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SUPERMEMBRANES AND THE SIGNATURE OF SPACETIME

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ABSTRACT

We consider extended objects with s space and t time world-volume dimensions moving in a spacetime with $S \geq s$ space and $T \geq t$ time dimensions. The requirements of spacetime supersymmetry and world-volume fermionic gauge invariance severely restrict the possible values of S and T. If we furthermore insist that the transverse group $SO(S-s,T-t)$ be compact to avoid ghosts, then $t=T$. The results may be interpreted as a set of superconformal field theories with $s+t \leq 6$ and $N \leq 8$ whose superconformal groups are in one-to-one correspondence with those in Nahm's classification. Although the choice $t=T=1$ is not uniquely singled out, it does seem to play a preferred role.

1. *Introduction*

If our senses are to be trusted, we live in a world with three space and one time dimensions. However, the revival of the Kaluza-Klein idea, brought about by supergravity and superstrings, has warned us that this may be only an illusion. In any case , there is a hope, so far unfulfilled, that the four-dimensional structure that we apparently observe may actually be predicted by a "Theory of Everything". Whatever the outcome, imagining a world with an arbitrary number of space dimensions has certainly taught us a good deal about the properties of our three-space-dimensional world.

In spite of all this activity, and in spite of the popularity of Euclidean formulations of field theory, relatively little effort has been devoted to imagining a world with more than one time dimension[F1]. This is no doubt due partially to the psychological difficulties we have in treating space and time on the same footing. As H.G. Wells reminds us in *The Time Machine*, "There is, however, a tendency to draw an unreal distinction between the former three dimensions and the latter, because it happens that our consciousness moves intermittently in one direction along the latter from the beginning to the end of our lives." There are also more justifiable reasons associated with causality . Nevertheless, one might hope that a Theory of Everything should predict not only the dimensionality of spacetime, but also its signature.

For example, quantum consistency of the superstring requires 10 spacetime dimensions, but not necessarily the usual (9,1) signature. The signature is not completely arbitrary, however, since spacetime supersymmetry allows only (9,1), (5,5) or (1,9). Unfortunately, superstrings have as yet no answer to the question of why our universe appears to be four-dimensional, let alone why it appears to have signature (3,1).

The whole debate about the dimensionality of spacetime has recently received a new impetus with the discovery of supermembranes [5,6,7]. We note , in particular, the following four observations:

1) The upper limit of D=11 on the dimensionality of spacetime permitted by supersymmetry now follows automatically from the supermembrane action, without any additional assumptions about the consistency of massless higher spin field theories . See Section 3.

2) The maximal D=11 supermembrane resolves the old dilemma of eleven-dimensional supergravity as to whether the Freund-Rubin [8] mechanism favours compactification to four or seven spacetime dimensions: it definitely favours four [9,10] !

3) On the resulting $(AdS)_4 \times S_7$ background, the supermembrane equations admit a stable vacuum solution for which the three-dimensional world- volume of the membrane occupies the $S_1 \times S_2$ boundary of $(AdS)_4$: the Membrane at the End of the Universe [10]. Fluctuations about this vacuum are described by a superconformal field theory with OSp(8/4) as the super conformal group. The superconformal invariance resolves the non-renormalizability problem and, in fact, guarantees a free-field theory [11,12]. The closure of the full quantum OSp(8/4) algebra has also been verified and, in this respect, gives rise to a consistent anomaly-free theory [13,14] .

4) A universe with three space and one time dimension is, of course, singled out if a membrane with two space and one time dimensions is to act as the AdS boundary at spatial infinity.

Of course, the eleven-dimensional supermembrane is not the only supersymmetric extended object allowed at the classical level, and the vacuum state discussed above is not the only stable solution. The purpose of the present paper, therefore, is to imagine a world with an arbitrary number S of space dimensions and an arbitrary number T of time dimensions and then to see how far classical supermembranes restrict not only S+T but S and T separately. To this end we also allow an (s,t) signature for the world-volume of the membrane where $s \leq S$ and $t \leq T$ but are otherwise arbitrary. The discussion will take place within the framework of the Green-Schwarz-type actions of the supermembranes introduced by Bergshoeff, Sezgin and Townsend [6] following the work of Hughes, Liu and Polchinski [5]. As described in Section 2, these display supersymmetry in spacetime and a fermionic gauge invariance or "Siegel symmetry" on the world-volume which, as we shall see, together place severe restrictions on S, T, s and t.

2. *The Classical Supermembranes*

Let us consider a d-dimensional extended object moving in a D-dimensional, N-extended superspace. In the case of the usual Minkowski signatures $t = T = 1$, Achucarro et al [7] have classified those values of (d, D, N) compatible with spacetime supersymmetry and world-volume fermionic gauge invariance. They found that the crucial constraint that follows from the latter is

$$D - d = \frac{nN}{4} \qquad d \geq 2 \qquad (1)$$

where n is the real dimension of the minimal spinor in D dimensions. This equation is satisfied only for N=1 and then only for certain values of d and D. The d=2 case is special because of the possibility of having left and right moving modes. Here we can have either

$$D - 2 = \frac{nN}{4} \qquad (2)$$

which is satisfied only for $N = 2$, or

$$D - 2 = \frac{nN}{2} \qquad (3)$$

which is satisfied for N=1. In all cases, these conditions are precisely the requirement of equality of bose and fermi degrees of freedom on the world-volume. Starting from D bosonic coordinates and taking into account d world-volume reparametrisations yields D-d boson degrees of freedom. Whereas $nN/4$ is the number of fermionic degrees of freedom when one takes into account the fermionic gauge invariance (which removes half of the fermionic coordinates) and the fact that they obey first order equations (which removes half again if $d > 2$). There are four types of solution as shown in Table 1, with $8+8$, $4+4$, $2+2$, or $1+1$

Bose $D-d$	Fermi $nN/4$	Algebra
8	8	$\mathbb{O}$
4	4	$\mathbb{H}$
2	2	$\mathbb{C}$
1	1	$\mathbb{R}$

Table 1. Bose-Fermi matching

degrees of freedom respectively. Since the numbers 1, 2, 4 and 8 are also the dimension of the four division algebras, these four types of solution are referred to as real, complex, quaternion and octonion respectively. The connection with the division algebras can, in fact, be made more precise [15,16]. The possible values of D and d are displayed in the "brane scan" of fig. 1 . Note that the four classical Green-Schwarz superstrings ($d=2$) in $D=3,4,6$ and 10 emerge as special cases. As described in [17,7], the diagonal lines relate different extended objects by the process of "simultaneous dimensional reduction" while the horizontal lines correspond to swapping the 3-index antisymmetric tensor field strength by its dual.

