Phase-transition sound of inflation at gravitational waves detectors

Yu-Tong Wang a, Yong Cai a,b,*, Yun-Song Piao a,c

a School of Physics, University of Chinese Academy of Sciences, Beijing 100049, China
b Department of Physics and Astronomy, University of Pennsylvania, Philadelphia, PA 19104, USA
c Institute of Theoretical Physics, Chinese Academy of Sciences, P.O. Box 2735, Beijing 100190, China

A R T I C L E   I N F O
Article history:
Received 14 September 2018
Received in revised form 13 December 2018
Accepted 13 December 2018
Available online 18 December 2018
Editor: M. Trodden

A B S T R A C T
It is well-known that the first-order phase transition (PT) will yield a stochastic gravitational waves (GWs) background with a peculiar spectrum. However, we show that when such a PT happened during the primordial inflation, the GWs spectrum brought by the PT will be reddened, which thus records the unique voiceprint of inflation. We assess the abilities of the GW detectors to detect the corresponding signal.
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1. Introduction

Inflation is the current paradigm of early universe [1], [2], [3], [4]. It predicts nearly scale-invariant scalar perturbation, which is consistent with the cosmic microwave background (CMB) observations, as well as the gravitational waves (GWs).

The primordial GWs background spans a broad frequency-band, \(10^{-18} - 10^{10} \) Hz, e.g., [5], [6]. It is usually thought that the discovery of primordial GWs will solidify our confidence that inflation has ever occurred. The primordial GWs at ultra-low frequency \(10^{-18} - 10^{-16} \) Hz may induce the B-mode polarization in the CMB [7], [8]. Recently, the combination of Planck, BICEP and Keck Array's 95 GHz data has put the constraint on the tensor-to-scalar ratio to \(r_{0.05} < 0.07\) (95% C.L.) [9]. However, if \(r \ll 0.001\), detecting the primordial GWs and verifying the inflation with CMB will be extremely difficult.

Recently, the LIGO/VIRGO Scientific Collaboration, using the laser interferometer, has observed the GWs signals consistent with the binary black holes coalescence [10], [11]. This opens a new window for exploring the inflation. It is well-known that the stochastic GWs background predicted by the slow-roll inflation is too small to be detected by the GWs detectors, such as Advanced LIGO and LISA, see [12], [13] for reviews on other possible sources. How can we verify the inflation with the GWs detectors?

The initial state of sub-horizon GWs mode \(\gamma_k\) in de Sitter space is \(\gamma_k \sim \frac{1}{a^2} e^{-ikr}\), i.e., the Bunch–Davis (BD) vacuum, where the conformal time \(\tau = \int dt/a\). Its power spectrum is \(P_T(k \gg aH) \sim k^2\). However, the inflation will stretch the sub-horizon GWs mode outside the horizon. When \(\gamma_k\) is leaving the horizon \((k = aH)\), \(\gamma_k \sim \frac{H}{\sqrt{2k}} e^{-ikr}\), hereafter \(\gamma_k\) becomes superhorizon and we have \(P_T \sim k^0\). Thus one significant effect of inflation (nearly exponential expansion) is that it reddens the sub-horizon spectrum \(P_T(k \gg aH)\). Could we find this reddening effect of inflation with the GWs detectors? It is possible, if a known sub-horizon GWs background, stronger than the BD state, was produced at some time (corresponding to the wavebands of GWs detectors) during inflation.

Phase transition (PT) is ubiquitous in the early universe. In this Letter, we find that when the first-order PT happened during inflation, the GWs spectrum brought by the PT will be reddened, which thus records the unique voiceprint of inflation. Especially, we show that Advanced LIGO and LISA have abilities to detect the corresponding signals. It should be mentioned that Ref. [14] discussed the detectability of such a signal in light of CMB experiments, see also [15]. Our result suggests a novel possibility of witnessing inflation, which has not been noticed before.

2. Inflation and primordial PT GWs

Historically, it is the first-order PT in particle physics that has motivated the idea of (old) inflation [1], see also extended inflation [16], hybrid inflation [17], [18] (ended by a first-order PT, see [19], [20], [21] for comparing it with the observations), chain inflation [22], [23], [24].

