The Shuvaev Transform and the Skewness Effect at small $x$

T. Teubner, A.D. Martin, C. Nockles, M.G. Ryskin, and A.G. Shuvaev

1- Department of Mathematical Sciences
University of Liverpool, Liverpool L69 3BX, U.K.

2- Department of Physics and Institute for Particle Physics Phenomenology
University of Durham, Durham DH1 3LE, U.K.

3- Petersburg Nuclear Physics Institute
Gatchina, St. Petersburg, 188300, Russia

We describe how in the small $x$ regime skewed parton distributions can be obtained from the diagonal ones in a parameter free way, using the so-called Shuvaev transform.

Numerical results based on input from global fit partons are presented, and an easy-to-use package for fast interpolation with grid files is provided by us. We briefly comment on the comparison with another approach to determine skewed partons.

1 Introduction

Generalised parton distributions (GPDs) are important for the description of diffractive processes. Examples range from diffractive vector meson production, over elastic (exclusive) production of heavy quarks, jets or the Higgs at the LHC, to inclusive diffraction. In perturbative QCD diffractive (semi-) hard scattering is mediated by a two-parton exchange. In the high energy limit (at small $x$) the two-gluon exchange dominates for the processes mentioned, whereas contributions from quarks enter at higher order. In Fig. 1 we show schematically how the two different plus momentum fractions $x_1$ and $x_2$ enter the GPD $H(x, \xi, \mu^2, t)$, where we are using the convention $x_{1,2} = x \pm \xi$. (In the following we will suppress the variables $\mu$ and $t$ as appropriate.) Approximating $H(x, \xi)$ by the diagonal PDF $H(x, 0)$ is only valid at leading ln $1/x$. However, as is well known and as we will see below, for realistic energies the effects from ‘skewing’ can be sizeable and lead, e.g. to an enhancement of elastic $\Upsilon$ production at HERA by nearly a factor of two. In general GPDs are non-perturbative objects and have to be determined from data in a way similar to the global fits of the diagonal partons. Unfortunately, while from the theoretical viewpoint deeply virtual Compton scattering (DVCS) seems to be the ideal process for this purpose, in practice the accuracy and range of the DVCS data is quite limited, especially in the high-energy regime. An unambiguous determination of the full form of $H_{g,q}(x, \xi)$ is therefore not possible at present, and most approaches rely on additional model assumptions, see [1]. Here we will take a different route and determine, in a parameter free way, the GPDs in the small $x$ regime from the diagonal partons alone. In the following we will describe how this is possible under certain additional assumptions motivated by the high-energy limit of QCD, and show how GPDs can be obtained using known (global fit) diagonal partons.

DIS 2009
2 Shuvaev transform in a nutshell

On first sight it may look impossible to reconstruct non-diagonal partons at small $x, \xi$ from diagonal input $H(x, \xi = 0)$ alone. However, it has been shown [2] that the anomalous dimensions governing the evolution of the Gegenbauer moments $G_N$ of $H(x, \xi)$ are equal to the anomalous dimensions of the conventional Mellin moments, $M_N = \int_0^1 x^N H(x, 0) \, dx$. This is a consequence of the conformal invariance of the evolution equations. (Note that $\xi$ is not changed during evolution.) Furthermore, the property of polynomiality, $G_N = \sum_{n=0}^N c_n^N \xi^{2n}$, allows the determination of all Gegenbauer moments $G_N$ from conventional PDFs, i.e. from $c_0^N = M_N$, with $O(\xi^2)$ accuracy. The Shuvaev transform is an integral transformation which determines the $x$ dependence of $H(x, \xi)$ for given small $x, \xi$ from the diagonal partons, $H(x, 0)$. Parametrically the accuracy is of $O(\xi^2)$ at leading order (LO). At next-to-leading order (NLO) the conformal invariance is broken by operator mixing of $O(\xi\alpha_s)$, therefore lowering the accuracy accordingly, which is however still sufficient for the small $\xi \ll 1$ region considered here. There is an important caveat, which seems to severely impair the validity of the Shuvaev transform. In order to get the $x$ distribution from the Gegenbauer moments $G_N$, an analytical continuation to complex $N$ is needed. So, for the solution to be unique there must be no singularities in the input distribution in the right-half plane. Hence we have to assume the additional condition of a Regge-based form of the low-$x$ input distribution, with no singularities in the right-half plane ($j > 1$) in the space-like ($\xi < |x|$) domain, although higher-spin resonances exist in the time-like ($\xi > |x|$) region. Note that any function $H^t$ with support in the time-like domain only (but with $H^t = 0$ for $|x| > \xi$) can be added to the result from the Shuvaev transform. However this is not relevant for predictions of diffractive cross sections using GPDs, since such calculations depend on $H(x, \xi)$ for $|x| > \xi$ only.

