QUARKS, GLUONS AND THE SPIN OF THE PROTON*

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ABSTRACT

Experimentally, quark helicities do not seem to contribute to the proton spin. I shall show how this result can be obtained by a direct computation in a chiral field-theoretical model of quark and gluon confinement, taking gluon-exchange effects into account. At the same time, I shall get an estimate for the gluon-helicity contribution.

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Thinking in terms of the naive quark model, we were used to the idea that all of the proton spin originates from the constituent quark helicities. This picture now has to be revised. Indeed, it has been observed recently [1,2], by combining EMC measurements of the polarized proton structure function [3] with information from neutron and hyperon decay, that very little of the proton spin is carried by quark helicities, the "experimental" value

$$\Delta q = \Delta u + \Delta d + \Delta s = 0.12 \pm 0.18$$  \hspace{1cm} (1)

resulting from a compensation between the positive contribution of the non-strange quarks

$$\Delta u + \Delta d = 0.31 \pm 0.12$$  \hspace{1cm} (2)

and a negative strange-quark contribution $\Delta s$. The question then arises: where does the spin of the proton come from?

An obvious possibility is angular momentum [4,5]. I shall show how $\Delta q$ and $\Delta g$ may be estimated, taking into account non-perturbative aspects of QCD, such as chiral symmetry breaking and confinement. Further details may be found elsewhere [6,7]. The first attempts [8,2] to explain the result (1) were made in the case of the Skyrme model [9,10] where it was shown [8] that, to zeroth order in the $1/N_c$ expansion, $\Delta q = 0$ in the chiral symmetric limit, while to first order [11], $\Delta q$ varies between 0.1 and 0.3, depending on the choice of parameters. A computation [12], in an SU(2) chiral quark-meson model, of the contributions of the non-strange quark helicities $\Delta u + \Delta d$ and mean orbital angular momentum $2 \langle L_z \rangle$ to the proton spin, led to results which were, over most of the range of parameters studied, consistent with the "experimental" determination (2).

It was suggested in Ref. [12] that the agreement with experiment should be improved by taking into account the effects of gluons. Now, gluons enter in this problem in three different ways, leading to contributions which I shall term the direct, anomalous, and indirect gluon contributions to the proton spin.

First, the sum rule

$$\Delta q + 2 \langle L_z \rangle_q + 2 \Delta g + 2 \langle L_z \rangle_g = 1$$  \hspace{1cm} (3)

shows that gluons may contribute directly to the proton spin, both through their helicity $\Delta g$, and their angular momentum $\langle L \rangle_z$ [2,13].

Second, there is an anomalous gluon contribution [14] to the flavour singlet axial current matrix element [13] so that in the left-hand side of Eqs. (1) and (2), $\Delta u$, $\Delta d$, $\Delta s$ should be replaced by the $q^2$-independent combinations [13]
\[ \Delta u = \Delta u - \frac{\alpha_s}{\pi} \Delta g \] 

(4)

e tc. The effect of the anomaly is thus to relax the experimental constraint (2) for the contribution of the non-strange quark helicities, which is replaced by the relation between the quark and gluon contributions

\[ \Delta u + \Delta d = 0.31 + 2 \frac{\alpha_s}{\pi} \Delta g \pm 0.12 \] 

(5)

Finally, it has been proposed [12,15] that gluon exchange between constituent quarks may affect the relative contributions of quark helicities and orbital angular momentum, and thus contribute indirectly to the proton spin. It is this last contribution which I shall in particular be interested in.
between QCD (in the limit $f \to 0$ and $\kappa \to 1$) and a non-linear $\sigma$-model (in the limit $\lambda^2 \to \infty$ and $g_s \to 0$) which admits topological soliton solutions [23] closely related [24] to those of the Skyrme model.

