EFFECT OF QUARK COLOR-HYPERFINE INTERACTIONS ON BARYON MASSES

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ABSTRACT

We consider the contribution from the color-hyperfine interaction to the masses of ground-state hadrons, with an emphasis on baryons. We use experimental information about how the color-hyperfine term depends on flavor to make predictions about the masses of baryons containing a heavy quark. Most of our predictions are in the form of inequalities.

1 Introduction

De Rújula, Georgi, and Glashow [1] pointed out in 1975 that the color-hyperfine interaction between two quarks arising from one-gluon exchange in QCD has the same form as the analogous interaction arising from one-photon exchange in QED. Since that work, many authors have calculated the color-hyperfine interaction in the Fermi-Breit approximation [2] and in higher order in QCD perturbation theory [3].

The magnitude of the color-hyperfine interaction energy is quite sensitive to the properties of specific models, and so some authors have concentrated on qualitative features. Cohen and Lipkin [4] and Richard and Taxil [5] discussed reasons why the color-hyperfine interaction in baryons leads to a \( \Xi^* - \Xi \) mass splitting which is larger than that of \( \Sigma^* - \Sigma \). These ideas were generalized [5,6] to baryons containing heavy quarks.

In this work we use experimental information on baryon mass differences to obtain qualitative information about how the color-hyperfine interaction varies with quark masses and obtain inequalities relating mass differences of unknown baryons. We also give results for mesons, which are well known, and we present them only for the purpose of comparison with baryons.

We assume that the Hamiltonian \( H \) describing a hadron in terms of its constituent quarks consists of a principal term \( H_0 \), which is independent of spin, plus a term \( H' \), which can be treated in lowest-order perturbation theory. The unperturbed Hamiltonian \( H_0 \) can be quite general, including relativistic motion and an interaction which depends on quark flavors. We do, however, require, that the ground-state wave function can be approximated by the product of a flavor wave function, a space wave function with no orbital angular momenta, and a spin wave function which is nonrelativistic in that each quark spin is described by two components. With these assumptions, the only term in \( H' \) which contributes to the ground-state energy is the color-hyperfine term.

This framework is rather general. Obviously, if we do not assume a particular form for the wave equation which governs the motion and a particular form for the quark interactions, we cannot calculate the masses of hadrons from the Hamiltonian of the system. But we can use experimental information about hadron masses to establish a pattern of how the color-hyperfine interaction energy depends on quark flavors, and to use the pattern to make predictions about the masses of as yet undiscovered baryons.

2 Expressions for hadron masses

We denote the mass of a ground-state meson containing quarks \( q_1 \) and \( q_2 \) by \( M_{q_1 q_2} \), if the meson has spin 0, and by \( M^*_{q_1 q_2} \) if the meson has spin 1. Likewise, we denote the mass of a ground-state baryon of spin 3/2 containing quarks \( q_1, q_2, \) and \( q_3 \) by \( M^*_{q_1 q_2 q_3} \). If a ground-state baryon has spin 1/2, we omit the asterisk on the symbol. If all quarks have different flavors, a second baryon of spin 1/2 exists, and we denote its mass by \( M_{q_1 q_2 q_3} \).

Expressions for these masses can be written in terms of the ground-state eigenvalues of \( H_0 \) and (in lowest-order perturbation theory) matrix elements of the color-hyperfine interaction with respect to the eigenfunctions of \( H_0 \). We denote the ground-state eigenvalue of \( H_0 \) by \( E_{q_1 q_2} \) for a meson and by \( E_{q_1 q_2 q_3} \) for a baryon. We denote the meson and baryon...
eigenfunctions by \(|q_1q_2\rangle\) and \(|q_1q_2q_3\rangle\) respectively.

The only part of the perturbing interaction \(H'\) which is relevant for ground-state hadrons is the color-hyperfine term, which we denote by \(I_M\) for mesons and \(I_B\) for baryons. We write \(I_M\) rather generally as

\[
I_M = A_{12} S_1 \cdot S_2,
\]

where \(S_i\) are quark spin operators and \(A_{12}\) is an operator which can act on quark flavor and space coordinates, but not on spin coordinates. Likewise, in rather general fashion we write \(I_B\) as

\[
I_B = A_{12,3} S_1 \cdot S_2 + A_{13,2} S_1 \cdot S_3 + A_{23,1} S_2 \cdot S_3,
\]

where the \(A_{ij,k}\) act on quark flavor and space coordinates. We must adopt a definite ordering of the quarks in our scheme. We adopt the following convention [7]: If two quarks are identical, they are considered the first two; if all three quarks are different, the quarks are ordered in the wave function from lightest to heaviest.