It is not difficult to repeat the above analysis for arbitrary signatures (s,t) and (S,T) of world-volume and space-time respectively. The action is

$$S = \int d^d\xi \{ \tfrac{1}{2}\sqrt{(-1)^t g}\; g^{ij} E_i{}^a E_j{}^b \eta_{ab} - \frac{(d-2)}{2}\sqrt{(-1)^t g} + \frac{1}{d!}\varepsilon^{i_1 i_2 \ldots i_d} E_{i_1}{}^{A_1} E_{i_2}{}^{A_2} \ldots E_{i_d}{}^{A_d} B_{A_d \ldots A_2 A_1} \} \tag{4}$$

where ξ^i $(i=1,2\ldots d)$ are coordinates for the world-volume with metric g_{ij} .

The $E_i^{\ A}(A=a,\alpha)$ are defined by

$$E_i^{\ A} = \partial_i z^M E_M^{\ A} \tag{5}$$

where $z^M=(x^m, \vartheta^\mu)$ are the coordinates of D-dimensional superspace ($m=1,2...D$) and $E_M^{\ A}$ is the supervielbein. The superspace d-form

$$B = \frac{1}{d!} E^{A_1} \ldots E^{A_d} B_{A_d \ldots A_1} \tag{6}$$

is the potential for a closed (d+1)-form H=dB. The Dirac matrices obey

$$\Gamma^a \Gamma^b + \Gamma^b \Gamma^a = 2\,\eta^{ab} \tag{7}$$

where

$$\eta^{ab} = \mathrm{diag}(\underbrace{-,-,\cdots,-}_{T}, \underbrace{+,+,\cdots,+}_{S}) \tag{8}$$

and $D=S+T$. Similarly g_{ij} has signature (s,t) where $d=s+t$ and $s\le S$ and $t\le T$.

The fermionic gauge invariance is

$$\delta z^M E_M^{\ \alpha} = 0 \qquad\qquad \delta z^M E_M^{\ \alpha} = (1+\Gamma)^\alpha{}_\beta \kappa^\beta(\xi) \tag{9}$$

where

$$\Gamma^\alpha{}_\beta = \frac{(-1)^{d(d+1)/4}}{d!\sqrt{(-1)^t g}} \varepsilon^{i_1 i_2 \ldots i_d} E_{i_1}^{\ a_1} E_{i_2}^{\ a_2} \ldots E_{i_d}^{\ a_d} (\Gamma_{a_1 a_2 \ldots a_d})^\alpha{}_\beta \tag{10}$$

where the parameter κ^β is a world-volume scalar but spacetime spinor. For $d>2$ the equation of motion for g_{ij} is

$$g_{ij} = E_i^{\ a} E_j^{\ b} \eta_{ab} \tag{11}$$

Using this equation, one can show that $\Gamma^2 = \mathbb{1}$ and hence $(1\pm\Gamma)/2$ are projection operators. Hence the fermionic gauge invariance allows us to gauge away half the ϑ components. In <u>flat</u> superspace, the constraints required for fermionic gauge invariance reduce to

$$H = \frac{i}{2(d-1)!} e^{a_{d-1}} e^{a_{d-2}} \ldots e^{a_1} d\bar\vartheta \Gamma_{a_1 \ldots a_{d-1}} d\vartheta \tag{12}$$

for the superspace (d+1)-form H and

$$T^A = (\tfrac{i}{2} d\bar\vartheta \Gamma^a d\vartheta, 0) \tag{13}$$

where

$$e^A = (\delta^a{}_m (dx^m - \tfrac{i}{2} \bar\vartheta \Gamma^m d\vartheta), d\vartheta^\alpha) \tag{14}$$

The closure of H is then equivalent to the Γ-matrix identity

$$(d\bar\vartheta \Gamma_a d\vartheta)(d\bar\vartheta \Gamma^{a\, b_1 \ldots b_{d-2}} d\vartheta) = 0 \tag{15}$$

for a commuting spinor $d\vartheta$.

It is not difficult to show that the solutions of this identity occur if and only if there is a matching between the bosonic and fermionic degrees of freedom and we again recover (1), (2) and (3). As in the case of Minkowski signature, one finds that after fixing the gauge for the world-volume diffeomorphisms and for the fermionic gauge invariance, there is a residual fermionic symmetry which is nothing but a world-volume supersymmetry.

So far, therefore, we have learned nothing new by allowing arbitrary signature. However, severe constraints on possible supermembrane theories will now follow by demanding space-time supersymmetry.

3. *Spacetime Poincaré Supersymmetry*

We restrict ourselves in this section to N=1 flat superspace, and require ϑ^{α} to be a minimal spinor i.e. Majorana, Weyl, Majorana-Weyl etc, whenever it is possible to impose such a condition. We furthermore assume invariance under the generalized super-Poincaré group super-IO(S,T), as required by the superspace construction of Section 2. Such supermembranes (and their compactifications discussed in Section 5) are the only ones currently known. In Section 7, however, we consider the possibility of other supermembrane theories obtained by requiring space-time supersymmetry but with a different supergroup. For the moment, however, we shall require super-Poincaré which means, in particular, that the anticommutator of two supersymmetry charges Q yields a translation

$$\{ Q, Q \} \sim P \tag{16}$$

This is only possible for certain values of S and T when Q is a minimal spinor.

Those values of S and T permitting minimal spinors have been determined by Kugo and Townsend [3]. See also the works of van Nieuwenhuizen [4], Coquereaux [18] and Freund [19]. For the Clifford algebra given by (7) and (8), there exist matrices A and B for which

$$\Gamma_a^{\dagger} = (-1)^T A\, \Gamma_a A^{-1} \quad , \; A A^{\dagger} = 1 \tag{17}$$

$$\Gamma_a = \eta\, B^{-1} \Gamma_a^{*} B \quad , \; B B^{\dagger} = 1, \quad B^{*}B = \varepsilon \tag{18}$$

where ε and η are given in table 2.

S-T mod 8	ε	η
0,1,2	+1	+1
6,7,8	+1	-1
4,5,6	-1	+1
2,3,4	-1	-1

Table 2. Values of ε and η.

We can choose a basis such that

$$\Gamma_a^{\dagger} = \begin{cases} -\Gamma_a & a = 1,\dots,T \\ +\Gamma_a & a = T+1,\dots,D \end{cases} \tag{19}$$

and

$$A = \Gamma_1 \Gamma_2 \dots \Gamma_T \quad . \tag{20}$$

The charge conjugation matrix is defined as $C = B^T A$. The properties of A and B then imply

$$\tilde{\Gamma} = (-1)^T \eta \; C \, \Gamma \, C^{-1} \tag{21}$$

$$\tilde{C} = \varepsilon \, \eta^T (-1)^{T(T+1)/2} C \qquad C^{\dagger} C = 1 \tag{22}$$

where the tilda denotes transpose. For D even, we can also define the projection operator

$$P_{\pm} = \frac{1}{2} \left[1 \pm (-1)^{(S-T)/4} \, \Gamma^{D+1} \right] \tag{23}$$

where

$$\Gamma^{D+1} \equiv \Gamma^1 \Gamma^2 \dots \Gamma^D \tag{24}$$

Using the above properties we find that we can have the minimal spinors given in Table 3.

