Enhanced $K_L \to \pi^0 \nu \bar{\nu}$ from Direct CP Violation in $B \to K \pi$ with Four Generations

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Recent CP violation results in $B$ decays suggest that $Z$ penguins may have large weak phase. This can be realized by the four generations (standard) model. Concurrently, $B \to X_s t^+ \ell^- \bar{\nu}$ and $B_s$ mixing allow for sizable $V^*_{ts} V_{tb}$ only if it is nearly imaginary. Such large effects in $b \to s$ transitions would affect $s \leftrightarrow d$ transitions, as kaon constraints would demand $V_{td} \neq 0$. Using $\Gamma(Z \to bb)$ to bound $|V^*_{td}|$, we infer sizable $|V^*_{ts}| \lesssim |V^*_{tb}| \lesssim |V_{us}|$. Imposing $\varepsilon \ K^+ \to \pi^+ \nu \bar{\nu}$ and $\varepsilon'/\varepsilon$ constraints, we find $V^*_{ts} V_{tb} \sim \times 10^{-4}$ with large phase, enhancing $K_L \to \pi^0 \nu \bar{\nu}$ to $5 \times 10^{-10}$ or even higher. Interestingly, $\Delta m_{B_d}$ and $\sin 2\Phi_{B_d}$ are not much affected, as $|V^*_{td} V^*_{tb}| \ll |V^*_{td} V_{tb}| \sim 0.01$.

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Just 3 years after CP violation (CPV) in the $B$ system was established, direct CP violation (DCPV) was also observed in $B^0 \to K^+ \pi^-$ decay, $A_{K^+ \pi^-} \sim -0.12$. A puzzle emerged, however, that the charged $\_b^+ \to K^+ \pi^0$ mode gave no indication of DCPV, and is in fact a very large, $A_{K^+ \pi^0} \gg 0$. Currently, $A_{K^+ \pi^0} - A_{K^+ \pi^-} \sim 0.16$, and differs from zero with 3.8σ significance.

The amplitude $M_{K^+ \pi^-} \simeq P + T$ is dominated by the strong penguin ($P$) and tree ($T$) contributions, while the main difference $\sqrt{2} M_{K^+ \pi^0} - M_{K^+ \pi^-} \simeq M_{EW} + C$ is from electroweak penguin (EWP, or $P_{EW}$) and color-suppressed tree (C) contributions which are subdominant. Thus, $A_{K^+ \pi^0} \sim A_{K^+ \pi^-}$ was anticipated by all models. As data indicated otherwise, it has been stressed that the $C$ term could be much larger than previously thought, effectively cancelling against the CPV phase in $T$, leading to $A_{K^+ \pi^0} \sim 0$. While this may well be realized, a very large $C$ (especially if $A_{K^+ \pi^0} > 0$) would be a surprise in itself.

In a previous paper, we explored the possibility of New Physics (NP) effects in $P_{EW}$, in particular in the 4 generation standard model (SM4, with SM3 for 3 generations). A sequential $t'$ quark could affect $P_{EW}$ most naturally for two reasons. On one hand, the associated Cabibbo-Kobayashi-Maskawa (CKM) matrix element product $V^*_{ts} V_{tb}$ could be large and imaginary; on the other hand, it is well known that $P_{EW}$ is sensitive to $m_t^2$ in amplitude, and heavy $t'$ does not decouple.

Using the PQCD factorization approach at leading order, which successfully predicted $A_{K^+ \pi^-} < -0.1$ (and $C$ not inordinately large), we showed that $A_{K^+ \pi^0} \gtrsim 0$ for sizable $m_t \gtrsim 300$ GeV and large, nearly imaginary $V^*_{ts} V_{tb}$. As the $m_t'$ dependence is similar, we also showed that data on $B \to X_s t^+ \ell^- \bar{\nu}$ and $B_s$ mixing concurred, in the sense that large $t'$ effect is allowed only if $V^*_{ts} V_{tb}$ is nearly imaginary. Applying the latter two constraints, however, $m_t'$ and $V^*_{ts} V_{tb}$ become highly constrained. In the following, we will take

$$m_t' \simeq 300 \text{ GeV}, \ V^*_{ts} V_{tb} \equiv r_{sb} e^{i\phi_{sb}} \sim 0.025 e^{i 70^0}, \quad (1)$$

as exemplary values for realizing $A_{K^+ \pi^0} - A_{K^+ \pi^-} \gtrsim 0.10$, without recourse to a large $C$ contribution.

