Two - Scale Technicolor

Kenneth Lane
Department of Physics
Boston University
Boston, MA 02215

Estia Eichten
Fermi National Accelerator Laboratory
P. O. Box 500, Batavia, IL 60510

Abstract

Walking technicolor theories naturally have fermions belonging to different technicolor representations and, therefore, at least two distinct energy scales below 1 TeV. We study a simplified two-scale version without ordinary color. We show that masses of light-scale technipions and technirhos are such that the latter tend always to decay to at least one $W^\pm$ or $Z^0$. This signature should be observable at the Tevatron collider for technirho masses up to 300 GeV.
This Letter is concerned with the possibility of relatively low-energy consequences of walking technicolor with striking experimental signatures.

Walking technicolor (TC) has been proposed\cite{1,2,3} as a way for extended technicolor (ETC)\cite{4,5} to generate reasonably large quark and lepton masses while suppressing flavor-changing neutral current interactions to a comfortable level. The TC coupling $\alpha(p)$ runs very slowly from the scale $\Lambda_T$ of technifermion chiral symmetry breaking up to some much greater energy which we shall assume to be the ETC scale $\Lambda_{ETC}$ itself. Over this large energy range, the anomalous dimension $\gamma_m$ of the fermion bilinear $\bar{T}T$ is fairly accurately given by the ladder-approximation formula\cite{6,3,8,7}

$$\gamma_m(\alpha(p)) = 1 - \sqrt{1 - \alpha(p)/\alpha_c} \approx 1.$$  \hfill (1)

Here $\alpha_c = \alpha(\Lambda_T)$ is the critical coupling at which spontaneous chiral symmetry breaking occurs. It, too, is well-represented by its ladder-approximation value $\alpha_c = \pi/3C_2(R)$, where $C_2(R)$ is the quadratic Casimir of the technifermion representation $R$. The large $\gamma_m$ greatly enhances quark and lepton masses for a fixed $\Lambda_{ETC}/\Lambda_T$:

$$m_{q,l} \approx \frac{\langle \bar{T}T \rangle_{\Lambda_{ETC}}}{\Lambda_{ETC}^2} = \frac{\langle \bar{T}T \rangle_{\Lambda_T}}{\Lambda_{ETC}^2} \exp \left[ \int_{\Lambda_T}^{\Lambda_{ETC}} \frac{d\mu}{\mu} \gamma_m(\alpha(\mu)) \right]$$  \hfill (2)

For light quarks, the lowest value of $\Lambda_{ETC}$ in Eq. (2) is set by the strength of ETC-generated $|\Delta S| = 2$ interactions. These have the form $\theta_{ij}^2/\Lambda_{ETC}^2 \langle \bar{s}\Gamma_i^\mu d \bar{s}\Gamma_j^\mu d + h.c. \rangle$, where $\theta_{ij}$ is a mixing angle and $\Gamma_i^\mu = \gamma^\mu \left( \frac{i+\gamma_5}{2} \right)$. If $\theta_{ij} \approx 0.1$, contributions to the $K_L - K_S$ mass difference are sufficiently small for $\Lambda_{ETC} \geq 100$ TeV\cite{8}. Such values of $\Lambda_{ETC}$ can lead to light fermion masses in the range of a few MeV to a few GeV\cite{1,2,3}.

For $\alpha(p)$ to walk, there must be many electroweak doublets of technifermions contributing to the TC beta-function at $p$ above a few $\Lambda_T$. There may be many doublets in the fundamental representation of the TC gauge group (here assumed to be $SU(N_{TC})$), a few doublets in higher-dimensional representations, or both. The last alternative is most likely for a phenomenologically acceptable theory. Whatever the ETC gauge group may be, to avoid very light axion-like mesons it must contain $SU(N_{TC}), SU(3)_{\text{color}}$ and some subgroup of quark/lepton flavors – perhaps genera-
tions. Furthermore, all fermions must belong to at most four chiral irreducible representations of the ETC group\(^5\). A very direct way to arrange this is that these ETC representations are nonfundamental (second-rank tensors, e.g.). When these are decomposed with respect to the ETC subgroups, they yield fermions transforming only under TC, under TC \(\otimes\) color, under TC \(\otimes\) flavor, etc. Hence, both fundamental and higher-dimensional TC representations occur.

