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Chameleon mechanism in f(R) modified gravitation model of polynomial-exponential form

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Abstract. In this paper, we first present briefly to Chameleon mechanism in modified gravity of f(R), then we apply it to modified gravity of polynomial exponential form to find constraints from solar system experiments to parameters of $\alpha$ and $\beta$ of the model. Results show that the Chameleon mechanism sets strict constraints on $\alpha$ and $\beta$ parameters as follows: $0 < \beta < 7.67 \times 10^{-15}$ and $0 < \alpha < 9 \times 10^{-75}$.

Key words: Chameleon mechanism; polynomial exponential modified gravity; solar system constraints.

1. Introduction
In 1998, Astrophysicists uncovered a surprise that the universe was accelerating rather than slowing down as expected [1,2,3]. To explain this event there are three main approaches. In the first approach, people think that the Einstein equation with cosmological constant is sufficient to present the acceleration of universe. However, detailed calculations show that the cosmological constant from theory is about 120 orders of magnitude greater than its experimental value. This is the problem of cosmological constant [4], so this approach is not supported by the majority of scientists. In the second approach, the acceleration of universe causes by fifth force field called Quintessence field [5]. However, to satisfy constraints from observations and experiments, artificial conditions are required on this force field, such as: the field potential must be very flat so that the field rolls very slowly into the potential hole; the mass of quintessence particle is very small about $10^{-31}$ eV. In the third approach, people change Einstein’s theory to describe the acceleration of universe today but do not need to dark energy. Modified gravity of f(R) belongs this class of theory. In the class of theories, action of Einstein-Hilbert R in general relativity theory is replaced by a function of f(R). Here f(R) is analytic function which satisfies some constraint conditions. The first authors to take this path are Soviet physicist Starobinski in the 1980s of the last century[6], followed by American physicist Sean Carroll[7]. These early approaches gave physicists more emotion and experience.

We also use a theory model in the class to research the evolution of universe. In our model, f(R) takes of polynomial-exponential form[8,9]

\[
S = \frac{1}{2\kappa^2} \int f(R) \sqrt{-g} d^4x + S_M(g_{\mu\nu}, \psi);
\] (1)
where
\[ f(R) = R + a + \frac{\alpha}{R^m} \left( 1 + bR^2 + cR^3 \right) e^{-\beta R^n}; \]  
with
\[ \alpha > 0, \beta > 0; m = n = 1; a = -2\Lambda; b = c = 1. \]

This is a small class in f(R) theory class but it also is quite general, and is also of interest to many cosmologists today. This class called the class of polynomial-exponential modified gravity.

The paper is constructed as follows: in section 2, we introduce the Chameleon mechanism in the class of f(R) modified gravity; in section 3, we present the Chameleon mechanism in the f(R) modified gravity of polynomial exponential form, then we calculate constraints on parameters of the model; section 4 is discussions and conclusion.

2. The Chameleon mechanism in f(R) modified gravity

We consider a static spherically symmetric compact object, it causes a perturbation on de Sitter background universe. The metric is as follows
\[ ds^2 = -[1 - 2\varphi(r)]dt^2 + [1 + 2\psi(r)]dr^2 + r^2d\Omega^2; \]  
where \( \varphi \) and \( \psi \) are post-Newtonian potentials satisfy
\[ |\varphi(r)|, |\psi(r)| << 1. \]

We only note small scales such that
\[ H_0.r << 1; \]
we expand Ricci scalar about constant curvature of de Sitter universe
\[ R(r) = R_0 + R_1. \]

The parameter of \( \gamma \) in Parameterized Post-Newtonian formalism (PPN)[10], is introduced as follows
\[ \gamma = -\varphi(r)/\Psi(r); \]
three following assumptions are required
- \( f(R) \) is analytic at \( R_0 \);
- \( mr << 1 \), i.e the mass of scalar field is very small and it has acting scale is great than the scale of solar system
- the pressure inside the object can be ignored, i.e, \( T = T_0 + T_1 \simeq -\rho \);

people prove that if the condition is satisfied, we have with an any f(R) theory
\[ \gamma = -\varphi(r)/\Psi(r) = 1/2; \]
while experiments in solar system give [11]
\[ |\gamma - 1| < 2.3 \times 10^{-5}; \]