Though currently the slow-roll inflation [2], [3] is popular, the PT as the birthright of inflation could always occur at certain time, see Fig. 1 (however, if the PT happened at the moment
corresponding to the CMB scale, it might be conflict with the observations of CMB and large scale structure). During the first-order PT, the lower-vacuum bubbles will nucleate with the rate \( \Gamma \sim e^{t/(t - t_c)} \), where \( t_c \) is approximately the duration of the PT. When \( \Gamma / H^2 > \frac{9}{2 \pi} \), the PT completed. The collisions of bubbles will yield a stochastic GWs background with a peculiar spectrum [26], [27], [28].

We will calculate the primordial GWs spectrum brought by the first-order PT during inflation, see Fig. 1 for the corresponding scenario. Tensor (GWs) mode \( \gamma_{ij} \) satisfies \( \gamma_{ii} = 0 \), and \( \partial_t \gamma_{ij} = 0 \), and the action of \( \gamma_{ij} \) is

\[
S^{(2)} = \int dt d^3x \frac{M^2_p}{8} \left( \frac{d \gamma_{ij}}{dt} - \nabla^2 \gamma_{ij} \right)^2 .
\]

where \( \tau = dt/a \). We have the propagating speed of GWs equal to the speed of light, or see [29], [30]. The Fourier series of \( \gamma_{ij} \) is

\[
\gamma_{ij}(\tau, \mathbf{x}) = \int \frac{d^3k}{(2\pi)^3} e^{-i(k \cdot x)} \sum_{s = \pm} \tilde{\gamma}_{ij}(\tau, \mathbf{k}) e^{is(k \cdot \mathbf{x})} .
\]

The wave equation of GWs mode \( \gamma_k \) is

\[
\frac{d^2 \tilde{u}_k}{dt^2} + \left( k^2 - \frac{a''}{a} \right) \tilde{u}_k = 0 .
\]

where \( a = \frac{M_p}{\sqrt{k^2/a}} \). Initially, the GW modes should be deep inside their horizon, i.e., \( k^2 \gg \frac{a''}{a} \), so the initial state is \( u \sim \frac{1}{\sqrt{2}} e^{-i k \tau} \).

When the PT happens, the collision of sub-horizon bubbles will bring a sub-horizon GW background, which will inevitably modify the initial state of GWs modes. The energy spectrum of PT GWs is given by [28]

\[
\Omega_{GW}^*(k) = \Omega_{GW}^{peak} \frac{(A + B) k^4_{peak}}{(A + B)_k} \left( \frac{k}{k_{peak}} \right)^{m-1} ,
\]

where \( m \) is the scaling power of the spectrum, and \( k_{peak} \) is the peak momentum of the spectrum. \( \Delta \chi_{inf} = V_{inf} - V_{inf} \) is the efficiency of converting vacuum energy into the bubble wall kinetic energy and \( k_a \) is the subscript ‘inf-a’ and ‘inf’ correspond to the quantities at and after the PT, respectively; the bubble is sub-horizon requires \( H_{inf} \beta < 1 \), and \( v_a \) is the bubble wall velocity.

The sub-horizon GWs mode modified by the PT could be written as

\[
u_a = C(k)e^{-i k \tau} ,
\]

which is the solution of Eq. (3) for \( k^2 \gg a''/a \gg a^2 H^2 \). We have that the information of PT is encoded in \( C(k) \). When \( k^2 \gg a^2 H^2 \), Eq. (3) suggests \( |C(k)|^2 = (k^2 + a^2 H^2)^2 |\gamma_k|^2 \). Thus with Eqs. (2) and (6), we have

\[
|C(k)|^2 = \frac{k^5}{\pi^2 M_p^2 H_{inf}^2 a^4} \sum \Omega_{GW}^* \Omega_{GW}^* ,
\]

where \( \rho_{inf} = 3M_p^2 H_{inf}^2 \). Thus we could get

\[
\Omega_{GW}^* = \frac{d \rho_{GW}}{\rho_{inf}^2} (d l n k)^2 = \frac{k^5}{3\pi^2 M_p^2 H_{inf}^2 a^4} |C(k)|^2 ,
\]

where \( \Omega_{GW}^* \) is set by (4). Hence, Eq. (8) is valid only in the sub-horizon limit.

