SUDAKOV EFFECTS IN HIGGS BOSON PRODUCTION AT THE LHC

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We discuss the resummation of Sudakov effects in a cross section from the viewpoint of its underlying factorization near the edge of phase space. We perform the resummation of Sudakov threshold logarithms in Higgs (Standard Model and Minimal Supersymmetric SM) production at the LHC, using an evolution equation in the Higgs mass that is derived from this factorization. We extend the class of universal large terms that is resummed to include additional universal contributions, which, when included, help to reproduce the exact result to within a few percent for the full allowed range of Higgs boson masses in the SM and MSSM. Using the analytic resummed formula as a generating functional for approximate perturbation theory, we show results for next-to-next-to-leading order corrections in Higgs production, and find they are potentially sizable.

1 Sudakov Factorization and Resummation

The intimate connection between resummation of large logarithms in amplitudes in quantum field theory, and the factorization of such amplitudes is well-known from multiplicative renormalization. The unrenormalized Green function in terms of fields $\phi_i$ factorizes as

$$G_{un}(p_i, \Lambda, g_0) = \prod_i \sqrt{Z_i} \left( \frac{\Lambda}{\mu} g(\mu) \right) G_{ren}(p_i, \mu, g(\mu))$$  \hspace{1cm} (1)$$

where $\Lambda$ is a UV cutoff, and $g_0/g(\mu)$ is the unrenormalized/renormalized coupling. $G_{un}$ does not depend on $\mu$, nor does $G_{ren}$ on $\Lambda$, so that

$$\frac{dG_{ren}}{d\mu} = - \sum_i \gamma_i(g(\mu))$$ \hspace{1cm} (2)$$

where $\gamma_i = (\mu d/\mu \log Z_i)$ can only depend on the renormalized coupling, by separation of variables. The solution to this evolution equation (here the renormalization group equation),
is the resummed Green function. In this example single logarithms are resummed. We wish to exhibit a similar paradigm for the double logarithmic, or Sudakov, case. The connection between factorization and Sudakov resummation was already pointed out by Mueller, Collins and Sen. We only provide a brief description here of a streamlined approach to Sudakov resummation for various reactions, that stresses common features.

We consider cross sections near the edge of phase space, where there is not much room for additional gluonic radiation, and that are color singlets at lowest order (true QCD processes can be treated in a similar fashion). Examples are the $e^+ e^-$ total cross section to hadrons near unit thrust, deep-inelastic scattering near unit Bjorken $x$, Drell-Yan production near threshold, etc. Let us call the edge of phase space in such cases the elastic limit. In the integral over virtual and real gluonic degrees of freedom, which takes place in the cross section, the most important momentum regions are, e.g. in Drell-Yan: (i) fast, almost collinear partons in the two incoming jets, (ii) far off-shell, short-distance partons that result from the collision of the two incoming jets, and produce the off-shell vector boson, and (iii) soft gluons that couple the two incoming jets with momenta $p_1$ and $p_2$. Near the elastic limit, the cross section factorizes into corresponding hard, a soft, and two jet functions that summarize these degrees of freedom.

$$\dot{\sigma}(N) = \int_0^\infty dw \ e^{-Nw} \sigma(w) = H(p_1/\mu, p_2/\mu, \zeta_i) \ \hat{S}(Q/\mu N, \zeta_i) \times \tilde{J}_1(p_1 \cdot \zeta_1/\mu, Q/\mu N) \ \tilde{J}_2(p_2 \cdot \zeta_2/\mu, Q/\mu N), \quad (3)$$

Let us discuss the variables that occur in the above equation. $Q$ is the hard scale of the process (the invariant mass of the vector boson produced in Drell-Yan) and $w$ is a dimensionless weight function that is defined to vanish in the elastic limit, is insensitive to collinear splittings of partons, and to additional soft radiation and is additive near the edge of phase space, i.e. $w = w_1 + w_2 + w_3$ where $w_i$ are contributions to the weight function from momenta in the jets, and $w_3$ is the contribution from soft momenta. In DY one can choose $w = 1 - z = 1 - Q^2/s$. Further, $N$ is the moment variable Laplace-conjugate to $w$, and is large near the elastic limit. Arbitrary variables, on which the physical cross section may not depend, are most generally the factorization scale $\mu$ and the vectors $\zeta_i$, which are necessary to define the jet functions $J_i$. The latter can be thought of as gauge-fixing vectors, as one may compute the jet functions in different gauges, as long as the total cross section does not depend on these gauges. It is the arbitrariness in these vectors that allows one to extend the earlier arguments for the single logarithm case to the Sudakov double logarithmic one.