In this model, a hadron is defined in the mean-field approximation as a state of $N$ constituent quarks of Dirac wave functions $q^i$, surrounded by mean chiral fields $\sigma, \phi$ and mean colour fields $E_a, B_a$, which satisfy a set of coupled stationary mean field equations with suitable boundary conditions. A hadron in the ground state corresponds intuitively to the case where all the constituent quarks are in the same spatial, spherically symmetric, ground state. However, the only known way to obtain a spherically symmetric solution of the mean-field equations is to assume further that the $N$ quarks are in the same spatial and spin-isospin "hedgehog" state [25] $q_0$, leading to a spherically symmetric mean scalar field $\sigma_0$ and to the hedgehog configuration for the mean pion field $\pi_0$.

Now, if all the quarks are in the same spatial state, the total colour charges of hadrons (colour singlets) are zero, as well as the chromoelectric fields $E_a$. Moreover, if the quarks are in the hedgehog state, the colour currents of the various quarks are (independently) zero, and so the chromomagnetic fields $B_a$ are zero. So at this stage the localized solutions of the theory are identical to those which may be obtained in the case $g_s = 0$ [17,26].

However, as the hedgehog assumption breaks rotational invariance, the mean-field hedgehog hadrons $|h>$ are not eigenstates of total angular momentum $j$. Nevertheless, they are eigenstates of grand spin $J+I$

$$\langle j + I \rangle |h> = 0 \quad ,$$

with zero mean spin and isospin

$$\langle \bar{j} \rangle = -\langle \bar{I} \rangle = 0 \quad .$$

It is therefore necessary to project [27] these states on (approximate) eigenstates of spin and isospin. Actually, because of the grand-spin symmetry, states with equal spin and isospin (as are the nucleon and delta) may be obtained from the hedgehog by a single Peierls-Yoccoz [28] angular momentum projection [29-31]. This can be used to compute gluonic effects only at the cost of intricate numerical computations.

So we choose the more simple semi-classical cranking method [32,27] to restore rotational invariance. In this method, which was previously applied to the Skyrme model [33], to the chiral bag model [34] and to chiral soliton bag
models [35,36], the hadronic spin and isospin are generated by time-dependent rotations (in space or isospace); the hadronic energy is modified by a spin-isospin dependent centrifugal term, leading to a mass splitting between the nucleon and delta. Cranking also affects other mean-field quantities, in particular the colour currents which are no longer zero and now generate non-vanishing chromomagnetic fields (the chromoelectric fields are of course still zero), thus mediating an attractive interaction between quarks (this interaction, which is due to one-gluon exchange, has not been taken into account in the quark-meson interaction, due to the exchange of $q\bar{q}$ and $g\bar{g}$ pairs, so that there is no problem of double counting [21]); this was applied in Refs. [37] and [38] to compute the gluonic contribution to the nucleon-delta mass difference in the lowest order of perturbation theory. Here, we apply the cranking method by taking into account self-consistently the mean chromomagnetic fields, using a non-trivial generalization of the self-consistent method of Ref. [39]. Finally, the cranking technique allows for a direct estimation of $\Delta q$ or, in the case of our two-flavour model, of $\Delta u+\Delta d$, as first observed in Ref. [12].

As suggested in Ref. [40], we may improve on the mean field cranking by cranking not the hedgehog but the coherent hedgehog [30] which is defined as product of valence quarks and a coherent superposition [41] of pions, as trial state. We isorotate a coherent hedgehog state $|h(t)\rangle$ to

$$|h'_\omega(t)\rangle = e^{-i\omega \hat{I}_t} |h(t)\rangle,$$

(12)

where $\omega$ is a time-independent vector and $\hat{I}_t$ is the isospin operator, and choose the cranked hedgehog $|h'_\omega\rangle$ so as to minimize the cranked energy $<H+\omega \hat{I}_t>'_\omega = <H+\omega \hat{I}_t>_\omega$ and choose the coherent hedgehog $|h'\rangle$ no longer extremizes the energy $<H>_t$, so that the quark and pion wave functions $q_0$ and $\phi_0$ are replaced by cranked wave functions $q'$, $\phi'$. The resulting cranked mean spin and isospin are, for slow rotations, linear in $\omega$,

$$\langle \hat{J}^x \rangle = \langle \hat{I}^x \rangle = J\omega + O(\omega^2),$$

(13)

thus defining the hadronic moment of inertia $J$, the cranking parameter $\omega = |\omega|$ being obtained by solving the physical constraint [40]

$$\langle \hat{J}^2 \rangle = \langle \hat{I}^2 \rangle = (\Delta I)^2(\omega) + J^2\omega^2 = j(j+1),$$

(14)

where $(\Delta I)^2$ is the mean square isospin fluctuation and $j$ is the hadronic spin-isospin.