Suppose \(N_1\) doublets of technifermions \(T_1\) and \(N_2\) doublets \(T_2\) belong to the (complex) TC representations \(R_1\) and \(R_2\), with dimensionalities \(d_1 < d_2\). Their characteristic chiral symmetry breaking scales satisfy \(A_1 < A_2\) \([8]\). Above \(p = A_2, \alpha(p)\) is presumed to remain nearly constant at \(\alpha_{\text{eq}} \equiv \pi/3C_2(R_2)\), i.e., \(\beta(\alpha(p)) \approx 0\). If \(N_1\) is large enough that \(\beta\) is also fairly small for \(A_1 < p < A_2\), then \(A_1 << A_2\). From now on we assume this hierarchy occurs and investigate its consequences.

In this Letter we consider a simplified two-scale version of walking technicolor which does not include ordinary color. Toy models we have constructed which do include \(SU(3)_c\) have so many colored technifermions that the asymptotic freedom of QCD is lost and \(\alpha_{QCD}\) starts to grow \([9]\). Thus, a scale hierarchy probably exists between colored and noncolored technifermions belonging to the same TC representations. This is a very interesting and essential complication whose study we shall take up in a later paper\([10]\).

The spectrum of low-lying \(T_iT_i\) mesons in two-scale technicolor includes spin-one technipions \(\rho_i\) and spinless technipions \(\pi_i\) which transform under the flavor group \(SU(2)_{EW} \otimes SU(N_i)\) as \((3,N_i^2 - 1)\), \((1,N_i^2 - 1)\) and \((3,1)\). The decay constants \(F_i\) of these technipions are related to the Fermi constant by

\[
F_i \equiv 2^{-1/4}G_F^{-1/4} = 246\text{GeV} = \frac{\sqrt{F_1^2 + F_2^2}}{N_2 F_1} \approx \sqrt{N_2 F_1}
\]  

(3)

if \(\Lambda_2 \gg \Lambda_1\). In the neglect of ETC interactions, \(M_{\rho_i} \approx 2\Lambda_i\) and \(M_{\pi_i} = 0\). ETC breaks all flavor symmetries except \((SU(2) \otimes U(1))_{EW}\), generating a current-algebra mass \(m_i\) for \(T_i\) just as for light fermions in Eq. (2). This gives mass to all technipions except the three involved in the Higgs mechanism,

\[
|W_L^+ \pi_L^0| = F_1^{-1} \sum_{i=1}^2 \sqrt{N_i F_i} |\pi_L^{\pm,0}(3,1)|.
\]  

(4)

Likewise, \(M_{\rho_i}\) is increased by \(2m_i\). If \(\Lambda_2 \gg \Lambda_1\), the upper scale is fixed by Eq. (3). Even a moderate uncertainty of the behavior of \(\alpha(p)\) when \(\Lambda_i \leq p \leq 2\Lambda_i\) leaves us
unable to estimate $\Lambda_1/\Lambda_2$ with better than a factor of two precision[8]. Therefore, we shall pin $\Lambda_1$ down by assuming that $M_{\rho_1}$ is in the phenomenologically interesting range 200-300 GeV. Numerical estimates of all other masses and scales will be worked out below.

