using semi-empirical methods.$^{14-17}$ The
ratio of measured flux at L to the calculated flux measures $P(\overline{\nu}_e \rightarrow \overline{\nu}_e)$. Neutrino
oscillation interpretations of the data thereby depend on the calculated spec-
trum about which there is some uncertainty.

Figure 1 shows a comparison of the measured $\overline{\nu}_e$ flux at L = 6 m and
L = 11.2 m with calculated spectra. We note that the Avignone-1978 calcul-
ated flux$^{16}$ accommodates best the L = 6 m measurements for $E_{\overline{\nu}_e} > 6$ MeV.
The data for $P(\overline{\nu}_e \rightarrow \overline{\nu}_e)$ obtained with the Avignone-1978$^{16}$ and Davis et al.$^{17}$
calculated spectra are shown in Fig. 2. The horizontal error bars in Fig. 2
take into account the finite size of the reactor core source. We observe
that $P(\overline{\nu}_e \rightarrow \overline{\nu}_e)$ seems to follow an oscillation pattern with one node in the
range of L/E covered by the measurements. The possibility of such a solution
in which a short wavelength oscillation occurs was not considered by Reines et al.\textsuperscript{3} in their analysis of the 11.2 m data based on a similar calculated spectrum.$^{15}$

The oscillation in Fig. 2 is well-described by the formula

\[ P(\bar{\nu}_e \rightarrow \bar{\nu}_e) = 1 - 0.44 \sin^2(1.27 \text{ L}/E). \]

This corresponds to a mass difference

\[ \delta m^2 = 1 \text{ eV}^2, \]

which we can arbitrarily identify as $\delta m^2_{13}$. A non-zero $\delta m^2_{12}$ with $\delta m^2_{12} \ll \delta m^2_{13}$ is required to bring $<P(\nu_e \rightarrow \nu_e)>$ down asymptotically to the solar neutrino result. The value of $\delta m^2_{12}$ is not tightly constrained, other than the indication from deep mine measurements of $<P(\nu_\mu \rightarrow \nu_\mu)>$ that $\delta m^2_{12} \sim 10^{-3} \text{ eV}^2$. A solution which accommodates all known constraints is

\[ \delta m^2_{13} \quad \delta m^2_{12} \quad \theta_1 \quad \theta_2 \quad \theta_3 \quad \delta \]

\[ \text{SOLUTION A:} \quad 1.0 \text{ eV}^2 \quad 0.05 \text{ eV}^2 \quad 45^\circ \quad 25^\circ \quad 30^\circ \quad 0^\circ. \]

The predictions for subasymptotic transition probabilities are shown in Fig. 3.

A more conservative interpretation of the reactor $\bar{\nu}_e$ data could be that $P(\bar{\nu}_e \rightarrow \bar{\nu}_e)$ falls to around 0.7-0.8 in the range of L/E considered, but that oscillatory behavior is not established. If so, two other classes of solution are possible: Class B, where $\bar{\nu}_e \rightarrow \bar{\nu}_e$ is suppressed by the onset of a long wavelength oscillation, that may have its first node well beyond L/E = 1 m/MeV; Class C, where $\bar{\nu}_e \rightarrow \bar{\nu}_e$ is suppressed by a short wavelength oscillation, that may have many nodes below L/E = 1 m/MeV. Illustrative solutions of these classes are as follows (we emphasize that their parameters are less constrained than in Class A).
\[
\delta m_{13}^2 \quad \delta m_{12}^2 \quad \theta_1 \quad \theta_2 \quad \theta_3 \quad \delta \\
SOLUTION \ B: \ 0.15 \text{ eV}^2 \quad 0.05 \text{ eV}^2 \quad 55^\circ \quad 0^\circ \quad 45^\circ \quad 0^\circ \ (6)
\]

\[
SOLUTION \ C: \ 10 \text{ eV}^2 \quad 0.05 \text{ eV}^2 \quad 45^\circ \quad 25^\circ \quad 30^\circ \quad 0^\circ .
\]