Comparing with $|V^*_{us} V_{ub}| \simeq 0.04$, $r_{sb} \sim 0.025$ is quite sizable. In our $b \to s$ study, we had assumed $V^*_{td} \to 0$ out of convenience, so as to decouple from $b \to d$ and $s \to d$ constraints. The main purpose of this note, however, is to show that, in view of the large $r_{sb}$ and $\phi_{sb}$ values given in Eq. (1), $V_{td} = 0$ is untenable, and one must explore $s \to d$ and $b \to d$ implications. The reasoning is as follows. Since a rather large impact on $V^*_{ts} V_{tb}$ is implied by Eq. (1), if one sets $V^*_{td} = 0$, then $V^*_{ts} V_{tb}$ would still be rather different from SM3 case. With our current knowledge of $m_t$, the $\varepsilon_K$ parameter would deviate from the well measured experimental value. Thus, a finite $V^*_{td}$ is needed to tune for $\varepsilon_K$.

We find that the kaon constraints that are sensitive to $t'$ (i.e., $P_{EW}$-like), viz. $K^+ \to \pi^0 \nu \bar{\nu}, K_L \to \mu^+ \mu^-$, $\varepsilon_K$, and $\varepsilon'/\varepsilon$ can all be satisfied. Interestingly, once kaon constraints are satisfied, we find little impact is implied for $b \to d$ transitions, such as $\Delta m_{B_d}$ and $\sin 2\Phi_{B_d}$. That is, $V_{td} = 0$ works approximately for $b \to d$ transitions, for current level of experimental sensitivity. The main outcome for $s \to d$ and $b \to d$ transitions is the enhancement of $K_L \to \pi^0 \nu \bar{\nu}$ mode by an order of magnitude or more, to beyond $5 \times 10^{-10}$.

With four generations, adding $V^*_{ts} V_{tb}$ extends the familiar unitarity triangle relation into a quadrangle,

$$V^*_{us} V_{ub} + V^*_{cs} V_{cb} + V^*_{ts} V_{tb} + V^*_{ts} V_{tb} = 0. \quad (2)$$

FIG. 1: Unitarity quadrangles of (a) Eq. (4), with $|V^*_{us} V_{ub}|$ exaggerated; (b) Eq. (17), where actual scale is $\sim 1/4$ of (a). Adding $V^*_{ts} V_{tb}$ (dashed) according to Eq. (4) drastically changes the invariant phase and $V^*_{ts} V_{tb}$ from the SM3 triangle (solid), but from Eq. (17), the dashed lines for $V^*_{ts} V_{tb}$ and $V^*_{td} V_{tb}$ can hardly be distinguished from SM3 case.

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Using SM3 values for $V_{ub}^* V_{cb}$, $V_{ub}^* V_{cb}$ (validated later by our $b \to d$ study), since they are probed in multiple ways already, and taking $V_{ub}^* V_{cb}$ as given in Eq. 11, we depict Eq. 2 in Fig. 1(a). The solid, rather squashed triangle is the usual $V_{ub}^* V_{cb} + V_{ub}^* V_{cb} + V_{ub}^* V_{cb} = 0$ in SM3. Given the size and phase of $V_{ub}^* V_{cb}$, one sees that the invariant phase represented by the area of the quadrangle is rather large, and $V_{ub}^* V_{cb}$ picks up a large imaginary part, which is very different from SM3 case. Such large effect in $b \to s$ would likely spill over into $s \to d$ transitions, since taking $V_{ub}$ as real and of order 1, one immediately finds the strength and complexity of $V_{ud}^* V_{us}$ would be rather different from SM3, and one would need $V_{ud}^* V_{us} \neq 0$ to compensate for the well measured value for $\varepsilon_K$.

