Higher-order corrections to QCD evolution and scaling violation at HERA

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Abstract

We discuss the logarithmic corrections to the QCD anomalous dimensions in the region of small \( x \), and illustrate their possible impact on the deep inelastic structure functions in the kinematic domain being explored at HERA.

Résumé

Dans cet article, nous discutons des corrections logarithmiques aux dimensions anormales de QCD dans la région des petits \( x \) et discutons leur impact possible sur les fonctions de structures de diffusion profondément inélastique dans le domaine cinétique exploré à HERA.

1. QCD evolution at small \( x \)

Tests of the standard model and searches for new physics at future high-energy hadron machines require a good understanding of the small-\( x \) limit of QCD (see, for instance, Refs. [1, 2] for an overview of this subject). The main source of experimental information on small-\( x \) physics is given at present by the measurement of the structure function \( F_2 \) at the HERA \( ep \) collider [3, 4]. In this contribution we present the results of a study aimed at quantifying the size of enhanced higher-order corrections to \( F_2 \) [5, 6]. Our work starts out from the BFKL equation [7], which provides the leading-logarithmic description of high-energy scattering in perturbative QCD. More precisely, we start from the observation that BFKL-type terms (and, possibly, sub-leading corrections to them) can be rewritten in the form of an infinite resummation of perturbative contributions to the anomalous dimensions and coefficient functions of the flavour-singlet operators which enter in the renormalization group analysis of deep inelastic scattering [8, 9]. This fact allows one to study QCD evolution at small \( x \) beyond fixed perturbative order, by including resummed kernels in the evolution equations and solving these equations numerically.

Schematically, one has the equations for the flavour-singlet parton densities \( f_g \) (gluon) and \( f_S \) (sea quark) (flavour non-singlet components are not enhanced at small \( x \))

\[
\begin{pmatrix}
\dot{f}_g \\
\dot{f}_S
\end{pmatrix} = \begin{pmatrix}
\gamma_{SS} & \gamma_{SG} \\
\gamma_{SG} & \gamma_{gg}
\end{pmatrix} \begin{pmatrix}
f_S \\
f_g
\end{pmatrix},
\]

(1)

where the kernels are given by the anomalous dimension matrix \( \gamma_{ab} \). The perturbative structure of \( \gamma_{ab} \) as a function of the QCD coupling \( \alpha_s \) and the moment variable \( \omega \) conjugate to \( x \) in the Mellin-Fourier transform can be written as

\[
\gamma_{ab}(\omega, \alpha_s) = \gamma_{ab}^{(1)}(\omega)\alpha_s + \gamma_{ab}^{(2)}(\omega)\alpha_s^2 + \sum_{k=3}^\infty \left( \frac{\alpha_s}{\omega} \right)^k A_{ab}^{(k)}
\]

\[
+ \sum_{k=2}^\infty \alpha_s \left( \frac{\alpha_s}{\omega} \right)^k B_{ab}^{(k)} + \mathcal{O}\left( \alpha_s^2 \left( \frac{\alpha_s}{\omega} \right)^k \right)
\]

(2)

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In addition to the standard one-loop [10] and two-loop [11] terms, \( \gamma^{(1)} \) and \( \gamma^{(2)} \), which form the basis of any next-to-leading-order analysis of hard scattering, in the small-\( x \) region one has to worry about towers of poles at \( \omega \to 0 \) (that is, towers of logarithms in the \( x \)-space at \( x \to 0 \)), present to all orders in \( \alpha_s \), which stem from multiple exchange of soft gluons at high energies, and may give contributions of comparable size to the fixed-order terms if \( \alpha_s/\omega \sim \mathcal{O}(1) \). The terms \( A^{(k)} \) correspond to contributions resummed by the BFKL equation [7]. The terms \( B^{(k)} \) correspond to next-to-leading corrections to the BFKL equation, and the full resummation of such terms is only known in the channel \( a = S \), that is, in the case of final fermion lines [9]. Terms of order \( \mathcal{O}(\alpha_s^2(\alpha_s/\omega)^k) \) are beyond the accuracy of our calculation, and depend on the prescription one uses to match resummation on to fixed-order perturbation theory. They are related, for instance, to the conservation of energy-momentum to higher-loop accuracy. For the purpose of this talk, using different prescriptions for these terms will just amount to a method of estimating the uncertainties associated with unknown sub-dominant corrections.