The inflation after the PT will occur with \( \rho_{inf} \simeq V_{inf} \sim V_{inf} - \Delta V_{inf} \). We assumed that the energy density \( \rho_{prelased} \simeq \Delta V_{inf} \) deposits in bubble walls has been efficiently released during the collisions of bubbles [32], [33], [34], and diluted with the expansion of universe. The sub-horizon GWs (with the wavelength \( \lambda \ll H_{inf}^{-1} \ll k_{peak}^{-1} \)) brought by the PT will be stretched outside the horizon \( 1/H_{inf} \). By requiring that the solution of Eq. (3) reduces to (6) in the sub-horizon limit, we obtain

\[
u_a(\tau) = -C(k) \sqrt{\frac{2\pi}{k_{peak}^2}} \frac{H_{inf}^{(l)}}{k_{peak}^2} \left( k_{peak}^2 - \frac{a''}{a} \right) ,
\]

which describes the perturbation modes both in sub- and super-horizon. On super-horizon scale, \( H_{inf}^2 \left( k_{peak}^2 - \frac{a''}{a} \right) \approx -\frac{1}{2} (H_{inf}^2 / k_{peak}^2) / (\sqrt{2}/(\pi k_{peak}^2) \right) \), the spectrum \( P_T \) of primordial PT GWs is

\[
P_T = \frac{4\pi^3}{\pi^2 M_p^2 a^4} |v_a|^2 = \frac{12a^2 H_{inf}^2 \Omega_{GW}^*}{k^3} .
\]

The calculation is equivalent to that of [14], so Eq. (9) has the same \( k \)-dependence as that in [14]. Thus we see that compared with \( P_{T,sub} \), \( P_T \sim k^{2 - 5} P_{T,sub} \) is redden by inflation (\( P_{T,sub} \sim k^{2 - 5} \Omega_{GW}^* / k^2 \)) is that for the sub-horizon PT GWs, see also [14]), which thus records the unique imprint of inflation. The information of the PT is encoded in \( P_{T,sub} \). It should be mentioned that if the inflation ended after the first-order PT [17], [18], the sub-horizons modes will stay inside the horizon all the time, so \( P_T = P_{T,sub} \) will preserve the initial sub-horizon spectrum and decrease with the expansion of the universe as \( P_{T,sub} \sim a^{-2} \), see [20]. We call the superhorizon GWs brought by the PT (but stretched by inflation) the primordial PT GWs.

We have \( v_{b1} = 1 \) for \( V_{inf} = V_{inf} - \Delta V_{inf} \), so that \( A \gtrsim 2.8 \) and \( B \simeq 1 \) in (4). In addition, the peak momentum of \( \Omega_{GW}^* \) is \( k_{peak} / (2\pi) \rho_{peak} = 0.62/(1.8 - 0.1v_{b1} + v_{b1}) \) [28], which suggests \( k_{peak} \simeq 1.48/a_1 \) for \( v_{b1} \simeq 1 \). Thus Eq. (9) becomes

\[
P_T \simeq 0.24 \frac{V_{inf}}{V_{inf}^*} \left( \frac{\Delta V_{inf}}{V_{inf}^*} \right)^2 \left( \frac{H_{inf}^*}{H_{inf}} \right)^6 \frac{3.8 \left( k_{peak}^* \right)^{-1.2}}{1 + 2.8 \left( k_{peak}^* \right)^{-3.8}} ,
\]

where \( V_{inf} \sim 3M_p^4 H_{inf}^2 / \Omega_{GW}^* \) is used. We see that \( P_T \sim k^{-5} \) for \( k \gg k_{peak} \) is strongly red, while \( k^{-1.2} \) for \( k \ll k_{peak} \) the wave-lengths of GW modes brought by the PT is initially sub-horizon.
suggests a cutoff frequency $k_{\text{cut off}} = a H_{\text{inf}} < k$ for $P_T$. We have $k_{\text{peak}}/k_{\text{cut off}} \simeq 1.4 \beta/3 H_{\text{inf}}$, which is consistent with the requirement of sub-horizon burst ($H_{\text{inf}}/\beta < 1$).