The forms of \(I_M\) and \(I_B\) are motivated by QCD perturbation theory, but are more general. Because we regard QCD as the underlying theory behind our considerations, we assume that \(A_{12}\) and \(A_{ij,k}\) depend on flavor only through the different masses of different flavors.

We introduce the meson color-hyperfine interaction energy \(R_{q_1q_2}\), defined by

\[
4 R_{q_1q_2} = \langle q_1q_2 | A_{12} | q_1q_2 \rangle.
\]

It is straightforward to write the expression for the mass of a meson as

\[
M_{q_1q_2} = E_{q_1q_2} - 3 R_{q_1q_2},
\]

\[
M_{q_1q_2}^* = E_{q_1q_2} + R_{q_1q_2}.
\]

It then follows from the well-known fact that vector mesons are heavier than their pseudoscalar counterparts that \(R_{q_1q_2} > 0\). These equations are simple and well known. They do not apply to light self-conjugate mesons of isospin zero, because QCD annihilation terms and SU(3) mixing effects, omitted in our framework, make spin-dependent contributions to the masses of such mesons.

We define the baryon color-hyperfine energies \(R_{q_1q_2q_3}\) by

\[
4 R_{q_1q_2q_3} = \langle q_1q_2q_3 | A_{ij,k} | q_1q_2q_3 \rangle.
\]

Using baryon spin wave functions defined by Franklin et al. [7], we obtain for the mass of a baryon of spin 3/2 the expression

\[
M_{q_1q_2q_3}^* = E_{q_1q_2q_3} + R_{q_1q_2q_3} + R_{q_1q_2q_3} + R_{q_2q_3q_1}.
\]

The expression for the mass of a spin 1/2 baryon in which the first two quarks have spin 1 is

\[
M_{q_1q_2q_3} = E_{q_1q_2q_3} + 3 R_{q_1q_2q_3}.
\]

If the first two quarks have spin 0, the expression is

\[
M_{q_1q_2q_3}^* = E_{q_1q_2q_3} - 3 R_{q_1q_2q_3}.
\]

Unlike the meson case, the baryon color-hyperfine matrix elements cannot all be written as functions of baryon masses, in general. However, in those cases in which the values of the matrix elements \(R_{q_iq_jq_k}\) can be extracted from the experimental values of the baryon masses, these matrix elements are always positive, as in the meson case.

3 Color-hyperfine matrix elements

We use the equations of the previous section to obtain some hadron spin-averaged energies and color-hyperfine energies in terms of known hadron masses. The values for mesons are given in Table 1 and for baryons in Table 2.

We see from Tables 1 and 2 that a pattern exists among the known hyperfine energies in hadrons. In mesons, the pattern is quite simple, namely, that as the mass of any constituent increases, the value of \(R_{q_iq_j}\) decreases.

There is one apparent exception to the pattern of Table 1: we see that \(R_{qC} < R_{qC}\), rather than \(R_{qC} = R_{qC}\), as we might expect. This exception indicates that there may be a pattern reversal in mesons containing heavy quarks. A possible reason for such a reversal is that in heavy-quark systems, the wave function is smaller than in systems containing a light quark. In QCD perturbation theory, the shrinkage of the wave function eventually overcomes the effect of the decrease in the operator \(A_{12}\) as a function of quark masses, so that for sufficiently heavy quarkonium, we expect the pattern to be reversed.

We now turn to the more interesting case of the baryons. Here, we must distinguish between the quarks \((q_1, q_2)\) whose coordinates appear before the comma in \(R_{q_1q_2q_3}\) and the quark \((q_3)\) whose coordinates appear after the comma. In QCD perturbation theory, the color-hyperfine operator is a sum of two-quark operators. The matrix element of one of these two-quark operators involves not only the two participating quarks (interacting quarks) but also the quark whose coordinates appear only
Table 1. Spin-averaged energy eigenvalues and color-hyperfine matrix elements obtained from experimental values of meson masses as given by the Particle Data Group [8]. All masses and energies are in MeV.