The next task is to check which of these possibilities admits the super-Poincaré algebra. The part of the superalgebra which is the same in each case is

$$[M_{ab}, M_{cd}] = -i \left(\eta_{bc} M_{ad} - \eta_{ac} M_{bd} - \eta_{bd} M_{ac} + \eta_{ad} M_{bc} \right)$$

$$[M_{ab}, P_c] = i \left(\eta_{ac} P_b - \eta_{bc} P_a \right)$$

$$[P_a, P_b] = 0$$

$$[M_{ab}, Q_\alpha] = -\frac{i}{2} \left(\Gamma_{ab} Q \right)_\alpha \tag{25}$$

We now examine the {Q,Q} anticommutator. Consider first S-T = 0,1,2 mod 8 for which Q_α is Majorana. The only possible form for the anticommutator is

$$\{ Q_\alpha, Q_\beta \} = (\Gamma^a C^{-1})_{\alpha\beta} P_a \; . \tag{26}$$

Since the left hand side is symmetric under interchange of α and β we require

$$\widetilde{(\Gamma_a C^{-1})} = \Gamma_a C^{-1} \tag{27}$$

but

$$\begin{aligned} \widetilde{(\Gamma_a C^{-1})} &= \tilde{C}^{-1} \tilde{\Gamma}_a \\ &= \varepsilon \; \eta^T (-1)^{T(T+1)/2} C^{-1} \tilde{\Gamma}_a \\ &= \varepsilon \; \eta^{T+1} (-1)^{T(T-1)/2} \Gamma_a C^{-1} \quad . \end{aligned} \tag{28}$$

Now from Table 1, $\varepsilon = \eta = +1$ for $S - T = 0,1,2 \bmod 8$ and hence

$$\widetilde{(\Gamma_a \; C^{-1})} = (-1)^{T(T-1)/2} \Gamma_a \, C^{-1} \tag{29}$$

which is compatible with (27) only if $T = 0,1 \bmod 4$. Now consider the sub-case $S - T = 0 \bmod 8$, $T = 0, 1 \bmod 4$. Define

$$Q_{\pm\alpha} = (P_\pm \; Q)_\alpha \; . \tag{30}$$

From (26) we have

$$\begin{aligned} \{ Q_{\pm\alpha} \, , Q_{\pm\beta} \} &= P_{\pm\alpha\gamma} \{ Q_\gamma , Q_\delta \} \; P_{\pm\delta\beta} \\ &= (P_\pm \; \Gamma^a \; C^{-1} \tilde{P}_\pm \,)_{\alpha\beta} P_a \end{aligned} \tag{31}$$

but

$$\begin{aligned} C^{-1} \tilde{P}_\pm &= C^{-1} (1 \pm \tilde{\Gamma}_D \tilde{\Gamma}_{D-1} \ldots \tilde{\Gamma}_1) \\ &= (1 \pm \Gamma_D \Gamma_{D-1} \ldots \Gamma_1) \; C^{-1} \\ &= (1 \pm (-1)^{D(D-1)/2} \; \Gamma^{D+1}) \; C^{-1} \end{aligned}$$

and therefore

$$\begin{aligned} P_\pm \Gamma_a C^{-1} \tilde{P}_\pm &= P_\pm (1 \mp (-1)^{T(2T-1)} \Gamma^{D+1}) \; \Gamma_a \, C^{-1} \\ &= \begin{cases} 0 & T = 0 \bmod 4 \\ P_\pm \Gamma_a \, C^{-1} & T = 1 \bmod 4 \end{cases} \end{aligned} \tag{32}$$

Thus splitting up $\{ Q , Q \} = \Gamma^a C^{-1} P_a$ into its chiral parts, we get

$$\begin{aligned} \{ Q_\pm , \; Q_\pm \} &= P_\pm \Gamma^a C^{-1} P_a \\ \{ Q_\pm , \; Q_\mp \} &= 0 \end{aligned} \tag{33}$$

for $T = 1 \bmod 4$ and

$$\begin{aligned} \{ Q_\pm , Q_\pm \} &= 0 \\ \{ Q_\pm , Q_\mp \} &= P_\pm \; \Gamma^a C^{-1} \; P_a \end{aligned} \tag{34}$$

for $T = 0 \bmod 4$. Thus for $S - T = 0 \bmod 8$, only for $T = 1 \bmod 4$ can we set $Q_- = 0$, say , and obtain $\{ Q , Q \} \sim P$ with Q a Majorana-Weyl minimal spinor.

We can proceed in this way to exhaust all the possible values of S and T admitting super-Poincaré symmetry. These are summarized in Table 3, where $\overline{Q} \equiv Q^{+} A$.

S - T mod 8	Minimal spinor type	T mod 4	anticommutator
1,2	Majorana	0,1	$\{ Q, Q \} = \Gamma^a C^{-1} P_a$
6,7	pseudo-Majorana	1,2	$\{ Q, Q \} = i\Gamma^a C^{-1} P_a$
0	Majorana-Weyl pseudo-Maj-Weyl	1	$\{ Q_+, Q_+ \} = P_+ \Gamma^a C^{-1} P_a$ $\{ Q_+, Q_+ \} = i P_+ \Gamma^a C^{-1} P_a$
3,5	Dirac	0,1 2,3	$\{ Q, \bar{Q} \} = \Gamma^a P_a$ $\{ Q, \bar{Q} \} = i \Gamma^a P_a$ $\{ Q, Q \} = \{ \bar{Q}, \bar{Q} \} = 0$
4	Weyl	1	$\{ Q_+, \bar{Q}_+ \} = P_+ \Gamma^a P_a$

Table 3. Minimal spinors for different values of S - T mod 8 and super-Poincaré algebras for different values of T mod 4.

Combining these results with those of Section 2 allows to draw the S / T plot of Fig. 2 whose points correspond to possible supermembrane theories. Once again we have used the symbols $\mathbb{O}$, $\mathbb{H}$, $\mathbb{C}$ and $\mathbb{R}$ to denote objects with 8+8, 4+4, 2+2 or 1+1 degrees of freedom, respectively. For pictorial reasons, we call this the brane-molecule.

Several comments are now in order ;

1) In the absence of any physical boundary conditions which treat time differently from space, and which we have not yet imposed, the mathematics will be symmetric under interchange of S and T. This can easily be seen from Fig. 2. For every supermembrane with (S,T) signature, there is another with (T, S). Note the self-conjugate theories that lie on the S = T line which passes through the (5, 5) superstring.