It is interesting to note that the maximal amplitude of $\Omega_{GW}^2(k)$ brought by the PT is at $k_{\text{peak}}$, see (4), so it is with $P_{T,\text{sub}}$. However, the maximum of $P_T$, i.e., $P_{T,\text{max}}$, is at $k = k_{\text{cut off}}$, due to the reddening effect of inflation. We have

$$P_{T,\text{max}} \simeq 1.4 \left( \frac{\Delta V_{\text{inf}}}{V_{\text{inf}}} \right)^2 \left( \frac{V_{\text{inf}}}{H_{\text{inf}} + \beta} \right)^{4.8}$$

at $k = k_{\text{cut off}}$ for $(H_{\text{inf}}/\beta)^{3.8} \ll 1$. We set $V_{\text{inf}}/V_{\text{inf}} = 0.1$, which is consistent with the requirement $V_{\text{inf}} \ll V_{\text{inf}}$. Thus, if $H_{\text{inf}}/\beta = 0.1$, we have $P_{T,\text{max}} \approx 1.8 \times 10^{-6}$, while if $H_{\text{inf}}/\beta = 0.3$, $P_{T,\text{max}} \approx 3.5 \times 10^{-4}$; in both cases, $(H_{\text{inf}}/\beta)^{3.8} \ll 1$ is safely satisfied. The amplitude of $P_{T,\text{max}}$ depends on the choice of $H_{\text{inf}}/\beta$. We see that $P_{T,\text{max}}$ could be far larger than the power spectrum of the standard slow-roll inflation, which has $P_T \lesssim 10^{-10}$. However, it is also possible that $H_{\text{inf}}/\beta < 0.013$, which indicates that $P_{T,\text{max}} \lesssim 10^{-10}$ might even be smaller than the amplitude of the GWs spectrum in the slow-roll regime. Therefore, to generate observable PT GWs, it might require fine-tuning to some extent to obtain sufficiently large $H_{\text{inf}}/\beta$ (see Appendix A for details).

3. Detectability

The signal-to-noise ratio (SNR) for the stochastic GWs background is [35]

$$\text{SNR} = \frac{T_{\text{obs}}}{\sqrt{T_{\text{min}}}} \left( \int df \frac{\Omega_{GW, PT}^2}{\Omega_{GW, \text{sens}}^2} \right)$$

where $T_{\text{obs}}$ is the total observation time, $T_{\text{min}}$, $f_{\text{max}}$ defines the bandwidth of the detector, $\Omega_{GW,PT}^2 = \frac{\sigma(f)}{2\pi^2} f^3 H_0^2$ and $\Omega_{GW,\text{sens}}^2 = \frac{S(f)}{2\pi^2} f^3 H_0^2$ for LIGO [36] and LISA [37], respectively. We assume the same noise spectrum $S(f)$ and the parameters $H_{\text{inf}}/\beta$ and $\rho_{GW}(f_0)$ of the LIGO, with the overlap reduction $\Gamma(f)$ [35]. $\Omega_{GW, PT}(f_0) = (1/\rho_f) \int \frac{d\rho_{GW}(f_0)}{df}$ is the energy spectrum of current GWs background, where $\rho_c = 3 H_0^2 / (8\pi G)$ is the critical energy density, and $\rho_{GW}(f_0)$ is the energy density of GWs at present. The spectrum $\Omega_{GW, PT}$ for primordial GWs background (10) is [38]

$$\Omega_{GW, PT}(f_0) = \frac{k^2}{12a_0^4 H_0^2} P_T(k) \left( \frac{3 \Omega_{GW}(k)^2 k}{k_0^2} \right) \left( 1 + 1.36 \frac{k_{\text{peak}}}{k_0} + 2.5 \frac{k_{\text{peak}}}{k_0} \right)^2$$

see also e.g., [31], [39], [40] for the details, where $\Omega_{GW}(k) = \rho_c / \rho_f$, $j_1$ is the spherical-Bessel functions of order one, $k_0$ is the conformal time today, $1/k_{\text{eq}}$ is the Hubble scale at matter-radiation equality.

