Neutralino dark matter and the little hierarchy problem in MSSM

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Abstract

The little hierarchy problem is a fine tuning problem about masses of scalar top and Higgs in MSSM. We considered the light Higgs scenario to reduce tuning about this problem and found parameter regions where tuning is best relaxed. To discuss cosmological appropriateness of this regions, we calculated thermal relic density of the lightest neutralino in the parameter region.

1 Introduction

The supersymmetric standard model is a hopeful candidate for the physics beyond the standard model. The supersymmetry is mainly motivated by the fine tuning problem about the Higgs mass correction. However, even in the supersymmetric standard model, there is a small level fine tuning about scalar top mass. It is sometimes called the little hierarchy problem.

1.1 The little hierarchy problem

We will focus in the case of the minimal supersymmetric standard model(MSSM). The little hierarchy problem in MSSM is related to stop and neutral Higgs masses. Before the detailed explanation about the little hierarchy problem, we briefly discuss the MSSM Higgs mass bound. We gained the standard model Higgs(φ) mass bound in the LEP2 experiment by measuring the interaction between two Z boson and φ (Fig. 1), the coupling of which we write g_{Zφ}. In MSSM, there are two Higgs doublets, up-type Higgs H_u and down-type Higgs H_d, so there are two Z-Z-Higgs interaction (Fig. 2, 3). One of them, Z-Z-H_u coupling is almost equal to g_{Zφ} because tan β ≫ 1. Therefore, H_u mass bound is almost the same as the standard model Higgs mass m_{H_u} < 114[GeV]. On the other hand, Z-Z-H_d coupling is much smaller than g_{Zφ}, so H_d mass is not limited by the LEP2 experiment because this coupling was not measured in LEP2. For this reason, m_{H_d} can be lighter than 114GeV. The stop mass is limited by this H_u mass bound. Physical Higgs H and φ are respectively the heaviest and the lightest Higgs in the mass eigenstates of H_u and H_d. The mass matrix of H_u, H_d is

\[
\begin{pmatrix}
H_d \\
H_u
\end{pmatrix}
\begin{pmatrix}
m_A^2 & (m_A^2 + m_t^2) \sin \beta \over 2 \\
-(m_A^2 + m_t^2) \sin \beta \over 2 & m_t^2 + \frac{3Y_t^4(H_u^0)^2}{4\pi}m_t^2 \log \frac{m_t^2}{m_i^2}
\end{pmatrix}
\]

(1)

where m_A, m_t, Y_t, m_i, and m_t are CP-odd Higgs mass, Z boson mass, top Yukawa coupling, stop mass, and top mass, respectively. Generally, m_A is supposed to be much higher than weak scale i.e. m_A ≫ m_t. m_h and m_H can be obtained by diagonalizing this mass matrix. Since diagonalization makes smaller/larger diagonal components more smaller/larger, m_h is lighter than the lower right component of (1)

\[
m_h^2 \leq m_t^2 + \frac{3Y_t^4(H_u^0)^2}{4\pi}m_t^2 \log \frac{m_t^2}{m_i^2}.
\]

(2)
Since \( m_{H_u} \lesssim 114 \text{[GeV]} \), this relation limits the stop mass \( m_{\tilde{t}} \lesssim 500 \text{[GeV]} \).

On the other hand, there is a condition to stabilize Higgs potential

\[
\frac{m^2}{2} \sim -\mu^2 + \frac{3\lambda^2}{4\pi^2}m_{\tilde{t}}^2 \log \frac{\Lambda}{m_{\tilde{t}}},
\]

where \( \Lambda \) is the cutoff scale. Since \( m_z \) is in weak scale and \( m_{\tilde{t}} \lesssim 500 \text{[GeV]} \), this relation needs the fine tuning. If we suppose \( m_{\tilde{t}} \simeq 500 \text{[GeV]} \), then the required tuning is

\[
\frac{m_{\tilde{t}}^2/2}{\frac{\lambda^2}{4\pi^2}m_{\tilde{t}}^2 \log \frac{\Lambda}{m_{\tilde{t}}}} \sim 0.005(0.5\%).
\]

We take this tuning innatural, and study the solution for this problem.