Note from Fig. 1(a) that the usual approximation of dropping $V_{us}^* V_{ub}$ in the loop remains a good one. To face $s \to d$ and $b \to d$ transitions, however, one should respect unitarity of the $4 \times 4$ CKM matrix $V_{CKM}$. We adopt the parametrization in Ref. 2 where the third column and fourth row is kept simple. This is suitable for $B$ physics, as well as for loop effects in kaon sector. With $V_{cb}$, $V_{tb}$ and $V_{t'b}$ defined as real, one keeps the SM3 phase convention for $V_{ub}$, now defined as

$$\arg V_{ub}^* = \phi_{ub}, \quad (3)$$

which is usually called $\phi_3$ or $\gamma$ in SM3. We take $\phi_{ub} = 60^\circ$ as our nominal value 5. This can in principle be measured through tree level processes such as the $B \to DK$ Dalitz method 7. The two additional phases are associated with $V_{ts}$ and $V_{td}$, and for the rotation angles we follow the PDG notation 8. To wit, we have

$$V_{td} = -c_{24} c_{34} s_{14} e^{-i\phi_{ub}}, \quad (4)$$
$$V_{ts} = -c_{34} s_{23} e^{-i\phi_{ub}}, \quad (5)$$
$$V_{tb} = -s_{34}, \quad (6)$$

while $V_{t'b'} = c_{14} c_{24} c_{34}$, $V_{tb} = c_{13} c_{23} c_{34}$, $V_{cb} = c_{13} c_{23} s_{23}$ are all real. With this convention for rotation angles, from Eq. 3 we have $V_{ub} = c_{34} s_{13} e^{-i\phi_{ub}}$.

Analogous to Eq. 11, we also make the heuristic but redundant definition of

$$V_{td}^* V_{t'd} \equiv r_{db} e^{i\phi_{db}}, \quad V_{ts}^* V_{t's} \equiv r_{ds} e^{i\phi_{ds}}, \quad (7)$$

as these combinations enter $b \to d$ and $s \to d$ transitions. Inspection of Eqs. 11, 12 gives the relations

$$r_{db} s_{34} = r_{ds} s_{34}, \quad \phi_{ds} = \phi_{db} - \phi_{sb}. \quad (8)$$

As we shall see, $s \to d$ transitions are much more stringent than $b \to d$ transitions, hence we shall turn to constraining $r_{ds}$ and $\phi_{ds}$.

Before turning to the kaon sector, we need to infer what value to use for $s_{34} = |V_{ts}|$, as this can still affect the relevant physics through unitarity. Fortunately, we have some constraint on $s_{34}$ from $Z \to b \bar{b}$ width, which receives special $t$ (and hence $t'$) contribution compared to other $Z \to q\bar{q}$, and is now suitably well measured.

Following Ref. 9 and using $m_{t'} = 300$ GeV, we find

$$|V_{tb}|^2 + 3.4|V_{tb}|^2 < 1.14. \quad (9)$$

Since all $c_{ij}$'s except perhaps $c_{34}$ would still likely be close to 1, we infer that $s_{34} < 0.25$. We take the liberty to nearly saturate this bound ($\Gamma(Z \to b\bar{b})$ is close to $1\sigma$ above SM3 expectation), by imposing

$$s_{34} \simeq 0.22, \quad (10)$$

to be close to the Cabibbo angle, $\lambda \equiv |V_{us}| \simeq 0.22$. Note that Eq. 10 is somewhat below the expectation of “maximal mixing” of $s_{34}^2 \sim 1/2$ between third and fourth generations. Combining it with Eq. 11, one gets $|V_{ts}| \sim 0.11 \sim \lambda^2$. Its strength would grow if a lower value of $s_{34} \lesssim \lambda$ is chosen, which would make even greater impact on $s \to d$ transitions.

Using current values 8 of $V_{ub}$ and $V_{cb}$ as input and respecting full unitarity, we now turn to the kaon constraints of $K^+ \to \pi^+ \nu \bar{\nu}$, $\varepsilon_K$, $K_L \to \mu^+ \mu^-$, and $\epsilon'/\epsilon$. The first two are short-distance (SD) dominated, while the last two suffer from long-distance (LD) effects.

Let us start with $K^+ \to \pi^+ \nu \bar{\nu}$. The first observed event 10 by E787 suggested a sizable rate hence hinted at NP. The fourth generation would be a good candidate, since the process is dominated by the $Z$ penguin. Continued running, including E949 data (unfortunately not greatly improving accumulated luminosity), has yielded overall 3 events, and the rate is now $B(K^+ \to \pi^+ \nu \bar{\nu}) = (1.47^{+0.23}_{-0.19}) \times 10^{-10}$. This is still somewhat higher than the SM3 expectation of order $0.8 \times 10^{-10}$.