2) There is, as yet, no restriction on the world-volume signatures beyond the original requirement that $s \leq S$ and $t \leq T$.

3) If we were to redraw the D/d brane-scan of Fig.1 allowing now arbitrary signature, there would be no new points on the plot, but rather the new solutions would be superimposed on the old ones. For example, there would now be six solutions occupying the (d=3, D=11) slot instead of one.

4) Perhaps the most interesting aspect of the brane-molecule is the mod 8 periodicity. Suppose there exist signatures (s,t) and (S,T) which satisfy both the requirements of bose-fermi matching and super-Poincaré invariance. Now consider (s',t') and (S', T') for which

$$s' + t' = s + t \tag{35}$$

$$S' + T' = S + T \tag{36}$$

As a consequence of the modulo 8 periodicity theorem for real Clifford algebras [20,18], the minimal condition on a spinor is modulo 8 periodic e.g. $S - T = 0$ mod 8 for Majorana-Weyl. So if, in addition to (35) and (36) we also have

$$S' - T' = S - T + 8n \qquad n \in \mathbb{Z} \tag{37}$$

then (s',t') and (S',T') satisfy bose-fermi matching. (36) and (37) imply

$$S' = S + 4n$$

$$T' = T - 4n \tag{38}$$

Since, from Table 3, the existence of a super-Poincare algebra with minimal spinors is modulo 4 periodic in T, we see from (38) that the super-Poincaré invariance is also satisfied. Thus given the vertical sequence$(S, T) = (10,1) \rightarrow (2,1)$, modulo 8 periodicity implies the existence of the two other vertical sequences of Fig 2, namely $(6,5) \rightarrow (0,5)$ and $(2,9) \rightarrow (0,9)$. The three horizontal sequences $(1,10) \rightarrow (1,2)$, $(5,6) \rightarrow (5,0)$ and $(9,2) \rightarrow (9,0)$ are similarily related via modulo 8 periodicity.

Note the special crossover points at (9,1), (5,5) and (1,9) which permit Majorana-Weyl spinors and which correspond to the top horizontal line in the brane-scan of Fig.1. Similarly Weyl spinors are permitted at the crossover points (5,1) and (1,5) corresponding to the middle horizontal line of Fig.1. It is curious that the fundamental extended objects at the top of the $\mathbb{H}$ and $\mathbb{C}$ sequences are chiral, while those at the top of the $\mathbb{O}$ and $\mathbb{R}$ sequences are not. We shall return to this in Section 7.

4. *Compact Transverse Group*

In the usual signature all extended objects appear to suffer from ghosts because the kinetic term for the X^0 coordinate enters with the wrong sign. These are easily removed, however (at least at the classical level) by the presence of diffeomorphisms on the world-volume which allow us to fix a gauge where only positive-norm states propagate e.g. the light cone gauge for strings and its membrane analogues [21, 22]. Alternatively we may identify the d world-volume coordinates ξ^i with d of the D space-time coordinates X^i $(i = 1, \ldots, D)$, leaving $D - d$ coordinates X^I $(I = 1, \ldots, D-d)$ with the right sign for their kinetic energy [10]. Of course, this only works if we have one world-volume time coordinate τ that allows us to choose a light-cone gauge or else set $\tau = t$.

In the same spirit, we could now require absence of ghosts (or rather absence of classical instabilities since we are still at the classical level) for arbitrary signature by requiring that the "transverse" group $SO(S-s, T-t)$ which governs physical propagation after gauge-fixing, be compact. This requires $T = t$. It may be argued, of course, that in a world with more than one time dimension, ghosts are the least of your problems. Nevertheless, it is an interesting exercise to see how transversality of the gauge group restricts the possible super-extended-objects. For example, the superstring in (9,1) survives with SO(8), but the superstring in (5,5) with SO(4,4) does not. What about the superstring in (1,9) ? Here we once again encounter the problem that, in the absence of any physical input, we cannot distinguish (S, T) signature from (T, S). Since positivity of the energy is only a convention in field theory, ghosts can still be avoided by choosing $S = s$ instead of $T = t$. To avoid this repetition, let us cut the Gordian knot and demand from now on that $S \geq T$. The possible ghost-free solutions are those shown in Fig.3. Note the special case $(S, T) = (2,1)$ which permits not only $(s, t) = (1,1)$ but also (2,0) since a single field with negative energy can, by convention, be ghost-free.

Perhaps the most striking aspect of Fig. 3 is that the majority of super-extended objects do indeed lie on the $T=1$ axis.

5 *Superconformal Invariance*

In a previous paper [11], we discussed how after compactification to an $(AdS)_4 \times S^7$ spacetime, the 8 + 8 physical degrees of freedom of the eleven-dimensional supermembrane with $(S,T) = (10,1)$ were described by a superconformal field theory with world-volume signature $(s,t) = (2,1)$ and with OSp(8/4) as the superconformal group. This followed from previous work on the interpretation of the 8 spin 0 bosons and the 8 spin 1/2 fermions as singleton [9,10] representations of OSp(8/4), which is an anti de Sitter supergroup as far as the four-dimensional spacetime is concerned, but which acts as a superconformal group on the $S^1 \times S^2$ boundary of $(AdS)_4$ which is then identified with the three-dimensional world-volume of the supermembrane. We then conjectured that this super-conformal interpretation could apply to all the 12 extended-objects on the brane-scan of Fig.1, by noting that (for T=1) the Freund-Rubin [8] compactification mechanism for which

$$H_{\mu_1 \mu_2 \cdots \mu_{d+1}} \quad \alpha \quad \varepsilon_{\mu_1 \mu_2 \cdots \mu_{d+1}} \tag{39}$$

naturally leads to an $(AdS)_{d+1} \times S^{D-d-1}$ spacetime, whose boundary $S^1 \times S^{d-1}$ could be identified with the d-dimensional world-volume of the corresponding super-extended object. The same conjecture was made independently by Nicolai, Sezgin and Tanii [12], who also pointed out that the twelve corresponding superconformal groups were precisely those supergroups that admitted (spin 0, spin 1/2) singleton representations! In (39) $H \equiv dB$ is the $(d+1)$-form field-strength of the bosonic d-form $B_{\mu_1 \mu_2 \cdots \mu_d}$ which appears in the Wess-Zumino term of the d-dimensional extended object. Thus the AdS spacetime always has one more dimension than the world-volume (four in the case of the eleven-dimensional supermembrane). This is another way of understanding the upper limit d=6 on the world-volume[F2] of a super-extended object[7]: $d + 1 = 7$ is the upper limit for an anti-de Sitter supergroup [11].