The spectrum $\Omega_{GW, PT}(f_0)$ is plotted in the left panel of Fig. 2, and the corresponding SNR in the right panel. $k_{\text{peak}}(f_{\text{peak}})$ and $H_{\text{inf}}/\beta$ determine the frequency band and magnitude of $\Omega_{GW, PT}$, respectively. The stochastic background $\Omega_{GW, PT}$ with $H_{\text{inf}}/\beta \simeq 0.3$ at $f_{\text{peak}} \approx 100$ Hz could be marginally reached by LIGO(O5), whereas the signal of $\Omega_{GW, PT}$ with $H_{\text{inf}}/\beta \lesssim 0.1$ at a lower frequency band $f_{\text{peak}} \sim 0.01$ Hz could be detected by LISA. We plot the contours of the SNR computed for LIGO(O5) with 2-yrs integration and the LISA with 4-yrs integration, respectively, in Fig. 3, which reflect the effects of the parameters $f_{\text{peak}}$ and $H_{\text{inf}}/\beta$ on the SNR.

We see that for $H_{\text{inf}}/\beta \gtrsim 0.3$, Advanced LIGO would have the ability to find the signal of $\Omega_{GW, PT}$ with SNR $\gtrsim 3$. However, the superpositions of GWs emitted by all binary black holes (BBH) systems and other compact binaries will also yield a stochastic GWs background $\Omega_{GW} \sim f^{-2/3}$, e.g., [11], [41]. In certain sense, this BBH background actually acts as a foreground. To what extent can the primordial PT GWs background be distinguished from the BBH foreground (once a stochastic signal is detected)? To clarify this point, we will estimate the abilities of Advanced LIGO to make a distinction between $\Omega_{GW, PT}$ and the BBH stochastic GWs background $\Omega_{GW, PL}$, which has the power-law behavior $\Omega_{GW, PL} = \Omega_0 (f/f_0)^{2/3}$ (with $f_0$ being a reference frequency) in the sensitive band of LIGO [11].

We calculate the maximal likelihood ratio. Following [42], the likelihood ratio

$$\mathcal{R} = \frac{\mathcal{L}^M(\Omega_{GW, PT} | \Omega_{GW, PT})}{\mathcal{L}^M(\Omega_{GW, PL} | \Omega_{GW, PL})},$$

where our $\Omega_{GW, PT}$ is regarded as the fiducial model, so that $\mathcal{L}^M(\Omega_{GW, PT} | \Omega_{GW, PT}) = N$, and the corresponding maximal likelihood for the power-law background $\Omega_{GW, PL}$ is

$$\mathcal{L}^M(\Omega_{GW, PT} | \Omega_{GW, PL}) = N \exp \left\{ -\frac{1}{2} \left( \frac{\left( \Omega_{GW, PT} | \Omega_{GW, PT} \right) - \langle \omega \rangle \Omega_{GW, PT}}{\langle \omega \rangle \omega} \right) \right\}$$

with the definitions $\langle A | B \rangle = 2 \int \frac{3H_0^2}{2\pi^2} \int df \Gamma^2(f) \left( \frac{A(f) B^*(f)}{T(f) S(f)} \right)$ [36,43] and $\omega(f) = \Omega_{GW, PL} \langle \omega \rangle / \Omega_0 = (f/f_0)^{2/3}$. When the likelihood ratio $R \approx 1$, both GWs backgrounds ($\Omega_{GW, PT}$ and $\Omega_{GW, PL}$) could be hardly distinguished, while a fiducial GWs background could be identified only for a large value of $\mathcal{R}$. Thus for the full identifiability of the primordial PT GWs background, we must require $\ln \mathcal{R} \gg 1$.

The maximal log-likelihood ratio $\ln \mathcal{R}$ with respect to the parameters $H_{\text{inf}}/\beta$ and $f_{\text{peak}}$ is plotted in Fig. 4. We see that for LIGO O5, the region with $H_{\text{inf}}/\beta \gtrsim 0.3$ and $f_{\text{peak}} \approx 100$ Hz could lead to $\mathcal{R} \gg 1$, which corresponds to $\text{SNR} \gtrsim 3$ (see Fig. 3). Note that the maximum of $\Omega_{GW, PT}$ is at $f_{\text{cut off}} \approx f_{\text{peak}}/1.4 \beta/3 H_{\text{inf}}$, as mentioned. Thus $f_{\text{cut off}} \approx f_{\text{peak}}/4 \approx 25$ Hz (for $H_{\text{inf}}/\beta \gtrsim 0.3$), which is just the most sensitive band of LIGO. Thus if the parameter space ($f_{\text{peak}}, H_{\text{inf}}/\beta$) of primordial PT GWs is that plotted in Fig. 3, the Advanced LIGO at its design sensitivity would identify the corresponding signal.

