On the heavy-quark states interactions with nucleons and nuclei

A. Kaidalov
Institute for Theoretical and Experimental Physics, 117259, Moscow, Russia

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Abstract

The method of multipole expansion in QCD is used to calculate the scattering amplitudes of heavy–quark $\langle QQ \rangle$–states on hadrons. The amplitudes near threshold and the total cross sections for $(QQ)h$–interactions at different energies are calculated. It is pointed out that these cross sections are small at energies of $(QQ)$–system $E < 50$ GeV. This result has serious implications for the models, which explain the $(QQ)$–states suppression in heavy ions collisions in terms of $(QQ)$–interaction with comoving hadrons.
Study of interactions of heavy quark system \((Q\bar{Q})\) with nucleons and nuclei can give a valuable information on both QCD forces at small and intermediate distances and on the space–time picture of high–energy processes. It is also important for understanding of a mechanism of the \(J/\psi\) -suppression in heavy ions collisions, which was proposed as a possible signal for the quark–gluon plasma formation.

The data on the \(A\)-dependence of \(Q\bar{Q}\)-production in hadron–nucleus collisions are not, in general, directly related to the cross section of \((Q\bar{Q})\ N\)–interaction. This was demonstrated in the case of diffractive photoproduction of \(Q\bar{Q}\) states in Refs. [1, 2]. Recently we have shown [3, 4], that the same is true for non-diffractive production of \(J/\psi\) for interactions of both hadrons and photons with nuclei. We stressed that it is not \(J/\psi\) but another \(c\bar{c}\)-state propagates a nucleus. Thus it is difficult to extract an information on \(J/\psi\ N\)–interaction cross section by studying its production on nuclei.

Here I want to emphasize that the \((Q\bar{Q})\)-hadron amplitudes can be determined theoretically. For this purpose it is convenient to use the method of multipole expansion in QCD [5, 6, 7, 8]. This approach uses the fact that for very heavy quarks the sizes of the \((Q\bar{Q})\)-bound states are small \(r \sim a_0 = 4/3m_Qa_s\), the binding energies \(E_0 = (3a_s/4)^2m_Q\) are large and these states are determined by the QCD-coulomb part of the potential. For \(J/\psi\) or \(\eta_c\) –states the radius is not so small, however applications of the multipole expansion to the \(c\bar{c}\) states [8, 9, 10] lead to very satisfactory results. I shall follow the approach developed in Refs. [7, 8] to calculate \((Q\bar{Q})\)-h scattering amplitudes. The amplitudes of \((\Phi \equiv Q\bar{Q})\)-h scattering are written as the series of matrix elements of the gluonic operators

\[
T_{\Phi h} = \sum_{n=2} d_n a_0^2 \frac{2-n}{2} < h|\frac{1}{2}(i)^n F^{0\nu}(D^0)^{n-2} F^0_\nu|h \>
\]

where \(F^{\mu\nu}\) is the gluonic tensor field, \(D^\nu\) is the covariant derivative and \(d_n\) are numbers, determined by the wave–function of the \((Q\bar{Q})\)-state. The values of \(d_n\) have been calculated in some approximation in Ref. [7].

For the forward \((Q\bar{Q})\)-h scattering amplitude the matrix elements entering Eq. (1) can be written as

\[
< h|\frac{1}{2}(i)^n F^{0\nu}(D^0)^{n-2} F^0_\nu|h > = A_n \lambda^n
\]

where \(\lambda = (s - M^2)/2M, M \equiv M_{(Q\bar{Q})}\).

Near the threshold of \((Q\bar{Q})\)-h scattering the dominant contribution is given by \(n=2\). It can be found [11, 12], using the low-energy QCD-theorem, based on the triangle anomaly relation between the gluonic field and the trace of the energy–momentum tensor \(\Theta_{\mu\nu}\) [13, 14]

\[
F_{\mu\nu} = \left( \frac{4\alpha_s}{\beta(\alpha_s)} \right) \Theta_{\mu\nu}
\]

where \(\beta(\alpha_s)\) is the Gell-Mann–Low function.

Using the property of the matrix elements of \(\Theta_{\mu\nu}\) at zero momentum transfer

\[
< h|\Theta_{\mu\nu}|h > \big|_{p_1=p_2=p} = 2p_{\mu}p_{\nu}
\]

one can obtain

\[
< h|\alpha_s E^i E^i|h > = \frac{4\pi m_h^2}{b} + O(\alpha_s)
\]

where \(b=9\) is the first coefficient of \(\beta(\alpha_s)\).
The amplitude of $J/\psi N$--elastic scattering near threshold is equal to

$$T_{\psi N} = \frac{112\pi^2 M_{\psi} m_N^2 a_0^3}{3^5 \alpha_s}$$

(6)

At higher energies the elastic scattering amplitudes and the total interaction cross sections, related to their imaginary parts, can be obtained, using the method of Ref. [8]. In this paper it was pointed out that the same coefficients $A_n$ enter into the expansion of the gluonic structure function $f_g(x)$ of the hadron $h$.