Consider now the decays of the light technirhos $\rho_1 = \rho_1(3,1)$ [11]. In an ordinary TC theory we expect $\rho_1 \rightarrow \pi_1(3,1)\pi_1(3,1), \pi_1(3,N_1^2 - 1)\pi(3,N_2^2 - 1), \pi_1W_L^\pm$, and $\pi_1Z_L^0$, and $W_L^+W_L^-$ and $W_L^+Z_L^0$[12]. The relative decay rates, uncorrected for phase space, are $(N_2F_2^2/F_1^2)^2 \approx 1, N_1^2 - 1, 2N_1N_2(F_1F_2/F_2^2)^2$, and $(N_1F_1^2/F_2^2)^2$. In a walking TC theory, however, the current mass $m_1$ of $T_1$ is enhanced just as is $m_{\eta,4}$ in Eq. (2). This substantially raises $M_{\pi_1}$ above naive expectations and, to some extent, $M_{\rho_1}$. Our numerical estimates below show that it is very likely that $2M_{\pi_1} > M_{\rho_1}$. Thus, we expect that $\rho_1$ always decays to at least one $W_L^+,Z_L^0$ and that $\rho_1$ is very narrow ($\Gamma \approx 500$ MeV). This $\rho_1(3,1)$ is produced in $\bar{p}p$ collisions via the subprocess

$$\bar{q}_i q_j \rightarrow W^\pm, Z^0, \gamma \rho_1^{\pm,0} \rightarrow (\pi_1W_L)^{\pm,0}; \pi_1^{\pm} Z_L^0$$ \hspace{1cm} (5)

We will find that the inclusive $W^\pm, Z^0$ cross sections at Tevatron energy are a few picobarns. We expect $\pi_1^{\pm} \rightarrow t\bar{t}, t\bar{t}$ and $\pi_1^{0} \rightarrow t\bar{t}$ (if kinematically allowed). Thus, the signal for process in Eq. (5) is an identified $W^\pm$ or $Z^0$ recoiling against a di-jet of invariant mass greater than half the invariant mass of the whole system. Although we think it less likely, it is possible that $M_{\pi_1} + M_{W,Z} > M_{\rho_1}$. Then $\rho_1$ will be exceedingly narrow ($\Gamma \leq 50$ MeV), its production rate will still be a few picobarns, and the signal will be that of weak gauge boson pair production.

Mass Estimates

We assume the $T_1$ transform according to the fundamental representation of $SU(N_{TC})$, the $T_2$ according to the second-rank antisymmetric tensor representations. $N_{TC}, N_1$ and $N_2$ will be chosen to make the $O(\alpha^2)$ beta-function very small and negative for $p > 2\Lambda_2$. Without an explicit ETC model, we can make only generic estimates of $m_i$ and $M_{\pi_i}$. For these we assume essentially no splitting of technifermion flavors[11]. Apart from this simplification, we shall try to estimate masses as carefully as possible, scaling up from QCD relations[13] where appropriate.

The principle guiding our calculations is that, for fixed input parameters $(N_{TC}, N_1, N_2, \Lambda_{ETC}, M_{\rho_1})$, the resulting current-algebra masses should be consistent with first-
order chiral perturbation theory ($\chi PT_1$), i.e., the current mass of $T_i$, renormalized at $\Lambda_i$, must satisfy

$$m_i(\Lambda_i) < \Lambda_i$$  \hspace{1cm} (6)

If this condition is not met, we can surmise only that technipions $\pi_i$ are too heavy to be regarded as true pseudo-Goldstone bosons. Our calculational rules are as follows:

1. We take $\Lambda_{ETC} \geq 100$ TeV and the ETC gauge boson mass $M_{ETC} = g_{ETC}\Lambda_{ETC}/2$

   where $g_{ETC} \simeq \sqrt{4\pi \alpha_s} = \sqrt{4\pi^2/3C_2(R_2)} \simeq 2$ for $N_{TC} = 5, 6$.