However, in the case of marginal sensitivity, it would be difficult to prove the origin of the signal with respect to other possible sources. Thus in order to link the signal detected by GWs observatories to the inflation physics, a multi-frequency detection is a key. More sensitive detectors (the Einstein Telescope [44], the Cosmic Explore [45]) are required. If $H_{\text{inf}}/\beta < 0.1$, we will have $\Omega_{GW, PT} < \Omega_{GW, PL}$ in the frequency band of LIGO, the BBH foreground $\Omega_{GW, PL}$ must be subtracted accurately [45], which is a challenging issue, since the large uncertainty in the stellar BH merger rate will inevitably affect the amplitude of BBH GWs foreground [11]. Additionally, since the reddened inflationary PT GWs spectrum satisfies $\Omega_{GW}(k < k_{\text{peak}}) \sim k^{-1.2}$ and $\Omega_{GW}(k > k_{\text{peak}}) \sim k^{-5}$, the $k < k_{\text{peak}}$ region could be approximately mimicked by the $k > k_{\text{peak}}$ region of the GWs spectrum due to a PT happened after inflation ($\Omega_{GW}(k > k_{\text{peak}}) \sim k^{-1.2}$). Thus the signal should be identified carefully with the multi-frequency detection before linked to the inflation physics.
Fig. 2. Left panel: The stochastic background $\Omega_{GW,PT}$ in the frequency bands of Advanced LIGO and LISA. In the LISA band, we set the parameter $H_{inf}/\beta = 0.075, 0.1, 0.125$, respectively, for $f_{peak} \approx 10^{-4}, 10^{-3}, 10^{-2}$ Hz. In LIGO band, we set the parameter $H_{inf}/\beta = 0.3, 0.35, 0.4$ for different $f_{peak} \approx 100$ Hz. Right panel: The SNR for $\Omega_{GW,PT}$ plotted in the left panel.

Fig. 3. Left panel: The contour of the SNR computed for LISA with respect to the parameters $f_{peak}$ and $H_{inf}/\beta$. Right panel: The contour of the SNR computed for LIGO(O5).

Fig. 4. Contour of the maximal log-likelihood ratio $\ln R$ given in (14) with respect to the parameters $H_{inf}/\beta$ and $f_{peak}$.

4. Conclusion

It is well-known that the first-order PT will yield a stochastic GWs background with a peculiar spectrum. However, we found that when such a PT happened during inflation, the GWs spectrum brought by the PT will be reddened, which thus records the unique voiceprint (the reddening-imprint) of inflation. We estimated the abilities of Advanced LIGO and LISA to detect the corresponding signal. Our result suggests that the GWs detectors with higher sensitivity might consolidate our confidence that the inflation has ever occurred.

The first-order PT is ubiquitous in the early universe. Though our calculation is not sensitive to the details of model, the corresponding model, e.g., along the line in Refs. [17], [18], is actually interesting for study. Rich phenomenology caused by the PT (during inflation), such as the primordial black holes [46], [47], [48], galactic nuclei [49], [50], [51], magnetic field, also need to be fully understood, which might be also observable.

Acknowledgements

YC would like to thank Burt Ovrut and Rehan Deen for hospitalities and discussions during his visit at Penn. This work is supported by NSFC, No. 11575188, 11690021, and also supported by the Strategic Priority Research Program of CAS, No. XDB23010100. YTW is supported in part by the sixty-second batch of China Postdoctoral Fund. YC is supported in part by the UCAS Joint PhD Training Program.
Appendix A. A model

In this Appendix, we will show a model with a first-order PT happened during slow-roll inflation, slightly similar to that proposed in the Appendix A of Ref. [52].