In this approach the amplitude of $(Q\bar{Q})h$--scattering is determined by the diagram of Fig. 1, where all the information about the matrix elements of Eq. (2) is contained in the function $f_g(x)$ and the amplitude of $(Q\bar{Q})g$--elastic scattering is explicitly calculated. The total cross section is given by the following expression

$$\sigma_{(Q\bar{Q})h}(s) = \left(93 \frac{mb}{GeV^2} \right) \frac{3\alpha_s a_0^2}{4} \int_{1/\phi}^{1} dx f_g(x) \frac{(x\phi - 1)^{3/2}}{(x\phi)^5}$$

(7)

where $\phi = \lambda/\epsilon_0 = (s - M^2)/2M\epsilon_0$.

It follows from Eq. (7) that the energy dependence of these total cross sections are given by a universal function of variable $\phi$.

For the gluonic structure function of the form

$$f_g(x) = A \left(\frac{1 - x^n}{x}\right)$$

(8)

the total cross sections tend to constant values as $s \to \infty$ (if we choose $f_g(x) \sim \frac{1}{x^{1+\alpha}}$ for $x \to 0$ then the cross sections would behave as $s^\Delta$ as $s \to \infty$), which for $\psi p$ and $\Upsilon p$ interactions ($n=5, A=3$) are equal to [8]

$$\sigma_{\psi p}^\infty = (3 - 5)mb, \quad \sigma_{\Upsilon p}^\infty = (0.8 - 1.2)mb$$

(9)

Here I want to emphasize that there is a strong preasymptotic rise of the cross sections as energy increases, shown in Fig.2.
In order to check this theoretical prediction I shall use the assumption of the generalised vector dominance, which relates \( \sigma_{(Q\bar{Q})p} \) to the total photoproduction cross section of the \((Q\bar{Q})\)-quarks. For example

\[
\sigma_{\gamma p \rightarrow c\bar{c}X}(s) = c\sigma_{\gamma p}^{(tot)}(s) \tag{10}
\]

\[
\left[ \frac{d\sigma(\gamma p \rightarrow \psi p)}{dt} \right]_{t=0} = \frac{c}{16\pi} \left( \sigma_{\psi p}^{(tot)}(s) \right)^2 (1 + \rho^2(s)) \tag{11}
\]

where \( \rho = \text{Re} T_{\psi p \rightarrow \psi p}(s,0)/\text{Im} T_{\psi p \rightarrow \psi p}(s,0) \)

The points in Fig.2 show the energy dependence of \( \sigma_{\gamma p \rightarrow c\bar{c}X} \) correspondingly normalised. It was shown in Ref.[8] that Eq.(11) also agrees with experiment.

One can obtain the model independent inequalities for \( \sigma_{(Q\bar{Q})h} \), which follow only from unitarity [15]:

\[
\sigma_{(Q\bar{Q})h}^{(tot)}(s) \geq \left[ \frac{d\sigma(\gamma N \rightarrow (Q\bar{Q})N)}{dt} \right]_{t=0} \frac{16\pi}{(1 + \rho(s)^2)\sigma_{\gamma N \rightarrow (Q\bar{Q})X}(s)} \tag{12}
\]

At high energies \( E > 100 \text{ GeV} \) it is reasonable to neglect in Eq.(12) by \( \rho^2(s) \) compared to 1 and, using the experimental data on the total cross section of charm photoproduction \( \sigma_{\gamma p \rightarrow c\bar{c}X} \) and \( \left[ \frac{d\sigma(\gamma p \rightarrow \psi p)}{dt} \right]_{t=0} \), we get at \( E = 100 \text{ GeV} \)

\[
\sigma_{\psi N}^{(tot)} \geq 2.8 \text{ mb} \tag{13}
\]

For light-quark states the relation (12) is close to equality [15]. If we assume that the same is true for \( \psi N \)-interaction we obtain \( \sigma_{\psi N}^{(tot)} = 2.8 \text{ mb} \) at \( E_{(lab)} = 100 \text{ GeV} \). This result is in a reasonable agreement with theoretical estimates given above.

Thus, using the model, based on the multipole expansion in QCD, we obtain the following estimates for asymptotic \( s \sim 10^3 \text{ GeV}^2 \) for \( \psi \) and \( s \sim 10^4 \text{ GeV}^2 \) for \( \Upsilon \) values of \((Q\bar{Q})h\) cross sections

\[
\sigma_{\psi N} = (3 - 5)\text{mb}, \quad \sigma_{\psi \Upsilon} = (2 - 3)\text{mb}; \tag{14}
\]

\[
\sigma_{\Upsilon N} = (0.8 - 1.2)\text{mb}, \quad \sigma_{\Upsilon \Upsilon} = (0.6 - 0.8)\text{mb} \tag{15}
\]
These cross sections decrease strongly as energy decreases and are very small near threshold. The energy dependence of $\sigma_{T^*}$ is shown in Fig.3. For energies $E_T < 200 \text{ GeV}$ $\sigma_{T^*} < 0.1 \text{mb}$, while $\sigma_{\varphi^*} < 0.2 \text{mb}$ for $E_\psi < 50 \text{ GeV}$. These results are important for interpretation of the heavy-quarkonia suppression in hadron-nucleus and nucleus-nucleus collisions. If cross sections of $(Q\bar{Q})$-interactions with hadrons are so small then it is not possible to explain the experimentally observed suppression of $J/\psi$ and $\Upsilon$ in the models of $(Q\bar{Q})$-interactions with comoving hadrons.

References