2. We start with $F_2 = N_2^{-1/2} F_*$ and iterate the calculation once $F_1$ is determined.

3. The TC-summed (but not flavor-summed) $T_i$-condensate renormalized at the $T_i$ chiral-symmetry breaking scale $\Lambda_i$ and the scale $\Lambda_i$ itself are given by

   $$\langle T_i T_i \rangle_{\Lambda_i} = 4\pi \kappa_i F_i^3$$ \hspace{1cm} (7)

   $$\Lambda_i = \pi\kappa_i^{1/2} F_i$$ \hspace{1cm} (8)

   $\Lambda_i$ is also essentially the dynamical mass of $T_i$. In QCD a factor $\kappa = 1.7$ is needed to make the $\bar{s}$-quark mass calculated from the $\chi PT_1$ formula $m_\bar{s} = f_{\pi}^2 m_N^2 / \langle \bar{q}q \rangle$ agree with the constituent quark model formula $m_\bar{s} = M_K - M_\rho = M_K - M_K^*$, where $M_\rho \cong 2\Lambda_q \cong 8.2 f_\pi$. We vary $\kappa_1 = \kappa_2 = \kappa$ between 1.0 and 2.0 in calculations. The condensates and current masses at any other scale $\Lambda$ are given by

   $$\langle T_i T_i \rangle_{\Lambda} = \langle T_i T_i \rangle_{\Lambda_i} \exp \left[ \int_{\Lambda_i}^{\Lambda} \frac{d\mu}{\mu} \gamma m_i(\alpha(\mu)) \right]$$ \hspace{1cm} (9a)

   $$m_i(\Lambda) = m_i(\Lambda_i) \exp \left[ - \int_{\Lambda_i}^{\Lambda} \frac{d\mu}{\mu} \gamma m_i(\alpha(\mu)) \right]$$ \hspace{1cm} (9b)

   with $\gamma m_i$ specified below.

4. The mass of $\rho_i$ consistent with $\chi PT_1$ is

   $$M_{\rho_i} = 2(\Lambda_i + m_i(\Lambda_i))$$ \hspace{1cm} (10)

$\Lambda_i$ is obtained from steps 2 and 3. $\Lambda_1$ will be determined by fixing $M_{\rho_1}$, calculating $m_1(\Lambda)$ and finding the value of $\Lambda_1$ which satisfies Eq. (10).
5. The TC coupling is taken to run as

\[ \alpha(p) = \begin{cases} 
\alpha_{\lambda_1} \left[ 1 + \frac{\alpha_{\lambda_1}}{\delta \pi} (11N_{TC} - 4N_1) \ln \frac{p}{2\Lambda_1} \theta(p - 2\Lambda_1) \right]^{-1} & \Lambda_1 < p < \Lambda_2 \\
\alpha_{\lambda_2} & \Lambda_2 < p < \Lambda_{ETC}
\end{cases} \]  

(11)

The anomalous dimensions \( \gamma_m(\alpha(p)) \) are then given by the ladder approximation formula in Eq. (1). Of course, we really do not know how \( \alpha(p) \) runs in the threshold regions \( p \approx \Lambda_i \) to few \( \Lambda_i \). The various approximations to \( \alpha(p) \) and \( \gamma_m(\alpha(p)) \) embodied in Eq. (11) can be corrected for by small adjustments in \( \Lambda_{ETC} \).

The current mass of \( T_2 \) arises from ETC terms of the form \( \bar{T}_2^* T_2 T_2 / \Lambda_{ETC}^2 \) and it is given by

\[ m_2(\Lambda_2) = m_2(\Lambda_{ETC}) \Lambda_{ETC} / \Lambda_2 \cong (\bar{T}_2^* T_2) \Lambda_{ETC} / \Lambda_2 \Lambda_{ETC} = 4F_2 / \pi \]  

(12)

This determines \( M_{\pi_2} = 2(\pi \kappa^{1/2} + 4/\pi)F_2 \) and, for all \( \pi_2 \) except the "massless" \( W_L^\pm, Z_L^0, \)

\[ M_{\pi_2} \cong \left[ m_2(\Lambda_2)(\bar{T}_2^* T_2) / F_2^2 \right]^{1/2} - 4\kappa^{1/2}F_2 \]  

(13)