<table>
<thead>
<tr>
<th>Quark content</th>
<th>Vector meson</th>
<th>Pseudoscalar meson</th>
<th>(E_{Q_1Q_2} )</th>
<th>(R_{Q_1Q_2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(qg)</td>
<td>(\rho(770))</td>
<td>(\pi(138))</td>
<td>612</td>
<td>158</td>
</tr>
<tr>
<td>(qs)</td>
<td>(K^*(894))</td>
<td>(K(496))</td>
<td>794</td>
<td>99.5</td>
</tr>
<tr>
<td>(qc)</td>
<td>(D^*(2009))</td>
<td>(D(1867))</td>
<td>1974</td>
<td>35.5</td>
</tr>
<tr>
<td>(sc)</td>
<td>(D_s^*(2113))</td>
<td>(D_s(1969))</td>
<td>2077</td>
<td>35.5</td>
</tr>
<tr>
<td>(cc)</td>
<td>(J/\psi(3097))</td>
<td>(\eta_c(2980))</td>
<td>3068</td>
<td>29.2</td>
</tr>
<tr>
<td>(qg)</td>
<td>(B^*(5330))</td>
<td>(B(5278))</td>
<td>5317</td>
<td>13</td>
</tr>
<tr>
<td>(bb)</td>
<td>(Y(9460))</td>
<td>(\eta_b(1))</td>
<td>?</td>
<td>?</td>
</tr>
</tbody>
</table>

*Not in main meson table of the Particle Data Group, and needs confirmation.
†Not yet discovered.
‖The \(\eta_b\) needs to be observed and its mass measured to enable us to obtain this quantity by the methods of this paper.

Table 2. Spin-averaged energy levels and color-hyperfine matrix elements obtained from experimental values of baryon masses. These come from the tables of the Particle Data Group [8] except for the mass of the \(\Sigma_c\), which is from ref. [9]. All energies and masses are in MeV.

<table>
<thead>
<tr>
<th>Quark content</th>
<th>Spin 3/2 baryon</th>
<th>Spin 1/2 baryon</th>
<th>Spin 1/2 baryon</th>
<th>(E_{Q_1Q_2Q_3} )</th>
<th>(R_{Q_1Q_2Q_3} )</th>
<th>(R_{Q_1Q_2Q_3} )</th>
<th>(R_{Q_1Q_2Q_3} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(qq)</td>
<td>(\Delta(1232))</td>
<td>(N(939))</td>
<td>*</td>
<td>1086</td>
<td>48.8</td>
<td>48.8</td>
<td>48.8</td>
</tr>
<tr>
<td>(qs)</td>
<td>(\Sigma^*(1385))</td>
<td>(\Sigma(1193))</td>
<td>(\Lambda(1116))</td>
<td>1270</td>
<td>51.2</td>
<td>32</td>
<td>32</td>
</tr>
<tr>
<td>(ss)</td>
<td>(\Xi^*(1530))</td>
<td>(\Xi(1318))</td>
<td>*</td>
<td>†</td>
<td>†</td>
<td>35.8</td>
<td>35.8</td>
</tr>
<tr>
<td>(qc)</td>
<td>(\Sigma_c(1))</td>
<td>(\Sigma_c(2452))</td>
<td>(\Lambda_c(2285))</td>
<td>?</td>
<td>?</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>(gc)</td>
<td>(\Xi_c(1))</td>
<td>(\Xi_c(2470))</td>
<td>??</td>
<td>??</td>
<td>??</td>
<td>??</td>
<td>??</td>
</tr>
</tbody>
</table>

*State does not exist.
†Cannot be determined from the data by the methods of this paper.
††Not yet discovered.
‡‡The \(\Sigma_c\) needs to be observed and its mass measured to enable us to calculate this quantity by the methods of this paper.
‡‡‡The \(\Xi_c\) and \(\Xi_c\) need to be observed and their masses measured to enable us to calculate this quantity by the methods of this paper.

in the three-body wave function (spectator quark) [5]. Although our operators \(A_{ij,k}\) are three-body operators, we will use the terminology in this case also that quarks \(q_i\) and \(q_j\) are the interacting quarks and \(q_k\) is the spectator quark.

We see from Table 2 that again there is a pattern: If the mass of an interacting quark increases, the color-hyperfine energy \(R_{Q_1Q_2Q_3} \) decreases. On the other hand, if the mass of a spectator quark increases, the corresponding color-hyperfine interaction increases. This can be understood from QCD perturbation theory: The masses of the interacting quarks appear in the denominator of the expressions for the color-hyperfine energies, and cause these energies to decrease with increasing quark mass. However, the only effect of increasing the mass of the spectator quark is to shrink the baryon wave function, thereby increasing the value of \(R_{Q_1Q_2Q_3} \). We assume here that the pattern for baryons applies to all baryons containing at most one heavy quark.
We let the symbol for a baryon denote its mass. The quark content of some of the conventional symbols is given in Table 2. The $\Omega_c$ is not listed in Table 2, and is composed of $ssc$. If a $b$ quark replaces a $c$ quark in a baryon, the subscript $c$ is replaced by $b$. Our approach leads to the result, in agreement with observation, that

$$\Sigma_c - \Lambda_c > \Sigma - \Lambda.$$  \hspace{1cm} (10)