The transform can be expressed by a double integral [3] and reads $(g(s) = \frac{1}{2\pi i} \int_0^{4(1-s)/(\pi(1-2s))} dz')$

$$H_q(x, \xi) = \int_1^{-1} dx' \left[ \frac{2}{\pi} \text{Im} \int_0^1 \frac{ds}{y(s) \sqrt{1 - y(s)}} \frac{d}{dx'} \left( \frac{q(x')}{|x'|} \right) \right],$$  

$$H_g(x, \xi) = \int_1^{-1} dx' \left[ \frac{2}{\pi} \text{Im} \int_0^1 \frac{ds(x + (1 - 2s))}{y(s) \sqrt{1 - y(s)}} \frac{d}{dx'} \left( \frac{g(x')}{|x'|} \right) \right]$$

for quarks and gluons. These integrals have to be understood as principal value integrals and can be solved numerically by standard methods for given input distributions. Note that for our aim we need only input for $|x| > \xi$, and not from the ERBL regime, $|x| < \xi$.

For many processes of interest $x_1 > x_2$, so $H(x, \xi \to x)$ is relevant. In the case of a pure power form of the diagonal PDF, e.g. $H_q(x, 0) = xg(x) \sim x^{-\lambda}$, the Shuvaev transform can be solved analytically for the limit of ‘maximal skewing’, $\xi \to x$, and one obtains the skewing enhancement factors

$$R^q := \frac{H(\xi, \xi)}{H(2\xi, 0)} = \frac{2^{2\lambda+3}}{\sqrt{\pi}} \frac{\Gamma(\lambda + \frac{5}{2})}{\Gamma(\lambda + 3 + p)},$$

where $\lambda(\mu^2)$ is the parton’s effective power and $p = 0 \ (1)$ for quarks (gluons). Figure 2 shows $R^q$ as a function of $\lambda$. In applications frequently this analytical result is used. We would like to stress that our numerical calculations do not rely on this approximation. In the following we will present the full numerical results and compare them to the analytical approximation.
3 Numerical results for $H(x, \xi)$ at small $x$

In Fig. 3 we give our numerical results for the ratio $R = H(x/2, x/2)/H(x, 0)$ (upper panel, solid line, three scales as indicated), based on the Shuvaev transform (2) and using MSTW08 [4] (diagonal) gluons as input. For comparison the analytical approximation (3) is shown as dotted lines with $\lambda$ evaluated at $x$ and plotted in the lower panel. One can see that the approximation of a pure power form of the diagonal PDF is very good for $x < 10^{-2}$, whereas at larger $x$ the deviations become sizeable. However, close to the input scale the MSTW08 gluon starts falling (and eventually turns negative) for small $x$, resulting in a skewing correction ratio smaller than one. In this regime the approximation of a pure power is poor.