It is not difficult to generalize the above set-up to arbitrary signature. Once again, we find that for each of the 20 super-extended objects appearing in Fig.3, there corresponds a Freund-Rubin compactification and a superconformal group admitting (spin0, spin 1/2) representions as shown in Table 4. We note the following

a) The spaces S_n, $(dS)_n$, $(AS)_n$ and $(AdS)_n$ are all n-dimensional spaces of constant curvature with

$$R_{\mu\nu\rho\sigma} = a^2 (g_{\mu\rho}g_{\nu\sigma} - g_{\mu\sigma}g_{\nu\rho}) \tag{40}$$

and hence Einstein spaces

$$R_{\mu\rho} = (n-1)\, a^2 g_{\mu\rho} \quad . \tag{41}$$

S and dS correspond to $a^2 > 0$ with T=0 or T=1 respectively, while AS and AdS correspond to $a^2 < 0$ with T=0 or T=1 respectively. The supergroups are given in Nahm's notation [23] where the first entry in square brackets denotes the bosonic subgroups and the second entry denotes the corresponding representation of the supercharge. There is one $\mathbb{O}$ case and four $\mathbb{H}$ cases with T > 1 which do not fit this description in the strict sense, since they do not admit these compactifications, but nevertheless correspond to the quoted supergroups. There are two ways to understand this. First, there is a group-theoretic way of understanding the connection between the object and the superconformal group which does not rely on any specific compactification. Namely, we identify the d world-volume coordinates with d of the D spacetime coordinates, leaving (D - d) bose + (D - d) fermi degrees of freedom as described in Section 2. The resulting kinetic terms will display an internal SO(D-d) symmetry and a conformal symmetry on the world- volume given by SO(s+1,t+1). So, for example, both the (10,1;2,1) and the (9,2;1,2) objects yield SO(3,2) $\times$ SO(8) as the bosonic group and hence, by supersymmetry, the same supergroup OSp(8/4). An alternative way to understand these five special cases, is to imagine the corresponding compactification taking place not vertically in Fig 2 as is usual, but rather horizontally off the graph into negative time dimensions (even though one, quite literally, runs out of time !)

b) As discussed in Section 2, the case of strings is special because of the ability to have both left and right movers on the worldsheet. So when we write the [supergroup]^2 as in Table 4, we have in mind a Type II string with supersymmetry for both left and right movers e.g. [SO(2,1)$\times$SO(8), (2, 8_c)] $\times$ [SO(2,1)$\times$SO(8), (2, 8_c)] in the case of type II A or [SO(2,1)$\times$SO(8), (2, 8_c)] $\times$ [SO(2,1)$\times$SO(8), (2, 8_s)] in the case of type II B. Alternatively we could have the heterotic case where only right movers, say, correspond to a supergroup SO(2,1)$\times$[SO(2,1)$\times$SO(8),(2,8)]. These supergroups and the corresponding singleton interpretations have been discussed before in the string literature by Gunaydin, Nilsson, Sierra and Townsend [24].

c) In which case, the reader may well ask why the corresponding compactifications $(AdS)_3 \times S_7$ and $(AdS)_7 \times S_3$ have not also appeared in the string literature. In fact they have, in a 1982 paper by Duff, Townsend and van Nieuwenhuizen [25] who considered Freund-Rubin compactifications of ten-dimensional supergravity. An interesting feature of these solutions was the part played by the dilaton. Its derivative acted as a conformal Killing vector on $(AdS)_3$ or S_3 [F3].

d) We have seen that for every super-extended-object there corresponds a superconformal group with (spin 0, spin 1/2) representations. In fact, the converse is also true since the supergroups appearing in Table 4 exhaust all such supergroups in Nahm's classification [23] . (With the exception of those involving $SO(1,1)$ which correspond to superparticles).

6. *Quantum Consistency*

So far our analysis has been completely classical, and a crucial question is which of the super-extended objects survive the test of quantum consistency. In the case of superstrings, for example, we know that only the $D=10$ cases are anomaly-free. For $d \geq 3$ in the usual signature, the question of quantum consistency has recently been addressed [22]. Here the corresponding analysis of the closure of the Lorentz and supersymmetry algebras is much more complicated, owing to the intrinsic non-linearity of the theories. However, Bars [27] and Bars and Pope [28] have recently applied a weaker consistency check by requiring that the degeneracies of the states of the theories should be compatible with the appropriate little groups for massless and massive modes. When applied to the twelve cases of Fig. 1, they find that all are quantum-mechanically inconsistent with the exception of the ten-dimensional superstring and the eleven-dimensional supermembrane. Similarly Bars, Pope and Sezgin [29] have shown that only in these two cases do the massless states correspond to the same massless supergravity background fields appearing in the Green-Schwarz action. In particular, only for the ten- dimensional superstring and the eleven-dimensional supermembrane does the massless spectrum contain a graviton. Of course, further work is required before being able to claim that the eleven-dimensional supermembrane is fully consistent, but it has passed all the tests so far performed. The tentative conclusion, therefore, is that the octonionic sequence of Fig. 1 is quantum consistent while the quaternionic, complex and real sequences suffer from incurable anomalies.

If this can be generalized to arbitrary signature, then only the nine extended objects marked ⊙ of Fig. 2 would survive, and only the three marked ⊙ of Fig. 3 would survive if we further demand $T=t$ and $S \geq T$. We note that of these three, two have $T=1$ and one has $T=2$. It would obviously be interesting to see how far this reductionist approach can be carried and whether one may ultimately be able to narrow things down completely to a unique "theory of everything".

An alternative strategy has been suggested by Townsend [30] in which one keeps all possible extended objects but regards the quaternionic, complex and real sequences not as fundamental but rather as "cosmic p-branes" i.e. as solitonic excitations of some other underlying theory.

7. *Further Possibilities*

It is interesting to ask whether we have exhausted all possible theories of extended objects with spacetime supersymmetry and fermionic gauge invariance on the world-volume. This we claimed to have done in Section 3 by demanding super-Poincaré invariance but might there exist other Green-Schwarz type actions in which the supergroup is not necessarily super-Poincaré? Although we have not yet attempted to construct such actions, one may nevertheless place constraints on the dimensions and signatures for which such theories are possible. We simply impose the constraints (2) and (3) of Section 2 and those in the first two columns of Table 3 of Section 3 but relax the constraints in the second two columns which specifically assumed super-Poincaré invariance. The results are shown in Fig. 4.

Although the possibilities are richer than those of Fig. 3, there are still severe constraints. Note in particular that the maximum space-time dimension is now $D = 12$ provided we have signatures (10,2), (6,6) or (2,10). These new cases are particularly interesting since they belong to the $\mathbb{O}$sequence and furthermore admit Majorana-Weyl spinors. In fact, twelve- dimensional supersymmetry algebras have been discussed before in the supergravity literature [31]. The RHS of the $\{ Q, \bar{Q} \}$ anticommutator yields not only a Lorentz generator but also a six index object so it is certainly not super-Poincaré. We conjecture (together with C. Hull and K. Stelle) that the (2, 2) extended object moving in (10, 2) spacetime may (if it exists) be related by simultaneous dimensional reduction [17 , 7] to the (1, 1) Type II B superstring in (9, 1) just as the (2,1) supermembrane in (10,1) is related to the Type II A superstring [17]. We are encouraged in this conjecture by the appearance of Majorana-Weyl spinors and self-dual tensors in both the twelve-dimensional and Type II B theories.