It means \( \gamma = 1 \). This puts an end to f(R) modified theories if there is no Chameleon mechanism as follows: It is thought that the second request is not always satisfied because the scalar field’s mass may be large due to it depends on the curvature of the universe i.e on matter density of the surrounding environment. This mechanism was first proposed in 2004 by two American physicists, J. Khoury and A. Weltman, to overcome the above difficulty[12]. The oscillation
of neutrino particles seems to support this mechanism because the mass of neutrino particles depends on the environment around them. Below we give a shortly review to this mechanism. We consider the following action

\[ S = \frac{1}{2\kappa^2} \int f(R)\sqrt{-g}d^4x + S_M(g_{\mu\nu}, \psi); \]  

(11)

where \( g \) is the determinant of \( g_{\mu\nu} \), \( f(R) \) is an unknown function of the scalar curvature \( R \) and \( S_m \) is the matter action. We use a conformal transformation

\[ \bar{g}_{\mu\nu} = pg_{\mu\nu}; \]

\[ \phi = \frac{1}{2\beta_1} \ln p; \]

(12)

(13)

where

\[ p = \frac{df}{dR} = f'(R), \quad \beta_1 = \sqrt{\frac{T}{6}}. \]

(14)

This transforms the above action in Jordan frame to the Einstein frame\[13\]

\[ S = \frac{1}{2} \int d^4x \sqrt{-g[R - g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - 2V(\phi)]} + S_M(\bar{g}_{\mu\nu}e^{-2\beta_1\phi}, \psi); \]

(15)

the scalar field \( \phi \) in the Einstein frame has a self-interacting potential

\[ V(\phi) = \frac{1}{2} e^{-2\beta_1\phi} \left\{ r[p(\phi)] - e^{-2\beta_1\phi} f(r[p(\phi)]) \right\}; \]

(16)

where \( r(p) \) is a solution of the equation

\[ f'(r[p]) - p = 0. \]

(17)

The conformal transformation makes the coupling of the scalar field with the matter sector. The strength \( \beta_1 \) of this coupling is the same for all types of matter field and it equals \( \frac{1}{\sqrt{6}} \). In such case, the scalar potential has an important role for consistency with local gravity experiments in solar system. The potential of the scalar field has to attribute an effective mass to the field which has a strong dependence on material density of environment. A theory that has such dependence called a chameleon theory. In a chameleon theory of \( f(R) \), the scalar field can be heavy enough in the environment of laboratories on the Earth so that the local gravity constraints suppressed, while it can be light enough in the low density cosmological environment to makes the cosmic acceleration. Variation of the action (15) with respect to \( g_{\mu\nu} \) and \( \phi \) gives the field equations

\[ G_{\mu\nu} = \partial_\mu\phi\partial_\nu\phi - \frac{1}{2}\bar{g}_{\mu\nu}\partial_\gamma\phi\partial_\gamma\phi - V(\phi)\bar{g}_{\mu\nu} + T_{\mu\nu}; \]

(18)

\[ \bar{\phi} - \frac{dV}{d\phi} = -\beta_1 T; \]

(19)

where

\[ \bar{T}_{\mu\nu} = \frac{-2}{\sqrt{-\bar{g}}} \frac{\delta S_m}{\delta \bar{g}^{\mu\nu}}; \]

(20)

and

\[ T = \bar{g}^{\mu\nu} T_{\mu\nu}. \]

(21)
Covariant differentiation of (18) and Bianchi identities give
\[ \nabla^\mu T_{\mu\nu} = \beta_1 T \partial_\nu \phi; \]  
(22)

Equation(20) indicates that the matter field is not generally conserved and feels a new force because of gradient of the scalar field. We now consider \( T_{\mu\nu} \) as the stress-tensor of dust with energy density \( \rho \) in the Einstein frame. In a static spherically symmetric space-time region, equation (19) gives
\[ \frac{d^2 \phi}{d\bar{r}^2} + \frac{2}{\bar{r}} \frac{d\phi}{d\bar{r}} = \frac{dV_{\text{eff}}(\phi)}{d\phi}; \]  
(23)

where \( \bar{r} \) is the distance from the center of symmetric body to point that we investigated in Einstein frame. We have
\[ V_{\text{eff}}(\phi) = V(\phi) - \frac{1}{4} \rho e^{-4\beta_1 \phi}; \]  
(24)