One proceeds by acting on this factorized form (3) with two differential operators with respect to the arbitrary variables just mentioned:

$$\mu \frac{d}{d\mu}, \quad p_i \cdot \zeta \frac{d}{dp_i \cdot \zeta} \quad (i = 1, 2). \quad (4)$$

Using separation of variables, and solving the (double) differential evolution equations obtained with the above operators, one arrives at:

$$\ln \dot{\sigma}(N) = C^{(0)} + (a_s/\pi) \left[ A^{(1)} \ln^2 N + B^{(1)} \ln N + C^{(1)} \right] \quad (5)$$

where $A^{(1)}, B^{(1)}, C^{(1)}$ have to be fitted from a next-to-leading order calculation. As expected, the resummed cross section is an exponential, with at most double logarithms, while higher powers of $\ln N$ in the exponent arise only from the expansion of the running coupling. In the next section we consider a specific and important example, viz. the resummation of Sudakov logarithms in Higgs production at the LHC.
The presently allowed SM Higgs mass ranges from about 70 GeV (from direct searches at LEP) to about 700 GeV from unitarity/triviality constraints. In the MSSM the lower limits on the two scalar Higgs bosons $h$ and $H$, and the pseudoscalar $A$ are about 60 GeV. The theoretical upper limit on the $h$ mass is about 130 GeV. At the LHC the dominant production process is gluon-gluon to Higgs via a top (and bottom, about 10%) quark loop. Two approximations are in order. First, we shall neglect initial states involving quarks (they contribute only 10% at NLO). Second, we would like to consider the gluon-gluon-Higgs coupling $\alpha_tq$ as effectively pointlike. This can be done by taking the infinite top mass limit, supplemented by low-energy theorems. Comparing the NLO infinite top mass limit result with the full analytic massive NLO result one finds a difference less than 10% for the full range for the SM, as well as for the MSSM provided $tg/3$ is not too large. We may then write for the $d$-dimensional partonic cross sections ($d = 4 - 2\epsilon$)

\[ \sigma_0 h, H = \frac{G_F a_r^2 N_C C_F}{2\pi^2} \left( 1 + \epsilon \right) \left( \frac{4\pi}{m_t^2} \right)^{2\epsilon} \]

\[ \sigma_0 A = \frac{G_F a_r^2 N_C C_F}{16\pi^2} \left( 1 + \epsilon \right) \left( \frac{4\pi}{m_t^2} \right)^{2\epsilon} \]

where $a_{s,B}$ is the bare strong coupling constant (with dimension $2\epsilon$) and $g^\phi_t(\phi = h, H, A)$ denote the modified top Yukawa couplings normalized to the SM coupling, which are given in 10. The effective coupling constants $\kappa_\phi$ are given to NNLO in 11,12. The correction factor may be expanded perturbatively

\[ \rho_\phi(z, M_H^2/\mu^2, \alpha(\mu^2), \epsilon) = \sum_{n=0}^{\infty} \alpha^n(\mu^2) \rho^{(n)}_\phi(z, M_H^2/\mu^2, \epsilon) \]

where we define $\alpha(\mu^2) = a_s(\mu^2)/\pi$. The lowest and next order components of $\rho$ are

\[ \rho^{(0)}_\phi(z, M_H^2/\mu^2, \epsilon) = \delta(1 - z) \]

\[ \rho^{(1)}_\phi(z, M_H^2/\mu^2, \epsilon) = \left( \frac{\mu^2}{M_H^2} \right) C_A \left\{ - \frac{z^2}{\epsilon} \left[ 1 + z^4 + (1 - z)^4 \right] + \delta(1 - z) \left[ \frac{11}{6\epsilon} + \frac{203}{36} + \frac{\pi^2}{3} - \frac{11}{6} z^2(1 - z)^2 - \frac{11}{3} z^2(1 - z)^2 \right] \right\} \]

We have implicitly redefined the scale $\mu$ by $\mu^2 \rightarrow \mu^2 \exp\left\{ -(1/4) - \gamma_E \right\}$, and scaled an overall $1/z$ into the parton distribution functions 11.