Finally one might ask what all this has to do with the real world. As discussed in the last section, one may adopt either a narrow reductionist view or else take a broader stance. The reductionist, on examining Figs 3 or 4, would declare that one point on the graph (e.g. the (1, 1) superstring in (9, 1) spacetime) is the theory of everything while all the other points are theories of nothing. Alternatively, one could entertain the idea that the real world requires more than one, or possibly all, of the theories permitted by the mathematics. In particular, we recall the observation at the end of Section 3 regarding the appearance of chiral theories. This suggests something along the lines of Table 5. Do the four forces in Nature correspond to the four division algebras?

Acknowledgments

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Division Algebra	Signatures (S,T ; s,t)	Compactification	Superalgebra
$\mathbb{O}$	(10,1;2,1) or (9,2;1,2)	$(AdS)_4 \times S^7$	$[SO(3,2)\times SO(8),(4,8)]$
	(9,1;1,1)	$(AdS)_3 \times S^7$	$[SO(2,1)\times SO(8),(2,8)]^2$
$\mathbb{H}$	(9,0;5,0)	$(AS)_6 \times S^3$	$SU(2)\times[SO(6,1)\times SU(2),(8,2)]$
	(9,1;5,1) or (5,5;1,5)	$(AdS)_7 \times S^3$	$SU(2)\times[SO(6,2)\times SU(2),(8,2)]$
	(8,1;4,1) or ((5,4;1,4)	$(AdS)_6 \times S^3$	$SU(2)\times[SO(5,2)\times SU(2),(8,2)]$
	(7,1;3,1) or (5,3;1,3)	$(AdS)_5 \times S^3$	$SU(2)\times[SO(4,2)\times SU(2),(4,2)+(\bar{4},\bar{2})]$
	(6,1;2,1) or (5,2;1,2)	$(AdS)_4 \times S^3$	$[SO(3,2)\times SO(4),(4,4)]$
	(5,1;1,1)	$(AdS)_3 \times S^3$	$[SO(2,1)\times SO(4),(2,4)]^2$
$\mathbb{C}$	(5,0;3,0)	$(AS)_4 \times S^1$	$[SO(4,1)\times U(1),4+\bar{4}]$
	(5,1;3,1)	$(AdS)_5 \times S^1$	$[SO(4,2)\times U(1),(4,1)+(\bar{4},\bar{1})]$
	(4,1;2,1)	$(AdS)_4 \times S^1$	$[SO(3,2)\times U(1),(4,2)]$
	(3,1;1,1)	$(AdS)_3 \times S^1$	$[SO(2,1)\times U(1),(2,1)+(2,\bar{1})]^2$
$\mathbb{R}$	(3,1;2,1)	$(AdS)_4$	$[SO(3,2),4]$
	(2,1;1,1)	$(AdS)_3$	$[SO(2,1),2]^2$
	(2,1;2,0)	$(dS)_3$	$[SO(3,1),(2,1)+(1,2)]$

Table 4. Superconformal groups.

Algebra	Symmetry	Subgroup	Chiral?	Force
$\mathbb{O}$	SO(8)	SU(3)	No	Strong
$\mathbb{H}$	SO(4)	SU(2)	Yes	Weak
$\mathbb{C}$	SO(2)	U(1)	Yes	E.M.
$\mathbb{R}$	$\mathbb{1}$	$\mathbb{1}$	No	Gravity

Table 5. Do the four forces of nature correspond to the four division algebras?

Footnotes

F1 Notable exceptions are provided by the work of Sakharov [1] and Araf'eva, Volovich and Dragovic [2], van Nieuwenhuizen [4] and Kugo and Townsend [3].

F2 Note that $d = 6$ is also the upper limit for renormalisable (spin 0, spin 1/2) interactions. So the possible resolution of the renormalisability problem for supermembranes suggested in [11] applies to all supersymmetric extended objects.

F3 These solutions were criticised in the "Ten into four won't go" paper of Freedman, Gibbons and West[26]. As far as we can tell, however, ten into three (or seven) went and is still going!

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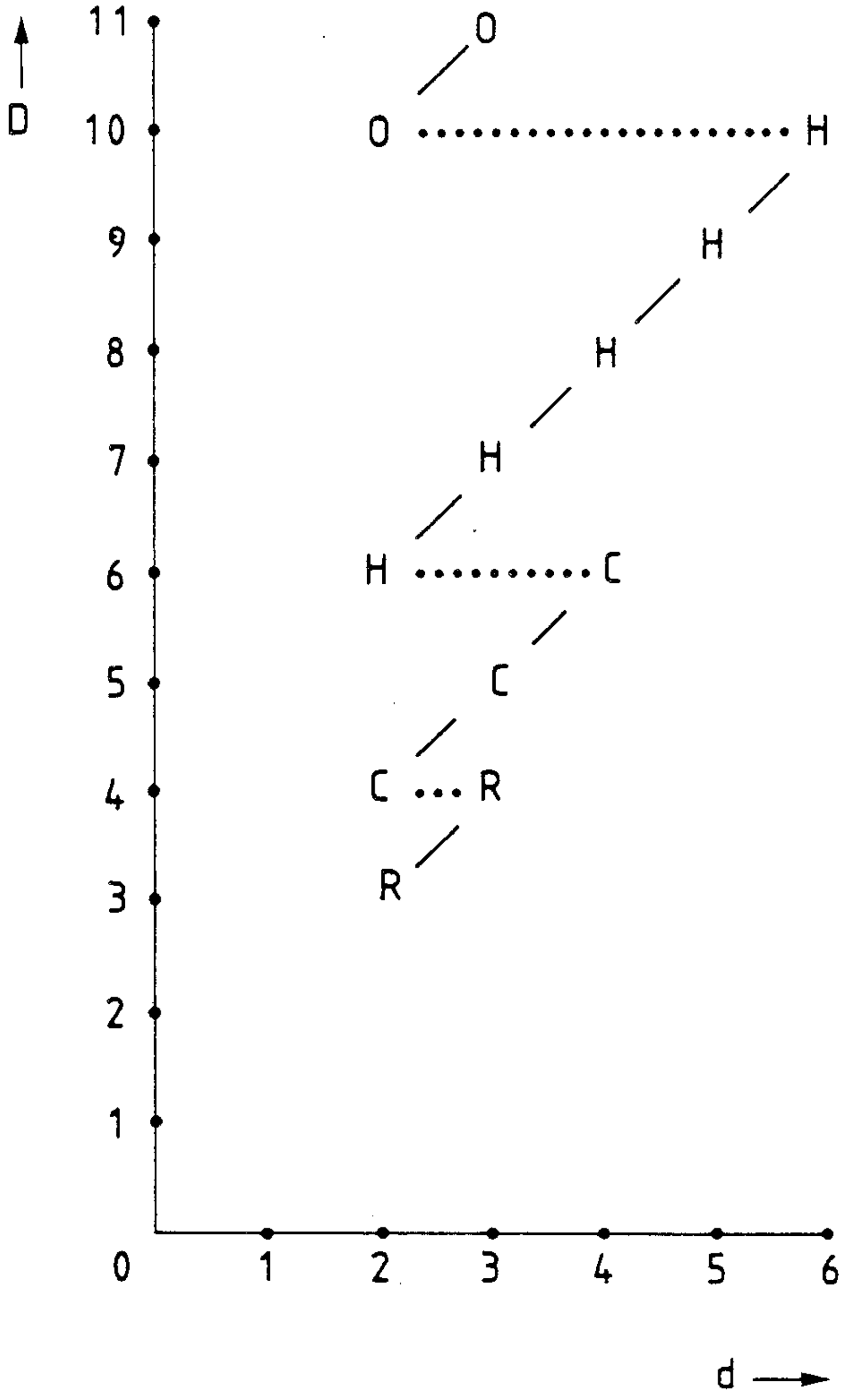