The effective potential is a $(\phi, \psi)$-landscape

$$V = V(\phi) + U(\phi, \psi),$$

where

$$U(\phi, \psi) = \frac{\sigma_1}{16} \left( \left( \psi - \frac{\mu}{\sqrt{\sigma_1}} \right)^2 - \frac{\mu^2}{\sigma_1} \right) - \frac{\sigma_2}{2\mu} \psi,$$

$\sigma_1, \sigma_2 = \text{const.}, \phi$ is the inflaton, $\psi$ is the scalar field responsible only for PT, $\mu$ depends on $\phi$ but is constant in the $\psi$-direction. Along the $\psi$-direction, $U$ has the minima at around $\psi \simeq 0$ (A) and $\psi \simeq \frac{2\mu}{\sqrt{\sigma_1}}$ (B). The height and width of the barrier between both minima are about $\mu^4/(16\sigma_1)$ and $2\mu/\sqrt{\sigma_1}$, respectively. Here, we set

$$\mu^2 = m^2_\psi \left[ 1 + \frac{\lambda \psi}{M_p} (\phi_\ast - \phi) \right],$$

with $m^2_\psi, \lambda, \lambda_\psi = \text{const}$, and $\phi_\ast$ being the PT point. As a result, the rolling of $\phi$ along its potential $V(\phi)$ will lower the height of the barrier along $\psi$-direction.

In the thin-wall approximation [53], the nucleating rate of bubble is $\Gamma \sim e^{-B}$, where

$$B = \frac{27\pi^2}{2\sigma^4_2} \simeq \frac{\pi^2 \mu^{12}}{\sigma^4_1 \sigma^2_2}.$$

According to Eq. (A.3), defining $x = \pi^2 m^2_\psi/(6\sigma^4_1 \sigma^2_2)$, we have

$$B \simeq x \left[ 1 + \frac{\lambda \psi}{M_p} (\phi_\ast - \phi) \right]^6 \simeq x \left[ 1 + 6\frac{\lambda \psi}{M_p} (\phi_\ast - \phi) \right].$$

around $\phi = \phi_\ast$. Thus $\Gamma \sim e^{-x}$ at $\phi = \phi_\ast$. Dependent on the parameters $m_\psi, \sigma_1, \sigma_2$, it is possible that when $\phi$ slowly rolled to the PT point $\phi_\ast$, the PT along $\psi$-direction could complete rapidly. After PT, the inflation will continue but $\phi$ will roll slowly along a lower potential $V(\phi, \psi) < V(\phi, \psi_A)$, see Fig. 5 for a sketch.

We could estimate the bound for $H_{inf}/\beta$, which appears in (11). Considering $\phi \approx -V'/(3H)$ in the slow-roll approximation in the $\phi$-direction, we have

$$\phi_\ast - \phi \simeq \frac{-V'}{3H} (t_\ast - t),$$

where we set $V' = \partial V/\partial \phi = \text{const.}$ for convenience. Combining Eqs. (A.5) and (A.6), we get

$$\Gamma \sim e^{-\beta x} \exp \left[ \frac{2\lambda \psi}{M_p H} \left( t - t_\ast \right) \right] \sim e^{\beta(t-t_\ast)},$$

which suggests $\beta = \frac{2\lambda \psi}{M_p H}$. Thus, around the PT point, we have

$$H_{inf}/\beta \approx \frac{M_p}{6\sqrt{2}\lambda \psi \frac{V'}{V_T^2}}.$$

where $H_{inf}$ is the Hubble parameter at PT, and $\epsilon = M^2_\psi (V'/V)^2/2$ is the slow-roll parameter along the $\phi$-direction, which should also be evaluated at the PT point. Generally, $\lambda_\psi < 1$. According to (A.6), it is possible to have $H_{inf}/\beta \simeq 0.1$ provided $\lambda_\psi$, $\epsilon$ and $\psi$ are sufficiently small.

However, it should be mentioned that if the PT is not completed at $\phi = \phi_\ast$, after PT, some regions will inflate along $V(\phi, \psi_A)$, while others along the lower potential $V(\phi, \psi_B)$, see multi-stream inflation [54], [55]. The relevant issue is also interesting for study.

References