Let us now construct a resummed expression for $\rho_\phi(z, M_H^2/\mu^2, \alpha(\mu^2), \epsilon)$, or rather for its Mellin transform $\tilde{\rho}_\phi(N, \ldots) = \int_0^\infty x^{N-1} \rho_\phi(x, \ldots)$. We could follow the methods outlined in the first section, but can in fact simplify further. Assuming the Higgs cross section factorizes near the elastic edge of phase space, one would arrive at a resummed cross section of the form (5). By
acting on this equation with \( \frac{d}{dM_H^2} \) one trivially obtains an evolution equation in the Higgs mass of the form:

\[
M_H^2 \frac{d}{dM_H^2} \bar{\rho}_\phi(N, M_H^2/\mu^2, \alpha(\mu^2), \epsilon) = \bar{W}_\phi(N, M_H^2/\mu^2, \alpha(\mu^2), \epsilon) \bar{\rho}_\phi(N, M_H^2/\mu^2, \alpha(\mu^2), \epsilon).
\] (13)

In order to solve eq. (13) we must impose a boundary condition, and find the evolution kernel. We may in fact use the boundary condition

\[
\bar{\rho}_\phi(N, M_H^2/\mu^2, \alpha(\mu^2), \epsilon) = \delta(1 - z).
\]

The solution to eq. (13) is then

\[
\bar{\rho}_\phi(N, M_H^2/\mu^2, \alpha(\mu^2), \epsilon) = \exp \left[ \int_0^{M_H^2/\mu^2} \frac{ds}{s} \bar{W}_\phi \left( N, \frac{\mu^2}{\mu^2}, \alpha(\mu^2), \epsilon \right) \right],
\] (14)

which may be expanded, using renormalization group invariance, as

\[
\bar{\rho}_\phi(N, M_H^2/\mu^2, \alpha(\mu^2), \epsilon) = \exp \left[ \int_0^{M_H^2/\mu^2} d\lambda \left\{ \alpha(\lambda, \alpha(\mu^2), \epsilon) W^1(1, 1/\lambda, \epsilon) + \ldots \right\} \right]
\]
with \( \nu(z) \) an arbitrary function. The one loop coefficient of the evolution kernel \( W_\phi \) can then be derived from a low order calculation of the correction factors \( \rho_\phi \) via

\[
\bar{W}_\phi(1)(N, 1, \epsilon) = (M_H^2/\mu^2)^\gamma M_H^2 \frac{\partial}{\partial M_H^2} \bar{\rho}_\phi(1)(N, M_H^2/\mu^2, \epsilon),
\] (16)

The general structure of the result is, in \( x \)-space

\[
W_\phi^{(1)}(z, 1, \epsilon) = \delta(1 - z) f_\phi^{(1)}(\epsilon) + \alpha \left( \frac{g^{(1)}(z, \epsilon)}{(1 - z)^{1/2}} \right) + h^{(1)}(z, \epsilon),
\] (17)

where the coefficient functions \( f_\phi^{(1)} \), \( g^{(1)} \), \( h^{(1)} \) are regular functions of their arguments at \( z = 1 \). We drop the term \( h^{(1)} \) as it is of order \( 1/N^4 \) in moment space. After rescaling to incorporate the factor \((1 - z)^{-2}\) and combining the plus distribution with the Mellin transform in eq. (15), we see that the relevant function to approximate is \((z^{N-1} - 1)g^{(1)}(z, \epsilon)\). We approximate the residue function \( g^{(1)}(z, \epsilon) \) in three schemes, which are defined by

scheme \( \alpha \) : \( \frac{1}{C_A} (z^{N-1} - 1)g^{(1)}(z, \epsilon) \rightarrow (z^{N-1} - 1) 2 \)
scheme \( \beta \) : \( \frac{1}{C_A} (z^{N-1} - 1)g^{(1)}(z, \epsilon) \rightarrow (z^{N-1} - 1) 2 - (1 - z)(2z^2 - 4z - 2z^3) \)
scheme \( \gamma \) : \( \frac{1}{C_A} (z^{N-1} - 1)g^{(1)}(z, \epsilon) \rightarrow (z^{N-1} - 1) 2 - (1 - z)(2z^2 - 4z - 2z^3) - 4z^{N-1}(1 - z) \). (18)

The minimal scheme \( \alpha \) involves replacing \( g^{(1)}(z, \epsilon) \) simply by \( g^{(1)}(1, \epsilon) \), whereas scheme \( \gamma \) includes in addition some \( O(\ln N/N) \) terms. Using the one-loop evolution kernel \( W_\phi^{(1)} \) we can now construct the resummed expressions for the Higgs production correction factor in the three schemes. Although these expressions are still divergent for \( \epsilon \rightarrow 0 \), the divergences may be cancelled by mass factorization and renormalization for which we chose the \( \overline{\text{MS}} \) scheme.