As expected from the analytical approximation, the skewing corrections are larger for quarks than for gluons. This can be seen in Fig. 4 where again the full numerical result is compared to the analytical approximation. In the upper (middle) panel the sum of the light quarks (anti-quarks) is shown; in the lower panel the values for the respective effective power $\lambda$ are given. Here the differences between full numerical result and analytical approximation are more pronounced, especially for the quarks. As for gluons, $\lambda$ is getting larger with growing scale $\mu$, resulting in larger skewing corrections.

In general, amplitudes for diffractive processes involve an integration over all possible values of $x_1$ and $x_2$. Hence the knowledge of $H(x, \xi)$ in a wide range of $x, \xi$ is required, and not only $H(x, \xi \to x)$. In Fig. 5 $H(x, \xi)$ is shown as a function of $x/\xi$ for $\xi = 10^{-3}$ for three scales as indicated (solid lines). This is to be compared, on the one hand, to the diagonal...
partons (dotted lines), and, on the other hand, to the diagonal parton multiplied by a skewing correction factor (dashed lines) as discussed above. It is clear that the off-diagonal $H(x, \xi)$ grows only gradually from the diagonal limit $H(x, 0)$ to the point of maximal skewing, $x = \xi$. Therefore a naive application of a skewing enhancement factor will in general overestimate the skewing corrections, especially for quarks.

The results for the Shuvaev transform discussed so far are obtained using global fit diagonal partons from MSTW. Results based on CTEQ partons are also available but cannot be shown here for lack of space, see [5] for details. The skewed quark and gluon distributions obtained from CTEQ6.6M [6] partons are similar to those shown here, but reflect the in general steeper growth for small $x$ of the CTEQ distributions compared to those from MSTW. For easy applications of the skewed partons presented here we provide a simple and fast interpolation routine in Fortran, which uses grid files similar to standard global fit partons, but taking into account the additional variable $x/\xi$. The routine, together with grid files for MSTW08 and CTEQ6.6M (and also grid files for the corresponding LO distributions) can be downloaded from [7]. Grid files for other PDFs can be provided upon request.

4 Comparison with an alternative approach

The Shuvaev transform discussed and used here allows to relate, at small $x$, skewed distributions, which in general contain new, non-perturbative information, to diagonal ones. An alternative approach is to fit both diagonal and skewed distributions from DIS and DVCS data, so constraining the additional parameters introduced to model the GPDs at the input scale, from where they are evolved to higher scales using pQCD. Such a program has been carried out in [1]. Their results for the ratio $r$ of skewed over diagonal partons, obtained in the $\overline{\textrm{MS}}$ scheme at NLO, are close to our results based on the Shuvaev transform using

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig4}
\caption{Skewing factors for MSTW08 quarks (upper) and anti-quarks (middle panel) for three different scales as indicated. Solid lines: full numerical results based on (1), dotted lines: analytical approximation (3). Lower panel: effective power $\lambda$ used for $R_a$.}
\end{figure}
Figure 5: $H(x, \xi)$ as a function of $x/\xi$ (solid lines) compared to the diagonal partons $H(x, 0)$ (dotted lines) and to $R^a \cdot H(x, 0)$ (dashed lines). MSTW08 [4] partons are used as input.

recent global fit diagonal PDFs as input, see Fig. 7 in the first Ref. of [1]. Large differences between the two approaches, as they appear in LO, are, in our opinion, not unexpected already from the poor description of the global data when determining the diagonal partons at LO [4].

5 Conclusions

The Shuvaev transform is consistent with a modelling of GPDs based on conformal theory and allow, in the small $x$ regime, the determination of skewed from diagonal partons. The results shown above are close to those obtained by fitting DVCS data at NLO [1]. Our approach has uncertainties in the diagonal PDFs used as input for the Shuvaev transform. The alternative approach [1] has limitations due to the DVCS data fitted and ambiguities in the modelling. We firmly believe that for small $x$ our parameter free predictions are a good approximation and can be used safely. Here we have discussed GPDs based on MSTW08 global partons; results using CTEQ are also available, see [5, 7].

We thank Dieter Müller and Markus Diehl for interesting discussions.

References


DIS 2009