The main features of the leading and next-to-leading resummation are well known (see for instance Ref. [9] for an extensive discussion). At leading level, the summation of the perturbative \( \omega \)-poles gives rise to a branch point singularity in the complex \( \omega \)-plane at a distance from the origin of order \( \alpha_s \). This behaviour in turn generates, in the \( x \)-space, a power-like growth at \( x \to 0 \). The next-to-leading summation in the quark sector does not change the position of the leading singularity, but contributes positive definite corrections to it. We will see in the following how important this enhancing effect may be at HERA.

2. Resummed results for the structure function \( F_2 \)

Given a boundary condition, as provided by a set of input distributions at a starting scale \( Q_0^2 \), we can solve the system of equations (1), with the kernels given by eq.(2), and thus obtain results for the evolution of the parton densities, which incorporate the effects of small-\( x \) dynamics [5, 6]. We can then use these results to calculate predictions for the structure function \( F_2 \). In figure 1 we report these predictions for the case of flat input distributions, such as the set MRSDO [12]. We show separately the effect of leading and next-to-leading resummation, and compare them with fixed-order results. We also plot the 1993 ZEUS data [3].

We see that BFKL terms (dot-dashed curves) have only little impact on \( F_2 \) in the HERA region. Conversely, next-to-leading terms from quark evolution (solid curves) affect the prediction significantly, and give rise to a steep increase in \( F_2 \) at low \( x \) (just about as steep as the behaviour observed in the HERA data), even though the initial distributions are flat. The predominance of the quark contributions over the leading ones was anticipated in Ref. [9], and can be understood in terms of \( F_2 \) coupling directly to quarks, and not gluons. The dotted curves in figure 1 correspond to a different prescription for the \( \mathcal{O}(\alpha_s^2(\alpha_s/\omega)^k) \) terms, and one may interpret the band between the dotted and the solid curves as an indicator of the theoretical uncertainty due to unknown sub-leading corrections.

In the case of steep input distributions, such as for example the set MRSD_—' [12], the impact of resummation turns out to be reduced, but still non-negligible [6]. Since modern input parametrizations, obtained by including the HERA data in the fits [2, 13], have a small-\( x \) behaviour intermediate between the two extreme ones considered above, we expect that resummation effects may play a role in this case also.

To have a more direct idea of the amount of scaling violation which is being brought in by the resummed
contributions, one can look at the predictions for $F_2$ as functions of $Q^2$ at fixed (small) values of $x$. Figure 2 shows that pronounced scaling violation effects can be generated at small $x$ as a consequence of perturbative resummation in the quark sector. The value of $F_2$ at the starting scale is essentially fixed by the input quark distribution (here chosen to be flat at low $x$), whereas the slope with $Q^2$ basically depends on the gluon density distribution (also assumed to be flat in this plot) and the quark evolution kernel: if the latter is taken into account with resummed accuracy, the slope becomes large, and the prediction exhibits strong scaling violation.

The $x$-dependence of the derivative of $F_2$ with respect to $Q^2$ is actually shown in figure 3. It may be interesting to note that, if one were to compare this result from resummed perturbation theory (solid line in figure 3) with the latest MRS fits [14], MRSG and MRSA', regarded as two possible good descriptions of the HERA data, one would find that that result is closer to the MRSG curve (see figure 2 of Ref. [14]), that is, closer to the description of the HERA data which entails more marked scaling violation.

To conclude, one should observe that the rise in $F_2$ measured at HERA can be accounted for by different mechanisms, in which the origin of the small-$x$ behaviour is ascribed chiefly either to the properties of the non-perturbative input distributions (as in standard next-to-leading-order analyses) or else to enhancement effects in the perturbative evolution (as in the resummed studies presented here). The fact that $F_2$ is not very sensitive to the leading BFKL dynamics makes it very difficult to discriminate between those mechanisms. From this point of view, investigations of other observables could be very useful. On the other hand, because corrections from quark evolution are fairly large, it is likely that small-$x$ resummation may prove to be helpful in improving our understanding of the HERA data. In this respect, further work is needed, to enlarge the spectrum of available resummed calculations, and to gain a better control of the subleading corrections at small $x$.

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