where \( \rho = e^{-4\beta_1 \phi} \rho \) is the relation that relates between the energy density in Jordan and Einstein frames. We now consider a spherically symmetric body which has the radius \( r_c \) and assume that the energy density is constant \( \rho_{\text{in}}(r < r_c) \) inside the body. We call the energy density outside the body by \( \rho_{\text{out}}(r > r_c) \). We denote the values at two minimums of the effective potential \( V_{\text{eff}}(\phi) \) inside and outside by \( \phi_{\text{in}} \) and \( \phi_{\text{out}} \), respectively. They satisfy relations \( V'_{\text{eff}}(\phi_{\text{in}}) = 0 \) and \( V'_{\text{eff}}(\phi_{\text{out}}) = 0 \), where prime indicates differentiation of \( V_{\text{eff}}(\phi) \) with respect to the argument of \( \phi \). Masses of small fluctuations about these minimums are given by \( m_{\text{in}} = \left[ V''_{\text{eff}}(\phi_{\text{in}}) \right]^{\frac{1}{2}} \) and \( m_{\text{out}} = \left[ V''_{\text{eff}}(\phi_{\text{out}}) \right]^{\frac{1}{2}} \).

Which depend on matter density of surrounding environment. The scalar field can has short-range effects in the solar system ( \( m > 10^{-3} eV \) for \( \lambda < 0.2 mm \) to avoid constraints by solar system experiments) due to the large material density in the solar system while it has a very small mass on cosmic scale due to very small density of universe and causes the current cosmic acceleration.

For dense compact objects which satisfy the thin shell following
\[ \frac{\Delta r_c}{r_c} = \frac{\phi_{\text{out}} - \phi_{\text{in}}}{6\beta_1 \phi_c} << 1; \]  
(25)

where \( \phi_c = \frac{M_c}{4\pi r_c^3} \) is Newtonian potential at \( r = r_c \), with \( M_c \) is the mass of the object. In this case, equation (23) with some appropriate boundary conditions gives the field profile outside the body[14]
\[ \phi(r) = -\frac{\beta_1}{4\pi} \frac{3\Delta r_c M_c e^{-m_{\text{out}}(r-r_c)}}{r} + \phi_{\text{out}}. \]  
(26)

3. The Chameleon mechanism in f(R) modified gravity of polynomial-exponential form

We consider the above action
\[ S = \frac{1}{2\kappa^2} \int f(R) \sqrt{-g} d^4x + S_M(g_{\mu\nu}, \psi); \]  
(27)

with
\[ f(R) = R - 2\Lambda + \frac{\alpha}{R} (1 + R^2 + R^3)e^{-\beta R}; \]  
(28)
where $g$ is determinant of $g_{\mu\nu}$, $S_m$ is the action of matter which depends on the metric $g_{\mu\nu}$ and an any scalar field $\psi$.

Transforming this action from Jordan frame to Einstein frame as equations (12), (13), (14), (15), (16), (17) and assuming that $\phi << 1$, one can find the solution of $V'_{\text{eff}}(\phi) = 0$. We have

$$ V'_{\text{eff}} = V(\phi) - \frac{1}{2} (\frac{\partial V}{\partial \phi})^2 = \frac{1}{2(1+3\beta_1\phi)} \left\{ (8\alpha\beta_1\phi - 4\alpha\beta_2\phi^2) - 8\alpha\beta_2\phi^2 + 8\alpha\beta_2 + 6\alpha\beta_1\phi + 20\alpha^2\beta_1\phi + \alpha^2\beta_1 - 2\alpha^2 \right\}; $$(29)

from

$$ V'_{\text{eff}} = 0; $$ (30)

we have

$$ \phi = \frac{-2\alpha\beta_1 + 7\alpha}{8\beta_1} \pm \sqrt{\frac{33\alpha^2 - 16\rho\alpha\beta^2}{64\beta^2 \beta_1^2}}. $$ (31)

In section 3, we shall consider the thin-shell condition together constraints from the equivalence principle and fifth force experiments on the model.