The final results for the resummed cross sections in moment space are given in 11, both for Higgs production and the Drell-Yan process. These two processes are very similar from the soft gluon point of view, the main difference being that Higgs production is driven by gluon
fusion, and Drell-Yan by quark-antiquark annihilation. Rather than evaluate the resummed answers numerically - which involves the difficult problem of treating the infrared renormalon - we expanded the resummed answers \(^{11}\) for Higgs production and Drell-Yan\(^{13}\) to NNLO in QCD perturbation theory, in the above three schemes. The answers are expressed in terms of plus-distributions \(D_\ell(x)\) and logarithms \(E_\ell(x)\) (which are integrable but large near the edge of phase space)

\[
D_\ell(x) = \ln\left(\frac{1-x}{1-x}\right), \quad E_\ell(x) = \ln\left(1-x\right),
\]

and constants. Scheme \(\gamma\) incorporates the \(E\) logarithms, which have not been included in resummed cross sections before. At any order, the leading \(E\) logarithms and those subleading ones that are related to the QCD running coupling are universal, as they arise directly from the splitting function. They occur in exact NNLO calculations for DY and DIS\(^{14}\), and we checked analytically that the scheme \(\gamma\) resummed answer for these processes, expanded to NNLO, reproduces them. In Fig. 1 we present the correction factors for SM Higgs production at the LHC, which coincide with the correction factors of MSSM scalar Higgs boson production for small \(\tan\beta\).

For MSSM pseudoscalar Higgs production we show similar results in\(^{11}\). In Fig. 1a the "partonic" K-factors, obtained from folding the correction factors \(\rho_k\) with NLO parton densities and using a NLO strong coupling for all orders of the cross sections, are presented. For comparison we show in Fig. 1b the corresponding NLO "hadronic" K-factors normalized to the LO cross sections evolved with LO parton densities and \(\alpha_s\). Whereas the former indicate the rate of convergence of the individual-order contributions within a fixed order calculation, the latter exhibit the convergence of the perturbative approach to the physical (hadronic) quantities. We observe from Fig. 1 that at NLO scheme \(\gamma\), remarkably, reproduces the exact NLO calculation almost exactly for the full range of the SM Higgs mass \(M_H \geq 70\) GeV (similar results are obtained for the neutral Higgses in the MSSM) schemes \(\alpha\) and \(\beta\) agree with the exact result only for \(M \geq 1\) TeV (the agreement of scheme \(\alpha\) in the intermediate Higgs mass range is accidental). Moreover, note that the NNLO corrections to the partonic cross sections in scheme \(\gamma\) are still very significant.

We checked\(^{11}\) in Drell-Yan that similarly good agreement is obtained in scheme \(\gamma\) in NLO and NNLO for the same kinematics\(^{14}\). Keep in mind that full NNLO predictions for hadronic cross

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**Figure 1**: a) Exact (solid line) and approximate two- (\(\alpha_\beta,\gamma\)) and three-loop (\(\alpha_\beta,\beta_\gamma,\gamma\)) partonic K-factors, convoluted with the NLO gluon-gluon luminosity \(d\sigma_{\text{LO}}^H/dt\), where \(t = M_H^2x\), in the heavy top-mass limit and in three different schemes, versus the scalar Higgs mass \(M_H\). We used NLO CTEQ4M parton densities\(^{15}\) and \(\alpha_s\), \((\alpha_s)^{(3)} = 0.202\) MeV) for the LO cross section and including the NLO contributions from \(\kappa_H\).

b) Hadronic NLO K-factor using LO CTEQ4L parton densities\(^{15}\) and \(\alpha_s\), \((\alpha_s)^{(3)} = 0.181\) MeV) for the LO cross section and including the NLO contributions from \(\kappa_H\).
sections require NNLO parton densities, which are not yet available. Should the size of the NNLO corrections to the physical cross section warrant concern about the convergence of the perturbative approach, our resummed answer provides a tool to control such large corrections.

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5. E. Gross, these proceedings.