We note that equivalent solutions to Eqs. (5) and (6) are obtained with \( \delta m_{13}^2 \leftrightarrow \delta m_{12}^2, \) \( \delta = \pi, \) and \( \theta_3 \rightarrow \frac{\pi}{2} - \theta_3 \) with \( \theta_1, \theta_2 \) unchanged. Table 1 presents a capsule summary of the present experimental limits on oscillations and summarizes predictions of solutions A, B and C for existing and planned experiments. For the L/E range of the CERN beam dump experiment, a \( \delta m^2 > 10 \text{ eV}^2 \) is required to yield an e/\( \mu \) ratio that is significantly less than unity.\(^{2b}\)

In solution C, which has a \( \delta m^2 \) in that range, the mixing angles are nearly the same as those contained in ref. 2b. Solution A has the same mixing matrix as solution C, but the smaller value of \( \delta m_{13}^2 \) leads to visible oscillations in reactor experiments rather than in high energy beam dump experiments.

**New reactor experiments:** Reactor measurements\(^3\) in the L/E range 5-20 m/MeV could provide information on \( \delta m_{12}^2. \) For \( \delta m_{12}^2 < 0.05 \text{ eV}^2 \) solutions A and C predict no appreciable deviation from a \( 1/r^2 \) fall-off of the average flux at L/E > 5 m/MeV.

**New meson factory experiments:** Since the decays of stopped \( \mu^+ \) mesons provide well-known \( \nu_e \) and \( \bar{\nu}_\mu \) spectra, meson factory experiments at L/E \( \sim 1-3 \text{ m/MeV} \)\(^{24}\) could confirm the existence of \( \nu_e \leftrightarrow \nu_e \) oscillations and place further constraints on \( \bar{\nu}_\mu \leftrightarrow \bar{\nu}_e \) oscillations.

**Summary:** Reactor \( \bar{\nu}_e \) data provide indications of neutrino oscillations with mass scale \( \delta m^2 = 1 \text{ eV}^2. \) Solar and deep mine results suggest that the other mass scale is in the range \( \delta m^2 > 10^{-3} \text{ eV}^2. \)
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24. Considerations are in progress by D. Cline and B. Burman for such a neutrino oscillation experiment at LAMPF.
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a San Onofre reactor experiment by Reines et al.\textsuperscript{3} in progress, with L = 25-100 m.

b Possible LAMPF experiment with E\textsubscript{\nu} = 30-50 MeV and L = 30-100 m.

c Brookhaven experiment\textsuperscript{18} in data analysis stage.

d $\nu_\text{e} \rightarrow \nu_\tau$ oscillations can lead to a e/\mu ratio different from unity in beam dump experiments (see e.g., ref. 2b and data of ref. 19).

e The excellent agreement of observed and calculated $\nu_\text{e}/\overline{\nu}_\text{e}$ flux at CERN and Fermilab indicates that most of the $\nu_\text{e}$ does not oscillate into $\nu_\tau$.

f Deep mine experiments\textsuperscript{7,8} have detected about 130 neutrino events (E \~ 10\textsuperscript{4}-10\textsuperscript{6} MeV, L \~ 10\textsuperscript{6}-10\textsuperscript{7} m). An unaccountably large number of multitrack events were observed in the Kolar gold field experiment;\textsuperscript{7} assuming that these are not attributed to $\nu_\mu$, the event rate is about half the expected rate. In the Johannesburg mine experiment\textsuperscript{8} a ratio 1.6 \pm 0.4 of expected to observed $\nu_\mu$ events was found. The analysis in ref. 9 of these experiments is consistent with $\langle P(\nu_\mu \rightarrow \nu_\mu) \rangle \sim 0.5$. A new deep mine experiment is operating at Baksan, USSR which is sensitive to $\nu_\mu$ flux through the earth.\textsuperscript{20}