We also obtain the predictions, not yet tested

$$\Sigma^*_b - \Sigma_b < \Sigma^*_c - \Sigma_c < \Sigma^* - \Sigma, \quad \Sigma_b - \Lambda_b > \Sigma_c - \Lambda_c,$$ \hspace{1cm} (11)

$$\Xi_b - \Xi_b > \Xi_c - \Xi_c, \quad \Xi^*_b - \Xi_b < \Xi^*_c - \Xi_c,$$ \hspace{1cm} (12)

$$\Omega^*_b - \Omega_b < \Omega^*_c - \Omega_c < \Xi^* - \Xi,$$ \hspace{1cm} (13)

$$2\Sigma^* + \Sigma - 3\Lambda < 2\Sigma^*_b + \Sigma_c - 3\Lambda_c < 2\Sigma^*_b + \Sigma_b - 3\Lambda_b,$$ \hspace{1cm} (14)

$$2(\Xi^* - \Xi) < 2\Xi^*_b + \Xi_c - 3\Xi_c < 2\Sigma^*_b + \Sigma_c - 3\Lambda_c,$$ \hspace{1cm} (15)

$$2\Xi^*_b + \Xi_c - 3\Xi^*_c < 2\Xi^*_b + \Xi_b - 3\Xi^*_b < 2\Sigma^*_b + \Sigma_b - 3\Sigma_b.$$ \hspace{1cm} (16)

A number of these inequalities have been obtained previously\cite{5-7}, although not with such general assumptions.

4 Discussion

The same techniques we have used so far can enable us to get additional predictions in a straightforward manner. We shall not write down the resulting inequalities here. In particular, we can use our methods to obtain predictions for baryons containing two or three heavy quarks. We believe the pattern of the change in color-hyperfine matrix elements with quark masses is reliable for baryons containing two $c$ quarks and a light quark and probably even for $ccc$, but the pattern is less certain for baryons containing two $b$ quarks or a $c$ and a $b$. There is genuine physics to be learned from measurements of the masses of baryons containing two or three heavy quarks, but it is difficult to foresee when such measurements will become feasible.

We have obtained results which are based on rather general considerations rather than on any specific model. We believe especially that the inequalities given here will be confirmed by experiment without exception, but also that other predictions we can make about baryons using these techniques are on a sound footing, and we look forward to the relevant measurements.

Acknowledgements

One of us (EP) thanks Indiana University for a fellowship of the Indiana University Institute of Advanced Study. This work was supported in part by the U.S. Department of Energy and in part by the Italian Ministry for Public Education.

References

DISCUSSION

Q. V. Gupta (Tata): How do your predictions compare with those of H. J. Lipkin who also used a similar approach a few years ago?

A. D. Lichtenberg: They are similar but our considerations are more general.

Q. Hitoshi Ito (Kinki Univ.) (1) I have examined the hyperfine interactions in the $q\bar{q}$ systems (contr. paper No. 202) by using a BS-like equation. I found very large relativistic effects in the wave function at the origin of the $\Gamma(t^+t^-)$ formula and also large renormalization effects, which we need in the relativistic wave function. The relativistic effects in the mass splittings are also large even in the $c\bar{c}$ system. My conclusion is that the Fermi-Breit approximation is wrong even in the charmonium. Higher-order $p/m$ corrections are necessary. (2) What do you think of the relativistic effects in the baryon systems? Your factors $-3$ and $1$ come from the lowest approximation.

A. D. Lichtenberg: The factors $-3$ and $1$ are for mesons. The factors for baryons are different and given in my talk. I agree that relativistic corrections are significant for mesons, but should be less important for baryons because the baryon wave functions are less singular near the origin and because the $q\bar{q}$ color hyperfine interaction is only one half as large as the $q\bar{q}$ interaction. That is why I emphasized baryons in my talk. However, the method also seems to have good predictive power for mesons.

Q. J. Lee-Franzini (SUNY, Stony Brook): Don't your considerations also apply to the hyperfine splittings of the $B_s$ and $B$ meson system?

A. D. Lichtenberg: Yes, in principle, but as both quarks become heavy, their wave function shrinks and the effect of the color-hyperfine interaction eventually becomes larger with increasing quark masses. This reversal may already be happening in the $B_s$ system. That is why we emphasized baryons, where the wave function is not as singular at the origin as in mesons, and the reversal is less likely to occur if only one quark is $c$ or $b$. 
