ABSTRACT

The influence of geometrical phase on neutrino spin-precession in the magnetic field is considered. At large geometrical phase velocity the depth of precession and therefore the probability of $\nu_L \rightarrow \nu_R$ transition are strongly suppressed in contradiction with J.Vidal and J.Wudka result. We propose the effect of resonant spin conversion induced by the geometrical phase, and consider its application to solar neutrinos.
Appearance of the Berry (geometrical) phase in the neutrino spin-precession is related to the rotation of the magnetic field, \( \vec{B} \), in the transverse (with respect to neutrino trajectory) plane. More precisely, the geometrical phase is determined by the angle of \( B \)-rotation, \( \phi \), and adds to the usual dynamic phase. It may influence on the solar \( \nu_{eL} \)-flux. And moreover, according to J. Vidal and J. Wudka paper, the geometrical phase, being much larger than the dynamic one, may solve the solar neutrino problem even at small field strength: \( B \lesssim 10^3 \text{G} \) and small magnetic moment: \( \mu_\nu \lesssim 10^{-13} \mu_B \) (\( \mu_B \) is the Bohr magneton); compare with usual VVO-scenario.

In this paper we show that the result by J. Vidal and J. Wudka is incorrect. New effect – the resonant spin conversion, induced by the geometrical phase, is proposed.

Consider the system of the left, \( \nu_L \), and the right, \( \nu_R \), neutrinos, \( \nu_S = (\nu_L, \nu_R) \), with magnetic moment, evolving in matter and in the transverse magnetic field. Let the field rotates on the neutrino way in the transverse plane: \( \vec{B} = B_x + i B_y = B e^{i \phi} \), \( \phi = \phi(t) \). Then the evolution equation for \( \nu_S \) can be written as

\[
\dot{\nu}_S = \hat{H} \nu_S, \quad \hat{H} = \begin{pmatrix} \nu/2 & \mu_\nu Be^{-i\phi} \\ \mu_\nu Be^{i\phi} & -\nu/2 \end{pmatrix}
\]  

where \( \nu(t) \) is the \( \nu_L \)- and \( \nu_R \)- levels splitting due to interactions with matter and due to possible mass difference:

\[
\nu = \sqrt{2} G_F n_{\text{eff}} - \frac{\Delta m^2}{2E}
\]

here \( G_F \) is the Fermi constant, \( \Delta m^2 = m^2(\nu_L) - m^2(\nu_R) \), \( E \) is the neutrino energy and \( n_{\text{eff}} \) is the effective concentration of particles interacting with neutrinos: \( n_{\text{eff}} = n_e - n_n \) for \( \nu_{eL} - \bar{\nu}_{\mu R} \) and \( n_e - n_n/2 \) for \( \nu_{eL} - \nu_{eR} \). \( n_e \) and \( n_n \) are the concentrations of electrons and neutrons correspondently. The imaginary part of \( \hat{H} \)
can be eliminated by the transformation \( \hat{\nu}_S' = \hat{U}\nu_S' \), where \( \hat{U} = \text{diag}[e^{-i\phi/2}, e^{i\phi/2}] \) and the evolution equation for \( \hat{\nu}_S' \) is:

\[
\frac{d\hat{\nu}_S'}{dt} = \begin{bmatrix}
V/2 & \mu_\nu B \\
\mu_\nu B & -V/2
\end{bmatrix} + \frac{\phi}{2} \begin{bmatrix}
-1 & 0 \\
0 & 1
\end{bmatrix} \hat{\nu}_S'
\]

(3)

\( \phi = \frac{d\phi}{dt} \) is the geometrical phase velocity. The mixing angle of \( \hat{\nu}_S', \theta_m' \), depends now on \( \phi \): \( \tan2\theta_m' = 2\mu_\nu B/(V - \phi) \). Since \( \hat{U} \) is diagonal, the transition probabilities for \( \hat{\nu}_S \) and \( \hat{\nu}_S' \) are the same.

For constant parameters \( n' \), \( B', \phi \) (\( B \) rotates steady) the equation (3) has the analytic solution. The survival probability (the probability of \( \nu_L \to \nu_L \) transition) can be written as \( P = \bar{P} + \left(A_p/2\right)\cos(2\pi t/l_p) \) where the depth, \( A_p \), and the length, \( l_p \), of precession are

\[
A_p = \sin^22\theta_m = \frac{(2\mu_\nu B)^2}{(V - \phi)^2 + (2\mu_\nu B)^2}
\]

(4)

\[
2\pi l_p^{-1} = \Delta E = \left[(V - \phi)^2 + (2\mu_\nu B)^2\right]^{1/2}
\]

(5)

and the average probability equals \( \bar{P} = 1 - A_p/2 \). Eqs. (4,5) generalize the results obtained in\(^3\) for precession in matter.