Deep mine experiments in construction\textsuperscript{21} will detect neutrinos of energies E\textsubscript{\nu} \~ 10\textsuperscript{2}-10\textsuperscript{3} MeV using very large water detectors placed in deep mines. At these energies the composition\textsuperscript{22} of the $\nu$-flux from $\pi$, $K$, and $\mu$ decays of the secondary cosmic ray component in the atmosphere is roughly ($2\nu_\mu + \nu_\text{e}$)/3. Upward events in the detector will have $L \approx 10\textsuperscript{6}-10\textsuperscript{7}$ m and downward events will have $L \approx 10\textsuperscript{4}$ m. The charged-current scattering of $\nu_\text{e}$ on electrons significantly modifies vacuum oscillation predictions only for deep mine events which have $E(\text{MeV}) \geq 10\textsuperscript{6} \delta m^2 (\text{eV}^2)$; see ref. 23.
<table>
<thead>
<tr>
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<th>L/E MeV</th>
<th>Present Limit</th>
<th>A</th>
<th>Solution B</th>
<th>C</th>
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<td>&gt; 0.85 e</td>
<td>1.0</td>
<td>1.0</td>
<td>0.9</td>
</tr>
<tr>
<td></td>
<td>M 12</td>
<td>0.3</td>
<td>$1.1 \pm 0.4$</td>
<td>0.95</td>
<td>1.0</td>
<td>0.8 mean</td>
</tr>
<tr>
<td></td>
<td>M b</td>
<td>1-3</td>
<td>0.6-1.0</td>
<td>0.8-1.0</td>
<td>0.8 mean</td>
<td></td>
</tr>
<tr>
<td>$P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$</td>
<td>M 12</td>
<td>0.3</td>
<td>&lt; 0.04</td>
<td>$10^{-4}$</td>
<td>$10^{-3}$</td>
<td>$10^{-3}$</td>
</tr>
<tr>
<td></td>
<td>M b</td>
<td>3</td>
<td></td>
<td>0.03</td>
<td>0.11</td>
<td>0.03</td>
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<tr>
<td>$P(\nu_\mu \rightarrow \nu_e)/P(\nu_\mu \rightarrow \nu_\mu)$</td>
<td>A 10,11</td>
<td>0.04</td>
<td>&lt; $10^{-3}$</td>
<td>$10^{-6}$</td>
<td>$10^{-5}$</td>
<td>$10^{-4}$</td>
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<tr>
<td></td>
<td>A 18 c</td>
<td>1-7</td>
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<td>0-0.8</td>
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</tr>
<tr>
<td>$P(\nu_e \rightarrow \nu_{\tau}$)</td>
<td>A d</td>
<td>0.04</td>
<td>&lt; 0.2 e</td>
<td>$10^{-3}$</td>
<td>$10^{-5}$</td>
<td>0.1</td>
</tr>
<tr>
<td>$P(\nu_\mu \rightarrow \nu_{\tau})/P(\nu_\mu \rightarrow \nu_\mu)$</td>
<td>A 13</td>
<td>0.04</td>
<td>&lt; 2.5 x $10^{-2}$</td>
<td>$10^{-5}$</td>
<td>$10^{-5}$</td>
<td>$10^{-3}$</td>
</tr>
<tr>
<td>$&lt;P(\nu_\mu \rightarrow \nu_\mu)&gt;$</td>
<td>D f</td>
<td>$10^{2-10^{3}}$</td>
<td>$\sim 0.5$</td>
<td>0.51</td>
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<tr>
<td>$&lt;P(\nu_c \rightarrow \nu_\mu)&gt;$</td>
<td>D g</td>
<td>$10^{3-10^{5}}$</td>
<td>0.48</td>
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<td>0.48</td>
<td></td>
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<tr>
<td>$&lt;P(\nu_c \rightarrow \nu_e)&gt;$</td>
<td>D g</td>
<td>$10^{3-10^{5}}$</td>
<td>0.42</td>
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<tr>
<td>$P(\nu_c \rightarrow \nu_\mu)$</td>
<td>D g</td>
<td>$10^{2-10^{2}}$</td>
<td>0.3-0.7</td>
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<tr>
<td>$P(\nu_c \rightarrow \nu_e)$</td>
<td>D g</td>
<td>$10^{2-10^{2}}$</td>
<td>0.2-0.6</td>
<td>0.2-0.6</td>
<td>0.2-0.6</td>
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Notation: S(solar), R(reactor), M(meson factory), A(accelerator), D(deep mine); $\nu_c = (2\nu_\mu + \nu_e)/3$. 
FIGURE CAPTIONS

Fig. 1  The $\bar{\nu}_e$ reactor flux measurements of Reines et al. at $L = 11.2$ m (ref. 3) and $L = 6$ m (ref. 4) compared with the calculated spectra of refs. 3, 4, 14-17.

Fig. 2  Transition probability $P(\bar{\nu}_e \rightarrow \bar{\nu}_e)$ versus $L/E$ deduced from the ratio of the observed to the calculated $\bar{\nu}_e$ reactor flux from refs. 16-17. The curve represents neutrino oscillations with an eigenmass difference squared of $\delta m^2 = 1$ eV$^2$ (SOLUTION A of Eq. (5)).