The current mass of \( T_1 \) arises mainly from \( \bar{T}_1^* T_2^* T_1 / \Lambda_{ETC}^2 \) terms which are necessary to avoid axion-like mesons\(^8\). The ETC bosons involved carry both TC and flavor. There are contributions to \( m_1 \) from both the dynamical and current masses of \( T_2 \) and they are comparable. We estimate (here \( \Sigma_3(p) \) is the full \( T_2 \)-propagator mass function)

\[ m_1(\Lambda_{ETC}) \cong \frac{(N_{TC} - 1)}{\frac{1}{2}N_{TC}(N_{TC} - 1)} \frac{\langle \bar{T}_1^* T_2 \rangle \Lambda_{ETC}}{\Lambda_{ETC}^2} + 4(N_{TC} - 1) \frac{g_{ETC}^2}{16\pi^2} \int_{\Lambda_2^2}^{\Lambda_{ETC}^2} p^2 dp^2 \frac{m_2(p)}{p^2 + \Sigma_3^2(p)} \frac{1}{p^2 + \Sigma_3^2(p)} \frac{1}{M_{ETC}^2} \]

\[ \cong \left[ \frac{2}{N_{TC}} + \frac{(N_{TC} - 1)g_{ETC}}{2\pi} \right] \frac{4\kappa^{1/2}F_2^2}{\Lambda_{ETC}^2} \]  

(14)

\( m_1(\Lambda_1) \) is obtained from Eq. (9b), and \( \Lambda_1 \) and \( F_1 \) follow from Eqs. (10) and (8), respectively. Finally, the masses of the \( \pi_1 \) pseudo-Goldstone bosons orthogonal to
$W_L^\pm, Z_L^0$ are given by

$$/\alpha M_{\pi_1} \cong \left[ m_1(\Lambda_1)(T_1 T_1)_{\Lambda_1}/F_1^2 \right] = [4\pi \kappa F_1 m_1(\Lambda_1)]^{1/2} \quad (15)$$

In Fig. 1 we plot $M_{\rho_1} - 2M_{\pi_1}$ versus $\kappa$ for $M_{\rho_1} = 200, 250, 300$ GeV and two sets of parameters which give results consistent with $\chi^2_{PT_1}$ and reasonable light fermion masses as well (see Eqs. (17) below). The input parameters are

- set a: $NTC = 5, N_1 = 6, N_2 = 2, \Lambda_{ETC} = 100$ TeV
- set b: $NTC = 6, N_1 = 12, N_2 = 1, \Lambda_{ETC} = 600$ TeV.

The plausibility of $M_{\rho_1} - 2M_{\pi_1} < 0$ is obvious from this figure\[14].

The current masses $m_f$ of light fermions $f$ (quarks and leptons) arise from ETC interactions of the form $f T_1 T_1 f/\Lambda_{ETC}^2$ and are generated by exchange of the same ETC bosons linking $T_1$ and $T_2$. Both the dynamical and the current mass of $T_1$ contribute to $m_f$. Taking account that the $f$ -self-energy graph contains a technicolor loop (introducing a factor of $N_{TC}$), these two contributions to $m_f$ are

$$m_{f,\text{dyn}}(\Lambda_{ETC}) \cong \langle T_1 T_1 \rangle_{\Lambda_{ETC}^2}$$

$$m_{f,\text{cur}}(\Lambda_{ETC}) \cong 4N_{TC}\frac{g_{ETC}^2}{16\pi^2} \int_{\Lambda_1^2}^{\Lambda_{ETC}^2} \frac{m_1(p)}{p^2 + \Sigma_1(p)/p^2 + \Lambda_{ETC}^2} \quad (16b)$$

where $m_1(p)$ is given by Eq. (9b) and $\Sigma_1(p)$ is the full $T_1$-mass function. Clearly, $m_{f,\text{cur}} \gg m_{f,\text{dyn}}$. This will be important for calculating light fermion masses in explicit ETC models. In our generic model we estimate (for $M_{\rho_1} = 250$ GeV, $\kappa \approx 1.5$)

$$m_{f,\text{cur}}(\Lambda_{ETC}) = \begin{cases} 950 \text{ MeV} \quad \text{(set a)} \\ 400 \text{ MeV} \quad \text{(set b)} \end{cases} \quad (17a)$$

$$m_{f,\text{dyn}}(\Lambda_{ETC}) = \begin{cases} 350 \text{ KeV} \quad \text{(set a)} \\ 250 \text{ KeV} \quad \text{(set b)} \end{cases} \quad (17b)$$