![Diagram](image)

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### 1.2 Solution for the little hierarchy problem

To reduce the fine tuning about the little hierarchy problem, we assume the CP-odd Higgs mass at the weak scale \( m_A \sim m_e \). Then, the upper left component of Higgs mass matrix (1) is smaller than the lower right one because of a mass correction. Now, light or heavy Higgs is composed mainly of \( H_d \) or \( H_u \), respectively, after diagonalizing Higgs mass matrix. Thereby, the new mass bound of Higgs mass eigenstates is \( m_H \geq 114 \text{[GeV]} \), and \( m_h \) can be lighter than \( 114 \text{[GeV]} \), from the result of LEP2.

Now, we can set the light Higgs mass below \( 114 \text{[GeV]} \), so stop mass can be lighter than before from eq.(2) and the fine tuning about eq.(3) can be reduced. Allowed regions in this scenario have been examined (11) and points in which the fine tuning is best reduced is \( (m_h \sim 90 \text{[GeV]}, m_H \sim 114 \text{[GeV]}, m_A \sim 95 \text{[GeV]}, m_{H^\pm} \sim 120 \text{[GeV]} \) where \( m_{H^\pm} \) is charged Higgs mass. We take A-term \( A \) where weak scale is \( 300 \text{[GeV]} \) and \( \mu \)-term is \( 250 \text{[GeV]} \). In the paper, the authors calculated allowed regions about inverse Higgs mass scenario in the case where \( A \sim 300, 325, 350 \text{[GeV]} \), and the point in which the fine tuning is best relaxed is almost the same in each case. We can reduce tuning about the little hierarchy problem to about 10% in this parameter point.

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### 2 The little hierarchy problem and dark matter

So far, we have considered the light Higgs scenario to reduce tuning on the little hierarchy problem, and seen the parameter point in which the tuning is best reduced. However, the parameter point cannot be experimentally checked until the LHC starts. Yet, we may be able to get some hints at the parameter point by considering the point cosmologically. In this research, we consider the thermal production of neutralino dark matter in the parameter region, and discuss the cosmological appropriateness of this point.
2.1 Thermal relic density of neutralino

The lightest supersymmetric particle (LSP) is a good candidate for dark matter because of R-parity when it is electrically neutral. One of LSP candidates is neutralino. Neutralino is linear combinations of four electrically neutral supersymmetric particles

$$\chi_l^0 = N_l B + N_{lW} W^3 + N_{lH_u} H_u + N_{lH_d} H_d$$

(4)

where $\chi_l^0$ is $l$th neutralino and $l$ runs from 1 to 4. $B$, $W^3$, $H_u$ and $H_d$ are bino, wino, up-type Higgsino, and down-type Higgsino, respectively. $N_-$ are their coefficients. Let us consider thermal production of the lightest neutralino. Neutralino was in thermal equilibrium in the early universe. After that, neutralino decouples from thermal bath and the number density of the neutralino in comoving volume was fixed. We can know thermal relic density on neutralino by solving Boltzmann equation

$$\frac{dn}{dt} + 3H(t)n = -\langle \sigma v \rangle (n^2 - n_{eq}^2).$$

(5)

Here $n$ and $H(t)$ is neutralino number density per comoving volume (eq. means one in thermal equivalence) and Hubble parameter. $\langle \sigma v \rangle$ is thermal averaged production of velocity and 2-body scattering cross section of neutralino. We thought only standard model fermion final states except for top quark because neutralino mass is less than 60[GeV] in the parameter point where we think about.

3 Conclusion

We have solved the Boltzmann equation, and found that the neutralino energy density in the parameter region where tuning on the little hierarchy problem is best reduced

$$\Omega h^2 = 8.6 \times 10^{-4}.$$ 

(6)

This is less than the observed relic density of neutralino, $\Omega_{obs}h^2 = 0.105 \pm 0.004$. So this parameter region is allowed if there are some components of dark matter other than neutralino. We are going to research the parameter region where neutralino can be the main component of dark matter and how large tuning on the little hierarchy problem there are.

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References