Fig. 3  Subasymptotic neutrino oscillations for all channels based on SOLUTION A in Eq. (5). Arrows on the right-hand side denote asymptotic mean values.
Fig. 1
Fig. 2
MASS AND MIXING SCALES OF NEUTRINO OSCILLATIONS

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ABSTRACT

The extraction of neutrino oscillation parameters from the ratio of rates \((\bar{\nu}_e d \rightarrow e^+ n n)/(\bar{\nu}_d \rightarrow \nu pn)\) is considered in the context of oscillations proposed to account for previous reactor data. The possibility that only \(\nu_e, \nu_\mu\) oscillations occur is shown to be barely compatible with present limits on \(\nu_\mu \rightarrow \nu_e\) transitions. Predictions for \(\nu_e, \nu_\mu \rightarrow \nu_\tau\) oscillations are made for future accelerator experiments.
Neutrino oscillations\textsuperscript{1-3} are of great interest because of the light they may shed on neutrino mass scales and mixing angles. The solar neutrino puzzle\textsuperscript{2-4} may indicate oscillations and a e/\mu discrepancy in the CERN beam dump experiment has also been speculated upon\textsuperscript{5}. Recently we presented\textsuperscript{6} a reexamination of all neutrino flux data, including the old reactor $\bar{\nu}_e$ measurements by Reines and collaborators.\textsuperscript{7} We drew the conclusion that the oscillation probability $P(\bar{\nu}_e \rightarrow \bar{\nu}_e)$ falls to 0.5 or lower in the neighborhood of $L/E = 1.5 \text{m/MeV}$, where $L$ is the distance from the source and $E$ is the energy. Interpreting this as an oscillation effect, we showed that neutrino-mixing with a leading mass squared difference of the order of $1 \text{eV}^2$ matched the reactor data in some detail (solution A of ref. 6). Qualitatively different classes of solutions were considered for comparison, with a $\delta m^2$ considerably smaller (solution B) or considerably larger (solution C) than $1 \text{eV}^2$.

Further evidence for neutrino oscillations from reactor experiments comes from the simultaneous consideration of charged current (CC) and neutral current (NC) deuteron disintegration reactions\textsuperscript{8}

\begin{equation}
\bar{\nu}_e d \rightarrow e^+ nn \quad \bar{\nu}_d \rightarrow \bar{\nu}_p n.
\end{equation}

The neutral current process is immune to oscillations, being the same for all types of antineutrinos in the standard theory and effectively monitors the initial $\bar{\nu}$ flux. The ratio of CC/NC rates is rather insensitive to theoretical uncertainties in the calculated $\bar{\nu}_e$ spectrum from the reactor (the principle uncertainty hitherto) and can be used to extract neutrino oscillation parameters. In the present letter we calculate the NC/CC ratios predicted for this experiment by the solutions of ref. 6 and discuss the constraint on $\delta m^2$ and mixing angles. The possibility of having oscillations only in the $\nu_e$, $\nu_\mu$ system is
shown to be barely compatible with reactor data and present experimental
limits on $\nu_\mu \rightarrow \nu_e$ transitions. For oscillations of three neutrinos, predic-
tions of $\nu_e, \nu_\mu \rightarrow \nu_\tau$ oscillations are made for future accelerator experiments.

The spectrum averaged cross sections for deuteron disintegration have
the form

$$<\sigma> = \int_0^\infty dE_r \int_{E_{\text{th}}}^\infty dE_{\nu} \rho(E_{\nu}) f(E_{\nu}) \frac{d\sigma}{dE_r}$$

(2)

where $\rho(E_{\nu})$ is the $\bar{\nu}_e$ flux at $L = 0$ and $f = P(\bar{\nu}_e \rightarrow \bar{\nu}_e)$ at $L/E_{\nu}$ for the CC
case and $f = \frac{1}{2}$ for the NC case. The variable $E_r$ is the energy of relative
motion of the final state nucleons; the recoil energy of the two-nucleon
system can be neglected to a 1% approximation. The differential cross
sections are\(^9\)

$$\frac{d\sigma}{dE_r} = \frac{g_A^2 G_F^2 M_N^{3/2}}{2\pi^3} J_d^2(E_r)(E_{\nu} - E_{\text{th}} + m)[(E_{\nu} - E_{\text{th}} + m)^2 - m^2]^{1/2} E_r^{1/2}$$

(3)

where $M_N$ is the nucleon mass and $m = m_e$ for the CC and $m = 0$ for the NC.