### 3.1. The thin shell condition

For a heavy and large object, the chameleon field is trapped inside the object so that its effect on other objects is only due to a thin shell near the outer surface of the object. The criterion for thin shell is given by equation of (25). When combining (25) and (31), we have

$$ \frac{\Delta r}{r_e} = \frac{\Delta r_{\text{in}} - \Delta r_{\text{out}}}{\rho_{\text{in}} r_e} << 1; $$

$$ \frac{\Delta r}{r_e} = \left( \pm \frac{\sqrt{33\alpha^2 - 16\rho_{\text{out}} \alpha\beta^2}}{64\beta^2 \beta_1^2} \mp \sqrt{\frac{33\alpha^2 - 16\rho_{\text{in}} \alpha\beta^2}{64\beta^2 \beta_1^2}} \right) \frac{1}{6\beta_1 \Phi_e} << 1; $$ (32)

where $\rho_{\text{in}}$ and $\rho_{\text{out}}$ are the energy densities inside and outside of the object in Jordan frame. In the weak approximation for a spherically symmetric object, the metric in Jordan frame has the form

$$ ds^2 = -[1 - 2X(r)]dt^2 + [1 + 2Y(r)]dr^2 + r^2 d\Omega^2; $$ (33)

where $X(r)$ and $Y(r)$ are functions of the variable $r$. The relation of $r$ and $r$ is $r = p^{1/2} r$. We assume that $m_{\text{out}} = \rho_{\text{out}} r_e << 1$ i.e, the Compton wave length $m_{\text{out}}^{-1}$ is much large than solar system scales, then we have $\gamma \approx r$. In this case, Chameleon mechanism gives the parameter $\gamma$ in Post-Newtonian formalism as follows

$$ \gamma = \frac{3 - \Delta r_e}{3 + \Delta r_e} \approx 1 - 2 \frac{\Delta r_e}{3 r_e}. $$ (34)

We obtain the thin shell condition for the Earth when applying (29). We consider the Earth as a solid sphere with the radius of $R_e = 6.4 \times 10^8$ cm and mean density $\rho_{\text{out}} \sim 10 g/cm^3$. We also consider the Earth is surrounded by an atmosphere layer with homogeneous density $\rho_a \sim 10 g/cm^3$ and thickness of 100km. When applying equation (32) and the constraint comes from Cassini tracking[15]

$$ |1 - \gamma| < 2.3 \times 10^{-5}; $$ (35)

we have

$$ \frac{\Delta R_e}{R_e} = \left( \sqrt{\frac{33\alpha^2 - 16\rho_{\text{out}} \alpha\beta^2}{64\beta^2 \beta_1^2}} - \sqrt{\frac{33\alpha^2 - 16\rho_{\text{in}} \alpha\beta^2}{64\beta^2 \beta_1^2}} \right) \frac{1}{6\beta_1 \Phi_e} < 3.45 \times 10^{-5}; $$ (36)
or
\[ \sqrt{\frac{33\alpha^2}{16\alpha\beta^2} - \rho_{out}} - \sqrt{\frac{33\alpha^2}{16\alpha\beta^2} - \rho_{in}} - \frac{165.6\beta^2}{4\sqrt{\alpha}} < 0; \]  
(37)

set \[ a = \frac{33\alpha^2}{16\alpha\beta^2}, \quad b = \frac{165.6\beta^2}{4\sqrt{\alpha}}; \]
therefore
\[ \sqrt{a - \rho_{out}} - \sqrt{a - \rho_{in}} - b < 0; \]  
(38)

when maintaining only first order of small parameter \( \beta \), we have
\[ \frac{256\beta^2}{33 \times (165.6)^2 \beta^2 \Phi_e^2 \times 10^{-10} < \frac{4}{(\rho_{in} - \rho_{out})^2} \Rightarrow \beta < \left( \frac{4.33(165.6)^2 \beta^2 \Phi_e^2 \times 10^{-10}}{256\rho_{in} - \rho_{out}} \right)^{\frac{1}{2}} \]  
(39)
\[ \beta < \left( \frac{4.33(165.6)^2 \beta^2 \times 10^{-10}}{256(4.33)^2} \right)^{\frac{1}{2}} = 1.23. \]  
(40)

This is the constraint condition on the parameter \( \beta \) of the model. We see that this condition is not too strict on \( \beta \) due to in the model we have assumed from the beginning that \( 0 < \beta < 1 \).

### 3.2. The equivalence principle

In this section, we consider constraints on the parameter \( \beta \) coming from possible violation of the equivalence principle. We assume that the Earth together its atmosphere is an isolated system.