Fig. 1. The brane-scan.

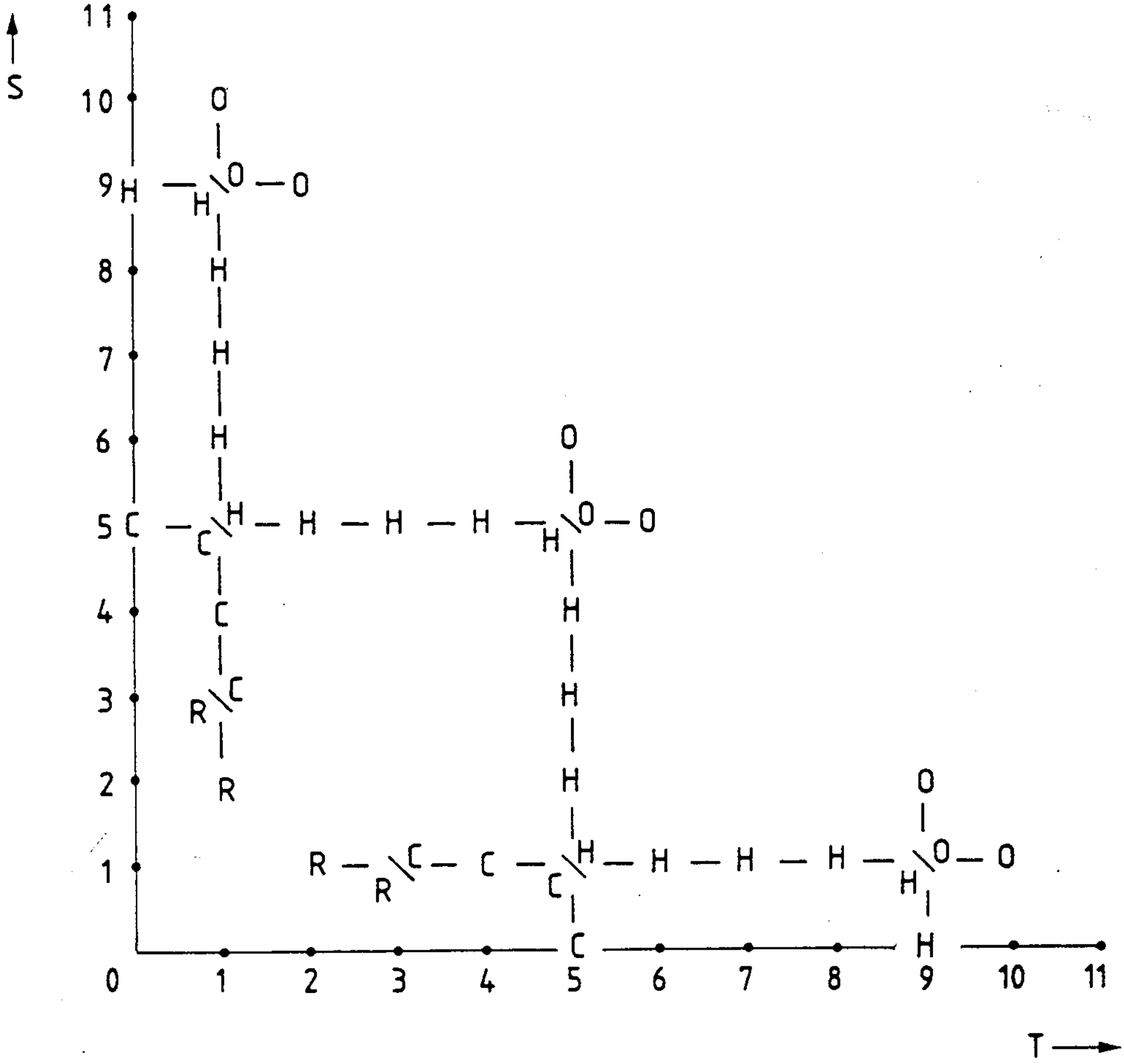

Fig. 2. The brane-molecule, assuming super-Poincaré invariance.