The relevant values of $m_f$ are those at 1 GeV, not 100 TeV, and these require knowing the scale-dependence of the anomalous dimensions $\gamma_m(\mu)$. This may be quite complicated for quarks and the question will be taken up in our study of the effects of ordinary color in multi-scale technicolor.
Technirho Production and Decay

The easiest way to access light technihadrons is production of $\rho_1(3,1)$ via the electroweak processes in Eq. (5) and its subsequent decay to technipion pairs $\pi_a \pi_b$. If ETC flavor splitting is small, the final states allowed by flavor symmetry are $\pi_1(3,1)\pi_1(3,1)$, $\pi_1(3,N_1^2 - 1)\pi_1(3,N_2^2 - 1)$, $\pi_1(3,1)W_L$ and $W_LW_L$. Here $W_L = W_L^\pm$ or $Z_L^0$. The cross sections for the parton reactions in Eq. (5) at subprocess center-of-mass energy $\sqrt{s}$ and scattering angle $\theta$ are given by

$$\frac{d\sigma(q_i,q_j \to p_1^{\pm,0} \to \pi_a \pi_b)}{d(cos \theta)} = \frac{\alpha_{EW}^2 M_{p_1}^4}{3 \delta (\delta - M_{p_1}^2)^2 + \delta \Gamma_{p_1}^2(\delta)} B_{ij}^{\pm,0} C_{ab} \frac{k_{ab}}{\delta^{3/2} 2} \sin^2 \theta$$

where $k_{ab}^2 = (\delta - (M_a + M_b)^2)(\delta - (M_a - M_b)^2)/4\delta$ and $\alpha_{EW} \approx 1/128$. The weights $C_{ab}$ for the four types of final state $\pi_a \pi_b$ are $(N_1F_1^2/F_2^2)^2$, $N_1^2 - 1$, $N_1N_2(F_1F_2/F_2^2)^2$ and $(N_1F_1^2/F_2^2)^2$. The electroweak factors $B_{ij}$ are

$$B_{ij}^{\pm}(\delta) = \frac{|K_{ij}|^2}{4 \sin^4 \theta_W} \left( \frac{s}{s - M_W^2} \right)^2$$

$$B_{ij}^0(\delta) = \delta_{ij} Q_i^2 \left[ 1 + \frac{2 \cos^2 \theta_W}{\sin^2 \theta_W} (T_{3i}/Q_i - \sin^2 \theta_W) \left( \frac{s}{s - M_2^2} \right) \right]^2$$

$$+ \left[ 1 - \frac{2 \cos^2 \theta_W}{\sin^2 \theta_W} \sin^2 \theta_W \left( \frac{s}{s - M_2^2} \right) \right]^2$$

In Eq. (19), $\theta_W$ is the weak mixing angle ($\sin^2 \theta_W \approx 0.22$), $K_{ij}$ is the Kobayashi-Maskawa matrix element (taken diagonal), and $Q_i$ and $T_{3i}$ are the electric charge and third component of weak isospin of quark $q_i$. The energy-dependent width used in the cross section is

$$\Gamma_{\rho_1}(\delta) = \frac{g^2}{6\pi} \sum_{a,b} C_{ab} k_{ab}^2 / \delta$$