The threshold energies are

$$E_{\text{th}}^{\text{CC}} = 4.030 \text{ MeV} + E_r$$

$$E_{\text{th}}^{\text{NC}} = 2.225 \text{ MeV} + E_r$$

(4)

In Eq. (2) the quantity $J_d$ is the overlap integral of deuteron wave functions
describing the $^3S$ ground state and the $^1S$ continuum state, given by\(^9\)

$$J_d = \frac{1.52 \times 10^{-3}(43.1 + 0.83 E_r) \text{MeV}^{-3/2}}{(E_r + 2.225)[E_r + (0.19 E_r + 0.27)^2]^{1/2}},$$

(5)
with $E_r$ in MeV units. With the exponential fall-off of $\rho(E_\nu)$ folded in, the dominant contribution to $<\sigma>$ comes from $E_r < 0.3$ MeV and $E_\nu - E_{\nu_{\text{th}}} \approx 0.5$-3.5 MeV. Thus oscillation effects can be measured in the range 4.6 - 7.6 MeV. We calculate the ratio

$$R_d \equiv \frac{<\sigma(\bar{\nu}_e d \to e^+ n n)>}{<\sigma(\bar{\nu}_e d \to \bar{\nu}_e p n)>}$$

(6)

with and without oscillations.

At $L \approx 11.2$ m, $R_d(\text{osc})/R_d(\text{no osc})$ measures $P(\bar{\nu}_e \rightarrow \bar{\nu}_e)$ over the range $L/E \approx 1.5 - 2.4$, which is the region in which our analysis of the $\bar{\nu}_e p \rightarrow e^+ n$ data shows an oscillation effect. Figure 1 shows the predictions, assuming that only one eigenmass-difference plays a significant role in the reactor range, for which

$$P(\bar{\nu}_e \rightarrow \bar{\nu}_e) = 1 - \sin^2 2\alpha \sin^2 (1.27 \delta m^2 L/E)$$

(7)

with $\delta m^2$ in eV$^2$ units and $L/E$ in m/MeV. The curves in Fig. 1 versus $\delta m^2$ represent mixing angles for which $\sin^2 2\alpha = 0.19$ ($\alpha =$ Cabibbo angle), 0.50, 0.80 and 1.0. These calculations are based on the Avignone-1978 reactor spectrum; closely similar results are obtained with the Davis et al. spectrum. Assuming ideal acceptance and allowing one standard deviation from the measured value of

$$R_d(\text{osc})/R_d(\text{no osc}) = 0.43 \pm 0.17$$

(8)

values of $\delta m^2$ are permitted in the ranges

$$0.3 < \delta m^2 < 1.1 \text{ eV}^2 \quad \delta m^2 > 1.7 \text{ eV}^2$$

(9)
for appropriate mixing angles $\alpha$. The solution classes A and C of ref. 6 satisfy these criteria. For the preferred class A solutions, our analysis $^6$ of the $\bar{\nu}_e p \rightarrow e^+ n$ data at $L = 11.2$ m gives the ranges

$$0.80 < \delta m^2 < 1.05 \text{ eV}^2$$

(10)

$$0.4 < \sin^2 2\alpha < 0.9.$$  

Results similar to Eqs. (9) and (10) were independently obtained in ref. 8. Figure 2 shows predictions for $R_d^{(osc)}/R_d^{(no \ osc)}$ versus $L$ for other reactor experiments, based on solution A with $\delta m^2 = 0.8 \text{ eV}^2$ and the spectrum of ref. 10.

Stringent limits $^{12,13}$ exist on $\nu_\mu \rightarrow \nu_e$ and $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ oscillations at $L/E \approx 0.04 \text{ m/MeV}$ and on $\nu_\mu \rightarrow \bar{\nu}_e$ oscillations at $L/E \approx 0.3 \text{ m/MeV}$. If oscillations occurred only between $\nu_e$ and $\nu_\mu$ states and if a single $\delta m^2$ is dominant below $L/E = 3 \text{ m/MeV}$ (as in solutions A and C of ref. 6), we can write

$$P(\nu_\mu \rightarrow \nu_e) = P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e) = P_o \sin^2(1.27 \delta m^2 L/E)$$

(11)

with the experimental bound

$$P_o < 0.3/(\delta m^2)^2.$$  

(12)

The corresponding bound on the mixing angle is

$$\sin^2 2\alpha < 0.3/(\delta m^2)^2.$$  

(13)

For $\delta m^2 \sim 1 \text{ eV}^2$, this is just on the borderline of admissibility by the existing reactor data.