The extremization of the cranked energy involves self-consistently the cranked mean fields $q'$, $\sigma'$, $\phi'$, $\hat{B}'_a$. To carry out this extremization, we expand
the cranked mean fields in powers of $\omega$ [35]:

\[
q' = q_0 + \omega q_1 + \ldots , \quad \sigma' = \sigma_0 + \omega \sigma_1 + \ldots , \\
\bar{B}_a' = \omega \bar{B}_a + \ldots , \quad \bar{\psi}' = \bar{\psi}_0 + \omega \bar{\psi}_1 + \ldots ,
\]

(it may be shown that $\sigma_1 = 0$ and $\bar{\psi}_1 = 0$ [35]). The corresponding expansion of the cranked energy is

\[
E' = \langle H + \bar{\psi}' \sum \bar{\psi}' \rangle = E_0 + \omega^2 E_2 + \ldots ,
\]

where the zeroth order energy is the hedgehog energy. Then, varying the resulting functional forms of $E_0$ and $E_2$, we obtain two systems of coupled differential equations which we solve numerically with appropriate boundary conditions. This gives us the zeroth and first order quark and meson wave functions.

The resulting mean angular momentum $\langle J_3 \rangle' = J \omega$ ($\omega$ is chosen along the third direction) generated by cranking may be broken down into quark spin, quark orbital and meson orbital components

\[
J \omega = (J_{qs} + J_{qe} + J_m) \omega ,
\]

where, for a baryon,

\[
J_{qs} = 3 \int d^3x \left( q_0^+ \bar{\sigma}_2 q_1 + q_1^+ \bar{\sigma}_2 q_0 \right) , \\
J_{qe} = 3 \int d^3x \left( q_0^+ \bar{\tau}_1 q_1 + q_1^+ \bar{\tau}_1 q_0 \right) , \\
J_m = \int d^3x \left( \bar{\sigma}_0 \hat{\sigma}_2 \right)^2 .
\]

This gives us immediately the non-strange quark helicity contribution to the proton spin

\[
\Delta u + \Delta d = J_{qs} / J .
\]

We show in Fig. 1 the variation with $\alpha_s = g_s^2 / 4\pi$ of $\Delta u + \Delta d$ for $m_\sigma = 600$ MeV, $f = 4.05$ and different values of the confining exponent $n$ occurring in Eq. (9) (the string tension [21], is infinite for $n < 1$). For $n = 2$, $\Delta u + \Delta d$ decreases quickly with increasing $n$ and $\alpha_s$, thus demonstrating that the effect of gluon exchange (the indirect gluon contribution) is crucial. A similar behaviour is obtained when $f$ and $m_\sigma$ are varied.

In order to obtain an estimate of $\Delta u + \Delta d$, we adjust the model parameters so as to reproduce, as far as possible, the nucleon and delta masses. To do so, we use the familiar cranking result [27] for the energy $E = \langle H \rangle'$ of the cranked hedgehog,

\[
E = E_0 + \frac{1}{2} J \omega^2 .
\]
Replacing the (uncranked) hedgehog energy $E_0$ by the centre-of-energy corrected hedgehog mass $M$ given by [42,43]

$$M^2 = E_0^2 - \langle p_q^2 \rangle - \langle p_\pi^2 \rangle,$$

where $\langle p_q^2 \rangle$ and $\langle p_\pi^2 \rangle$ are the quark and pion translational fluctuations, and $\omega$ by its value obtained from (14), we obtain

$$E = M + \frac{1}{2\sigma} \left[ J(\gamma + 1) - (\Delta E)^2(\omega) \right]$$