where $g^2 = 3g_{\rho\pi\pi}^2 / N_{TC}$ and $g_{\rho\pi\pi}^2 / 4\pi = 2.97$. We have used Eqs. (18)-(20) together with the EHLQ distribution functions to compute the final-state invariant-mass distributions and total cross sections for $\rho_1^{\pm,0}$ production in hadron collisions. We believe the most likely range for the $\pi_1$ mass is $M_{p_1}/2 < M_{\pi_1} < M_{p_1} - M_{W,L}$. Of immediate interest, therefore, are the inclusive
$W^\pm$ and $Z^0$ distributions for the Tevatron $\bar{p}p$ collider at $\sqrt{s} = 1.8$ TeV. We have calculated these distributions for the parameters of set a, with $M_{\rho_1} = 250$ GeV, $M_{\sigma_1} = 130$ GeV, $F_2 = 170$ GeV and $F_1 = 21.7$ GeV. The final state mesons are required to have rapidity $|y_{\rho_1}| < 1.5$. The total inclusive cross sections due to $\rho_1$ production in this example are

$$\sigma(W^+ + X) = \sigma(W^- + X) = 2.2 \text{ pb}$$
$$\sigma(Z^0 + X) = 1.8 \text{ pb}$$

The width of the $\rho_1$ is about 450 MeV in this case and the peak cross sections are about $3 \text{ pb}$/GeV. The total $\rho_1^0$ and $\rho_1^\pm$ cross sections are 1.5 pb and 2.3 pb [10].

For this choice of masses, 85-90% of the total cross sections is due to $\pi_1 W_L$ production. If, as naively expected [8], ETC interactions couple $\pi_1^{\pm,0}$ most strongly to the heaviest quarks and leptons, they decay into $b\bar{b}$ and $t\bar{t}$ (if kinematically allowed). Just how "jetty" these decays will be requires a detailed Monte Carlo simulation. An integrated luminosity of 20 pb$^{-1}$ should be sufficient to begin seeing these events with $W^\pm$ and $Z^0$ tagged by their leptonic decay modes. Since the total invariant mass of the system is very sharply peaked at $M_{\sigma_1}$, the backgrounds from standard electroweak and QCD processes should be negligible.

As an example of the possibility that $M_{\sigma_1} > M_{\rho_1} - M_W$, we computed the various cross sections for $M_{\sigma_1} = 175$ GeV, with all other parameters unchanged. Now the final states are all $W^+W^-$ and $W^\pm Z^0$ and the $\rho_1$ width has shrunk to about 35 MeV [12]. The total $W^\pm$ and $Z^0$ inclusive cross sections are 3.5 pb and 4.0 pb. The $\rho_1^0$ and $\rho_1^\pm$ cross sections are 1.5 pb and 2.0 pb.

To test the sensitivity of these production rates to model assumptions, we have repeated the calculations for the parameters of set b, with $M_{\rho_1}$ and $M_{\sigma_1}$ as above and $F_2 = 237.5$ GeV, $F_1 = 18.7$ GeV. In all cases these cross sections are about 20% higher than those for the parameters of set a.

Finally, the cross sections for production of a 250 GeV $\rho_1$ at the CERN $\sqrt{s}$ collider with $\sqrt{s} = 630$ GeV are all about 30 times smaller than the corresponding ones at $\sqrt{s} = 1800$ GeV. This suppression is expected since the typical parton $z$-value to produce $\rho_1$, $x \simeq M_{\rho_1}/\sqrt{s}$, is quite large for the CERN collider.

In summary, we have argued that walking technicolor theories possess at least
two symmetry-breaking energy scales and that the lightest scale may be accessible in ongoing experiments. Large anomalous dimensions \( \gamma_m \) enhance light-scale technipion masses, probably above half the corresponding technirho mass. For technirho masses \( \leq 300 \text{ GeV} \), \( \rho_{\pm,0}^\pm \to W^\pm \text{ or } Z^0 \text{ + anything} \) should be observable at the Tevatron collider. Along the way we have found several interesting features of walking technicolor theories. Two notable ones are: First, current masses of light-scale technifermions and of ordinary fermions get contributions from both the dynamical and current mass of heavier fermions higher up the ETC ladder. For quarks and leptons, the current-mass contribution appears to be most important by far. Second, inclusion of ordinary color in a walking technicolor theory is an essential and very interesting complication which we intend to study carefully. It is already clear that the loss of color asymptotic freedom near \( 1 \text{ TeV} \) helps unify color with technicolor at the ETC scale, enhances ETC-generated quark masses through larger anomalous dimensions and introduces a host of new, colored technirho and technipi mesons. Some of these technirhos will be strongly produced in hadron collisions through their coupling to a single colored gluon.