Far away from the Earth, the matter density is assumed to be homogeneous with the density \( \rho_G \sim 10^{-24} \text{gr/cm}^3 \). We now consider conditions for the Earth to satisfy the thin shell condition. We take the thickness of atmosphere to be the thickness of thin shell about \( \Delta R_a \sim 100 \text{km} \) and \( R_a \sim 4000 \text{km} \), we obtain \( \frac{\Delta R_a}{R_a} < 1.5 \times 10^{-2} \). We also have relations
\[ \frac{\Delta R_a}{R_a} = \frac{\varphi_a - \varphi_e}{6\beta \varphi_e}; \]  
(41)

and
\[ \frac{\Delta R_e}{R_e} = \frac{\varphi_G - \varphi_a}{6\beta \varphi_a}; \]  
(42)

where \( \varphi_a \), \( \varphi_e \) and \( \varphi_G \) are field values at local minimums of effective potential in regions \( r < R_e, R_a > r > R_e \) and \( r > R_a \) respectively. Because of the Newtonian potential inside a spherically symmetric object with the mass density \( \rho \) is \( \Phi \propto \rho R^2 \), we have \( \Phi_e = 10^4 \Phi_a \). Where \( \Phi_a \) and \( \Phi_e \) are Newtonian potentials on the surface of the Earth and atmosphere, respectively. This gives
\[ \frac{\Delta R_e}{R_e} \approx 10^{-4} \frac{\Delta R_a}{R_a}; \]  
(43)

such that, the condition for atmosphere to have a thin shell is
\[ \frac{\Delta R_a}{R_a} < 1.5 \times 10^{-6}. \]  
(44)

The tests of the equivalence principle measure the difference of free fall accelerations of the Earth and the Moon towards to the Sun. The constraint on this difference of two accelerations is given by the relation[16]
\[ \left| \frac{a_m - a_e}{a_N} \right| < 10^{-13}; \]  
(45)
where $a_m$ and $a_e$ are the accelerations of the Moon and the Earth, and $a_N$ is Newtonian acceleration. The Sun and the Moon all have the thin shell and the field profile outside of these objects are given by formula (26) with respective quantities. The accelerations $a_e$ and $a_m$ are given by

$$a_e \approx a_N \left\{ 1 + 18 \beta_1^2 \left( \frac{\Delta R_e}{R_e} \right)^2 \frac{\Phi_e}{\Phi_s} \right\};$$

$$a_m \approx a_N \left\{ 1 + 18 \beta_1^2 \left( \frac{\Delta R_e}{R_e} \right)^2 \frac{\Phi_e^2}{\Phi_s \Phi_m} \right\};$$

where $\Phi_e = 6.9 \times 10^{-10}$, $\Phi_m = 3.14 \times 10^{-11}$ and $\Phi_s = 2.12 \times 10^{-6}$ are Newtonian potentials on the surface of the Earth, the Moon, and the Sun respectively. When differentiating two free-fall accelerations, we have

$$\frac{|a_m - a_e|}{a_N} = (0.13) \beta_1^2 \left( \frac{\Delta R_e}{R_e} \right)^2;$$

combining with (45) and (48), we obtain result

$$\frac{\Delta R_e}{R_e} < 2.14 \times 10^{-6};$$

therefore

$$\beta < 0.076.$$  

This value is smaller than that in equation (40) about two orders of magnitude.

3.3. Fifth force
A fifth force is parameterized by a Yukawa potential following

$$U(r) = -\frac{m_1 m_2 e^{-r/\lambda}}{8\pi r};$$

where $m_1$, $m_2$ are masses of two bodies, $r$ is distance between them, $\varepsilon$ is the strength of the interaction and $\lambda$ is the range which is of the order of the size of vacuum chamber[17], namely $\lambda \approx R_{vac}$. The tightest bound on the strength of interaction is $\varepsilon < 10^{-3}$ [18]. Inside the chamber, one considers two identical bodies with uniform densities $\rho_c$, radius $r_c$ and masses $m_c$. If two these bodies satisfy thin shell condition, their field profile outside these bodies are given by

$$\phi(r) = \frac{\beta_1}{4\pi} \frac{3\Delta r_e m_c e^{-r/R_{vac}}}{r_c} + \varphi_{vac};$$