Applying similar considerations to oscillations of three neutrinos, probability conservation leads to the predictions
\[ P(\nu_e \rightarrow \nu_\tau) = P(\bar{\nu}_e \rightarrow \bar{\nu}_\tau) = [\sin^2 2\alpha - P_o] \sin^2 (1.27 \delta m^2 L/E). \tag{14} \]

\[ P(\nu_\mu \rightarrow \nu_\tau) = P(\bar{\nu}_\mu \rightarrow \bar{\nu}_\tau) = \left[ P_o/(4 \sin^4 \alpha) \right] P(\nu_e \rightarrow \nu_\tau). \]

Thus three neutrino oscillations can be tested by detecting \( \nu_\tau, \bar{\nu}_\tau \) produced in a \( \nu_e, \nu_\mu \) beam that is free of \( \nu_\tau \) and \( \bar{\nu}_\tau \). The \( \nu_\tau, \bar{\nu}_\tau \) are detected through the interactions

\[ \nu_\tau n \rightarrow \tau^- X^+ \tag{15} \]

\[ \bar{\nu}_\tau p \rightarrow \tau^+ X^0. \]

Figure 3 shows predictions for \( P(\nu_e \rightarrow \nu_\tau) \) for the L/E range of high energy accelerators (the two curves for solution A corresponding to \( P_o \) between 0 and 0.3, with \( \delta m^2 = 1 \text{ eV}^2 \)). \( P(\nu_e \rightarrow \nu_\tau) \) depends critically on \( P_o \) and can be larger than \( P(\nu_e \rightarrow \nu_\tau) \).

Solution C has been of primary interest in connection with the e/\( \mu \) ratio of beam dump experiments.\(^5\) By increasing the scale of the \( \delta m^2 \), it is possible to construct an alternate version of solution C (solution C') which can explain both reactor and beam results by having a short wavelength oscillation (such as \( \delta m^2_{13} \approx 50 \text{ eV}^2 \)) superimposed on a long wavelength oscillation (\( \delta m^2_{12} \approx 1 \text{ eV}^2 \)). Representative six-quark mixing angles for such a solution are

\[ \theta_1 = 30^\circ \quad \theta_2 = 50^\circ \quad \theta_3 = 55^\circ \quad \delta = 0^\circ. \tag{16} \]

Solution C' gives \( R_d(\text{osc})/R_d(\text{no osc}) = 0.59 \) at \( L = 11.2 \text{ m} \); the minimum value of \( P(\bar{\nu}_e \rightarrow \bar{\nu}_e) \) in the reactor range is 0.47 when averaged over the short wavelength oscillation. Predictions of \( P(\nu_e \rightarrow \nu_\tau) \) for this solution are also given in Fig. 3.
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FIGURE CAPTIONS

Figure 1  Neutrino oscillation results for the quantity $R_d(\text{osc})/R_d(\text{no osc})$ with $R_d$ as defined in Eq. (6).

Figure 2  Predictions for $R_d(\text{osc})/R_d(\text{no osc})$ versus distance $L$ from the reactor core, based on solution A with $\delta m^2 = 0.8$ $\text{eV}^2$.

Figure 3  Predicted $\nu_e \rightarrow \nu_\tau$ transition probability for the L/E range of high energy accelerators. The $\delta m^2$ values are $1 \text{ eV}^2$ for solution A, $10 \text{ eV}^2$ for C, $50 \text{ eV}^2$ and $1 \text{ eV}^2$ for C'.

$\sin^2 2\alpha$

$R_d \equiv \frac{\langle \sigma (\nu_e \rightarrow e^\pm n) \rangle}{\langle \sigma (\nu_d \rightarrow \nu_i p) \rangle}$

$\delta m^2 (\text{eV}^2)$

$R_{(\text{osc})}/R_{(\text{no osc})}$