We are grateful to Sekhar Chivukula for very useful discussions, particularly on the consistency of chiral perturbation theory. We also thank Lawrence Hall for a stimulating conversation on technirho phenomenology. K.L.'s research was supported in part by Boston University and by the Department of Energy under Contract No. DE-AC02-86ER40284.
References and Footnotes


8. S. Raby, S. Dimopoulos and L. Susskind, Nucl. Phys. B169, 373 (1980). If we chose to fix $A_1$ instead of $M_{\rho_1}$, then according to our calculational rules below, $A_2/2A_1 = \exp\left[6\pi(a_{e1}^{-1} - a_{e2}^{-1})/(11N_{TC} - 4N_1)\right]$. For the parameters of set a (set b), this gives $A_2/2A_1 = 2.0$ (5.8). These are to be compared with the values we compute, e.g., for $M_{\rho_1} = 250$ GeV and $\kappa = 1.5$. They are $A_2/2A_1 = 3.8$ (6.0).


10. One consequence of a growing $\alpha_{QCD}$ is that color and technicolor may be easily unifiable at the ETC scale.

11. From now on we make the simplifying assumption that ETC interactions do not induce large flavor splittings in the TC sector. For the opposite extreme
of large $T_i$-flavor breaking, our estimates apply to the lightest $\tilde{T}_i T_i$ mesons. Observations of the final states in $\rho_1$ production will help determine the amount of flavor splitting.

12. Now $\pi_1^{\pm,0}(3, 1)$ is the pseudo-Goldstone boson orthogonal to $W^\pm_L, Z^0_L$. The decay $\rho_1(3, 1) \rightarrow \pi_1(3, 1)\gamma$ occurs at a low level. We estimate the partial width to be 10 MeV (3.4 MeV) for $M_{\rho_1} = 250$ GeV and $M_{\pi_1} = 130$ GeV (175 GeV). We thank L. Hall for pointing this mode out to us.


14. For the parameters of set a (set b) with $M_{\rho_1} = 250$ GeV and $\kappa = 1.5$, we obtained $F_2 = 170$ (237) GeV, $m_2 = 216$ (302) GeV, $M_\rho_1 = 1740$ (2430) GeV, $M_{\pi_1} = 832$ (135) GeV, $F_1 = 22.3$ (19.7) GeV, $m_1 = 39$ (49) GeV and $M_{\pi_1} = 128$ (135) GeV. For set b, because $N_2 = 1$, no pseudo-Goldstone $\pi_2$ states exist.


16. For $M_{\rho_1} = 300$ GeV, $M_{\pi_1} = 155$ GeV, $F_2 = 168$ GeV and $F_1 = 26.4$ GeV, all the cross sections are about 50% of those reported here.
Figure Caption

Figure 1: \( M_{\rho_1} - 2M_{\pi_1} \) versus \( \kappa \) for \( M_{\rho_1} = 200 \text{ GeV} \) (dashed line), \( 250 \text{ GeV} \) (solid line), \( 300 \text{ GeV} \) (dot-dashed line). (a) \( N_{TC} = 5, N_1 = 6, N_2 = 2, \Lambda_{ETC} = 100 \text{ TeV} \); (b) \( N_{TC} = 6, N_1 = 12, N_2 = 1, \Lambda_{ETC} = 600 \text{ TeV} \).
Figure 1a

\[ \frac{\sigma}{\sqrt{\gamma}} \left( \frac{M}{2m} \right) \left( \frac{1}{\mu} - \phi \right) \]
Figure 1b