If the matter effect and mass splitting are small, \( V \ll \phi \), then the depth (4) can be rewritten as \( A_p = \phi_{\text{dyn}}/\left(\phi^2 + \phi_{\text{dyn}}^2\right) \), where \( \phi_{\text{dyn}} = 2\mu_\nu B \) is the dynamic phase velocity. At \( \phi \gg \phi_{\text{dyn}} \) (the case considered in\(^3\)) the depth of precession turns out to be very small: \( A_p \approx (\phi_{\text{dyn}}/\phi)^2 \ll 1 \), i.e. large geometrical phase velocity suppresses the depth of precession and consequently the \( \nu_L \to \nu_R \) transition in spite of large total phase. In the opposite case \( \phi \ll \phi_{\text{dyn}} \) the depth is not suppressed but the geometrical phase does not exceed the dynamic one. In paper\(^3\) the dependence of the precession depth on \( \phi \) is missed, so the depth turns out to be unsuppressed even at large \( \phi \). (The error comes from omission of
the proportional to $\phi$ nondiagonal terms in equation for the eigenstates of the Hamiltonian $\hat{H}$ (1)).

Geometrical phase may induce the resonant conversion. At $V = \dot{\phi}$ or

$$\sqrt{2G_F n}^{\text{eff}} - \frac{\Delta m^2}{2E} = \phi$$

the mixing $\theta_m$ becomes maximal, so the equality (6) is nothing but the resonance condition$^6)$. According to (6) the geometrical phase velocity shifts the resonance of spin-flavor precession$^7$) in $E$ or $n^{\text{eff}}$ scales. Essentially new situation realizes for Dirac or Zeldovich-Konopinski-Mahmoud (ZKM) neutrinos. In this case the $\nu_L - \nu_R$ mass splitting is zero ($\Delta m^2 = 0$), and at $\dot{\phi} = 0$ there is no level crossing inside the Sun at all: $n^{\text{eff}} > 0$. Nonzero geometrical phase velocity can compensate the matter effect and the resonance condition turns out to be fulfilled: $\sqrt{2G_F n}^{\text{eff}}(t) = \dot{\phi}(t)$. This condition does not depend on the neutrino energy, and moreover it can be satisfied only at definite direction of $\vec{B}$-field rotation, namely at $\dot{\phi} > 0$, when $n^{\text{eff}} > 0$. Such an asymmetry is related to the difference in interactions of the left (helicity $-1/2$) and the right (helicity $+1/2$) components of neutrinos.

The level crossing$^7$) at monotonous density change ($n^{\text{eff}}$) and at slowly enough change of $\dot{\phi}$ results in the resonant spin conversion: $\nu_L \rightarrow \nu_R$. Its efficiency depends on the adiabaticity$^6)$. The adiabaticity condition ($\Delta E \gg \dot{\theta}_m$) in resonance $(V = \dot{\phi})$ is

$$\frac{2(2\mu_{\nu B})^2}{\sqrt{2G_F n}^{\text{eff}} - \dot{\phi}} \gg 1$$

At $\dot{\phi}$ the geometrical phase disappears from (7). At $n^{\text{eff}} < 0$ the negative $\dot{\phi}$ improves the adiabaticity.

Consider the application to the Sun. The rotation of $\vec{B}$ on the way of neutrino may be stipulated by that the magnetic force lines
of the toroidal field form the helixes, which wind around torii\(^3\). Let us evaluate the angle of rotation, \(\Delta \phi\), needed for resonant conversion. Suppose the resonance takes place in the layer at \(R = 0.8 R_s\) (or \(R = 0.9 R_s\)), where the density is 0.06 \(g/cm^3\) (0.015 \(g/cm^3\), \(R_s\) is the Sun radius. Then it follows from (6) (at \(\Delta m^2 = 0\)), that \(\dot{\phi} = 2.2 \times 10^{-10} \text{cm}^{-1}\) (0.55 \(10^{-10} \text{cm}^{-1}\)). Consequently for the distance \(\Delta R = 0.1 R_s\) and for steady rotation one finds \(\Delta \phi = \frac{\phi}{2} \Delta R = \pi/2\) (\(\pi/8\)).

If \(\dot{\phi}\) is constant, the effect of conversion induced by the geometrical phase coincides with that, calculated for the spin-flavor conversion\(^8\) at \(\Delta m^2 /2E = \dot{\phi}\). So for the corresponding survival probabilities, \(P_{\text{geom}}\) and \(P_{S-f}\), one has \(P_{\text{geom}}(\phi) = P_{S-f}(E/\Delta m^2 = 1/(2\dot{\phi})), \) and \(P_{\text{geom}}\) does not depend on \(E\).

If the magnetic field twists in different directions in southern and in northern hemispheres, then the resonant conversion takes place only in the first or in the second half-year and the annual variations are expected. If the directions are the same, then one expects the semiannual variations. Also the anticorrelations of \(\nu\)-signals with solar activity can be reproduced\(^5\).

In conclusion, in presence of rotating field the depth of precession depends on the geometrical phase velocity, so that at large \(\dot{\phi}\) the depth is strongly suppressed: \(\lambda_p \propto 1/\dot{\phi}^2\) and the transformation \(\nu_L \rightarrow \nu_R\) is very small in contrast with result of\(^4\). No essential gain in \(\mu_\nu B\) is achieved with \(\phi\). Geometrical phase may induce the resonant spin conversion of Dirac or ZKM neutrinos inside the Sun.

**Acknowledgement.**

The author is grateful to Professor Ya.A.Smorodinski, who in
1986 for the first time pointed out on the possibility of Berry phase effect in mixed neutrino system.

References