the interaction potential is

$$V(r) = -2\beta_1^2 \left( \frac{3\Delta r_e}{r_c} \right)^2 \frac{m_c^2 e^{-r/R_{vac}}}{8\pi r};$$

the bound on the strength of interaction becomes

$$2\beta_1^2 \left( \frac{3\Delta r_e}{r_c} \right)^2 < 10^{-3};$$

combining (54) and (32), we have

$$\frac{\Delta R_e}{R_e} = \left( \sqrt{\frac{33\alpha^2 - 16\rho_{out}\alpha\beta_1^2}{64\beta_1^2 \beta_1^2}} - \sqrt{\frac{33\alpha^2 - 16\rho_{in}\alpha\beta_1^2}{64\beta_1^2 \beta_1^2}} \right) \frac{1}{6\beta_1 \Phi_e} < \frac{10^{-3/2}}{3\sqrt{2}\beta_1};$$
\[ \beta < \left( \frac{4 \times 33 \beta^2 \Phi_e^2 \times 10^{-3}}{2 \cdot (\rho_{in} - \rho_{out})^2} \right)^{\frac{1}{2}} \] ; \tag{56}

\[ \beta < \left( \frac{4 \times 33 \times \frac{1}{5} \cdot (6.95 \times 10^{-10})^2 \times 10^{-3}}{2 \times (9.5 \times 10^3)^2} \right)^{\frac{1}{2}} = 7.67 \times 10^{-15}. \tag{57} \]

3.4. The constraint on the parameter of alpha

From experiments in solar system, Gu and Lin\cite{19} have obtained the constraints on Chameleon theories of f(R) as follows

\[-10^{-15} \leq F_R \leq 0; \tag{58}\]

with \( R \sim 3 \times 10^5 H_0^2 \) and

\[ 0 \leq RF_{RR} \leq 0.4; \tag{59} \]

for

\[ R \geq 3 \times 10^5 H_0^2; \tag{60} \]

where \( F(R) = f(R) - R = 2\Lambda + \frac{\alpha}{R}(1 + R^2 + R^4)e^{-\beta R} \) and \( F_R = dF/dR; F_{RR} = d^2F/dR^2; \)

with

\[ H_0^2 \approx 10^{-35} \Rightarrow R \approx 3 \times 10^{-30} m^{-2}. \tag{61} \]

In equation (28), we have approximately

\[ e^{-\beta R} = 1 - \beta R; \tag{62} \]

therefore

\[ F_R = \alpha \left[ -3\beta R^2 - \frac{1}{R^2} - 2(\beta - 1) R + 1 \right]; \tag{63} \]

\[ F_{RR} = 2\alpha \left[ -\beta + \frac{1}{R^3} - 3\beta R + 1 \right]. \tag{64} \]

Since \( R \ll 1 \), therefore in expressions(63),(64), we have

\[ F_R \approx -\frac{\alpha}{R^2}; \tag{65} \]

\[ F_{RR} \approx \frac{2\alpha}{R^3}; \tag{66} \]

when combining (65),(61) and (58), we have

\[ \alpha \leq 9 \times 10^{-75}; \tag{67} \]

from (66),(61),(60) and (59), we have

\[ \alpha \leq 1.8 \times 10^{-60}; \tag{68} \]

thus, the constraint on the parameter of alpha is

\[ \alpha \leq 9 \times 10^{-75}. \tag{69} \]

From the constraints on beta parameter in formula (57) and on the alpha parameter in the formula (69), we see that these parameters are very small. Therefore we expect that the difference between this model and General relativity theory is observed only in regions that cosmic curvature is very large as in very early time of universe\cite{20} and in regions of edge of black holes. In solar system, where the cosmic curvature is not large, predictive results from this model and Einstein’s theory are indistinguishable.
4. Conclusion
In this paper, we have obtained constraints on the parameters of $\alpha$ and $\beta$ of the model based on solar system experiments. The constraints on the parameter $\beta$ required by the experiments of the thin shell condition of the Earth and the Equivalence principle are not too rigorous. The requirement from the absence of the fifth force caused by the scalar field puts a quite strict constraint on the parameter $\beta$ and this is also the general status of many f(R) gravity modified models for this experiment. The constraint on the parameter $\alpha$ from experiments in solar system is rigorous. The above constraints are only based on experiments in the solar system. There are indications that the constraints from the inflation of the universe to these parameters are even more severe, which will be examined again in a near future article.

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