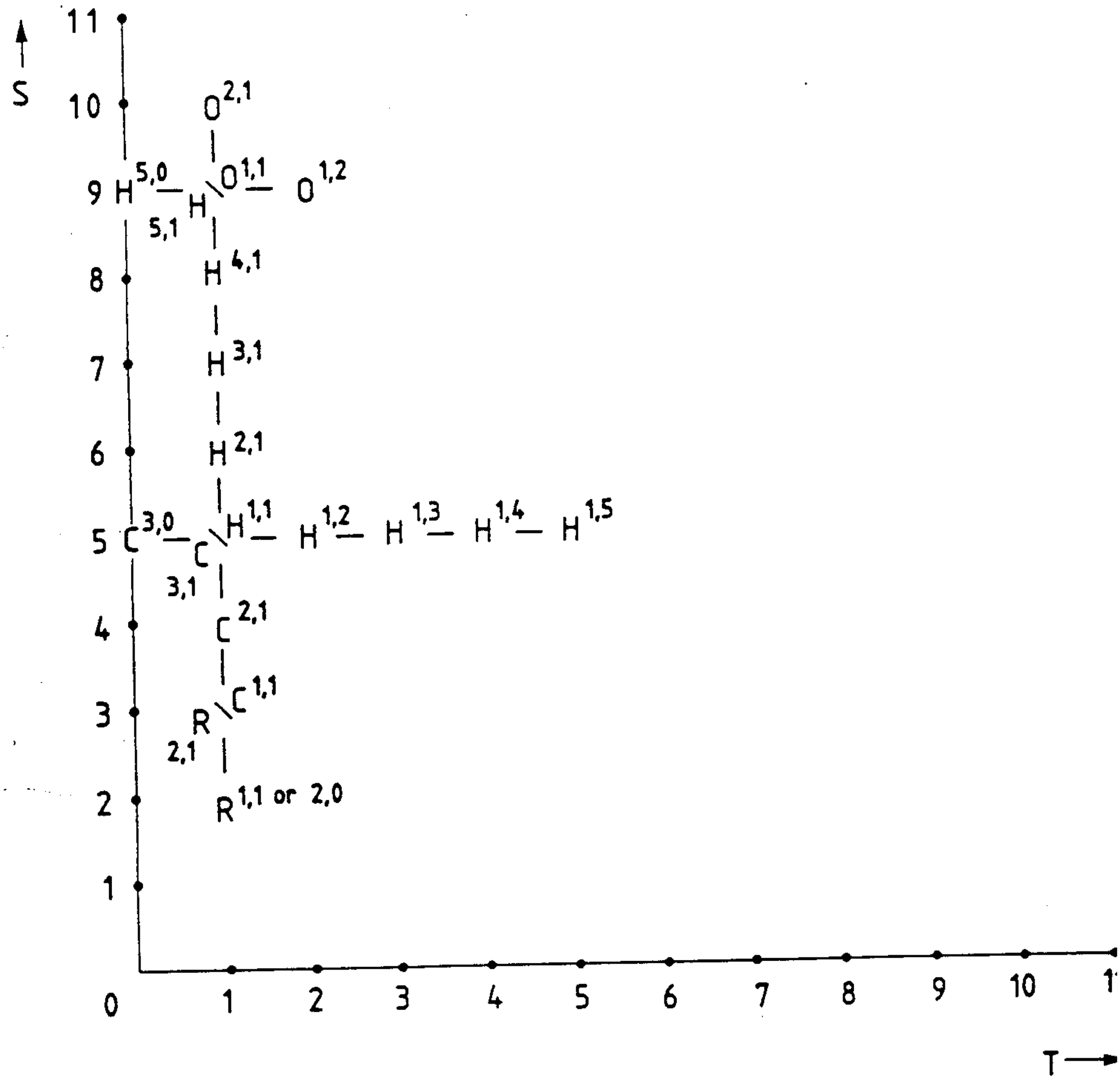

Fig. 3. The brane-molecule, assuming compact transverse group and $S \geq T$. The superscripts denote (s,t).

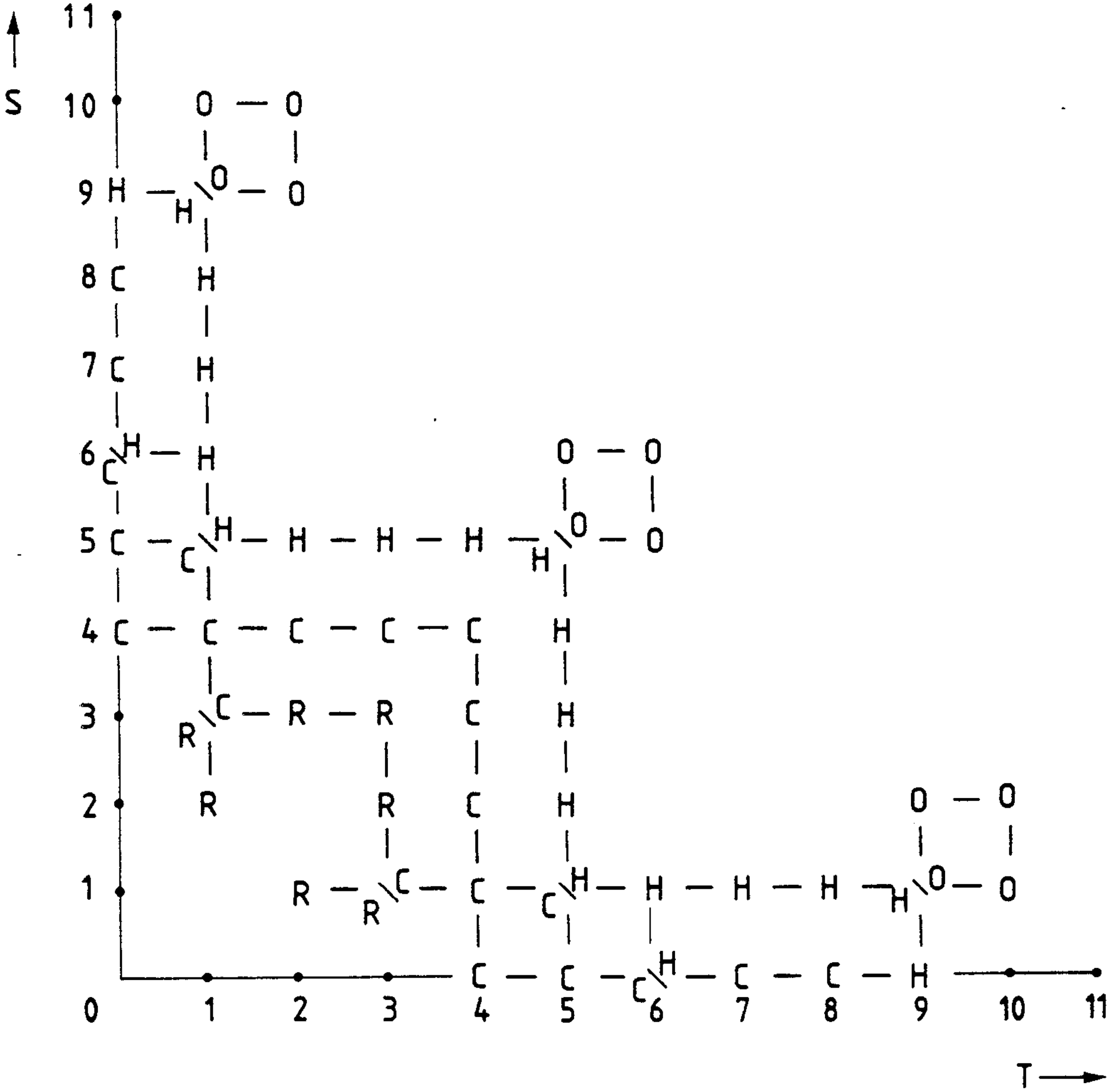

Fig. 4. The brane-molecule, without assuming super-Poincaré invariance.