for the fully quantum-corrected baryonic energy. From (22) we obtain the mean baryonic mass $\bar{E}$, which is lower than the physical mean nucleon-delta mass for all values of the model parameters, the choice $f = f_c(m_\sigma)$ (the critical value of the quark coupling constant below which there is no soliton solution to the mean field equations [17]) leading for a given value of $m_\sigma$ to the best possible value of $\bar{E}$, and the nucleon-delta mass difference $\Delta E$. We adjust the parameters $\alpha_s$ and $n$ so that the relative nucleon-delta mass splitting $\Delta E/E$ attains its experimental value (0.27). Bounds on the possible values of $m_\sigma$ follow from the study of the stability of the baryonic states. Forbidding the delta to decay into three free quarks leads to $m_\sigma > 600$ MeV, while allowing it to emit a pion yields $m_\sigma < 800$ MeV.

We are now able to give the variation of $\Delta u + \Delta d$ as a function of $\alpha_s$ (Fig. 2). We see that the non-strange quark contribution decreases not only with decreasing $\alpha_s$ (i.e., increasing $n$), but also with increasing $m_\sigma$. For $600$ MeV < $m_\sigma$ < $800$ MeV and values of $\alpha_s$ which lie between 0.25 and 0.85, as suggested by experiment (for $Q = 1000$ MeV) [44], we then obtain

$$0.21 \leq \Delta u + \Delta d \leq 0.32$$

(23)

corresponding to the shaded area in Fig. 2. These values are in good agreement with the "experimental" value (2). The quark orbital angular momentum contribution accounts for roughly one half of the proton spin.

As discussed in Refs. [12] and [6], these results may actually be used to estimate the total quark helicity contribution $\Delta q = \Delta u + \Delta d + \Delta s$ and the gluonic helicity contribution $\Delta g$ to the proton spin. Indeed, inserting our results (23) into relation (5), we derive

$$-0.11 \leq \frac{\Delta g}{\Delta q} \leq 0.07$$

(24)

consistent with $\Delta g = 0$. Then, going over to the three-flavour world, we assume that, because SU(2)$\times$SU(2) is a better symmetry than SU(3)$\times$SU(3) [45], our two-
flavour computations give substantially correct values for the non-strange contribution $\Delta u + \Delta d$ (as has been checked, for other observables, in Ref. [46]), while SU(3)$\times$SU(3) symmetry breaking generates a non-negligible strange contribution $\Delta s$, for which the experimental value [2] is

$$\Delta s = -0.19 + \frac{\alpha_s}{\alpha_s^0} \Delta q \pm 0.06 \quad (25)$$

Combining this with the previous results, we obtain

$$-0.03 \leq \Delta q = \Delta u + \Delta d + \Delta s \leq 0.13 \quad (26)$$

consistent with $\Delta q = 0$. We conclude that all the proton spin is in fact due to orbital angular momentum (as speculated in Ref. [5]). To arrive at this conclusion, it was essential to take into account the effect of gluon exchange between quarks [15], which does not contribute to $\Delta q$ [6], but strongly enhances the relative non-strange quark orbital contribution.

As our results (26) rest on the assumption that SU(2)$\times$SU(2) predictions for $\Delta u + \Delta d$ remain valid in the three-flavour case, it would be worthwhile complementing this investigation by a more direct computation of $\Delta s$. This should involve the effect of polarized sea quark pairs, which could be taken care of in an SU(3)$\times$SU(3) extended version of our model [47].

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REFERENCES

Fig. 1 The non-strange quark helicity contribution to the proton spin $\Delta u + \Delta d$ as a function of $\alpha_s$ for $m_0 = 600$ MeV, $I = 4.05$ and various values of $n$.

Fig. 2 The non-strange quark helicity contribution to the proton spin $\Delta u + \Delta d$ as a function of $\alpha_s$ for various values of $m_0$ and $f = f_c(m_0)$ (solid lines) and for various values of $n$ (dashed lines); the shaded area corresponds to $600$ MeV $< m_0 < 800$ MeV and $0.25 < \alpha_s < 0.85$. 