Defining $\lambda_q \equiv V_{qu} V_{qs}^*$ and using the formula 12

$$B(K^+ \to \pi^+ \nu \bar{\nu}) = \kappa_+ \left( \frac{\lambda_q}{|V_{us}|} \right)^2 \left( \frac{\lambda_q}{|V_{us}|} \right)^2 \frac{\eta_0(x_{t})}{x_{t}^2} \left( \frac{\lambda_q}{|V_{us}|} \right)^2 \left( \frac{\lambda_q}{|V_{us}|} \right)^2 \left( \frac{\lambda_q}{|V_{us}|} \right)^2$$

we plot in Fig. 2 the allowed range (valley shaped shaded region) of $r_{ds} - \phi_{ds}$ for the 90% confidence level (C.L.) bound of $B(K^+ \to \pi^+ \nu \bar{\nu}) < 3.6 \times 10^{-10}$. We have used $\lambda_q \equiv (4.84 \pm 0.06) \times 10^{-11} \times (0.224/|V_{us}|)^8$ and $\lambda_q \equiv (0.39 \pm 0.07) \times (0.224/|V_{us}|)^4$. We take the QCD correction factors $\eta_0(x_{t}) \sim 1$, and $X_{t}(x_{t})$ evaluated for $m_t = 166$ GeV and $m_{t'} = 300$ GeV. We see that $r_{ds}$ up to $7 \times 10^{-4}$ is possible, which is not smaller than the SM3 value of $4 \times 10^{-4}$ for $|V_{td}^2|$. The SD contribution to $K_L \to \mu^+ \mu^-$ is also of interest. The $K_L \to \mu^+ \mu^-$ rate is saturated by the absorptive $K_L \to \gamma \gamma \to \mu^+ \mu^-$, while the off-shell photon contribution makes the SD contribution hard to constrain. To be conservative, we use the experimental bound of $B(K_L \to \mu^+ \mu^-)_{SD} < 3.7 \times 10^{-9}$. It is then in general less stringent than $K^+ \to \pi^+ \nu \bar{\nu}$, although the generic constraint on $r_{ds}$ drops slightly. We do not plot this constraint in Fig. 2.

The rather precisely measured CPV parameter $\varepsilon_K = (2.284 \pm 0.014) \times 10^{-3}$ is predominantly SD. It maps
out rather thin slices of allowed regions on the $r_{ds}$-$\phi_{ds}$ plane, as illustrated by dots in Fig. 2, where we use the formula of Ref. [14] and follow the treatment. Note that $r_{ds}$ up to $7 \times 10^{-4}$ is still possible, for several range of values for $\phi_{ds}$. This is the aforementioned effect that extra CPV effects due to large $\phi_{d}$ and $r_{d}$ now have to be tuned by $t'$ effect to reach the correct $\varepsilon_K$ value. We have checked that $\Delta m_K$ makes no additional new constraint.

The DCPV parameter, Re $(\varepsilon'/\varepsilon)$, was first measured in 1999 [14], with current value at $(1.67 \pm 0.26) \times 10^{-3}$ [8]. It depends on a myriad of hadronic parameters, such as $m_s$, $\Omega_{1B}$ (isospin breaking), and especially the non-perturbative parameters $R_6$ and $R_8$, which are related to the hadronic matrix elements of the dominant strong and electroweak penguin operators. With associated large uncertainties, we expect $\varepsilon'/\varepsilon$ to be rather accommodating, but for specific values of $R_6$ and $R_8$, some range for $r_{ds}$ and $\phi_{ds}$ is determined.

We use the formula

$$\text{Re } \frac{\varepsilon'}{\varepsilon} = \text{Im} (\lambda_{ct}) P_0 + \text{Im} (\lambda_{cs}^d) F(x_t) + \text{Im} (\lambda_{tt}^d) F(x_{t'}),$$

where $F(x)$ is given by

$$F(x) = P_X X_0(x) + P_Y Y_0(x) + P_E E_0(x).$$

The SD functions $X_0$, $Y_0$, $Z_0$, and $E_0$ can be found, for example, in Ref. [15], and the coefficients $P_i$ are given in terms of $R_6$ and $R_8$ as

$$P_i = r_i^{(0)} + r_i^{(6)} R_6 + r_i^{(8)} R_8,$$

which depends on LD physics. We differ from Ref. [15] by placing $P_0$, multiplied by Im $(\lambda_{ct})$, explicitly in Eq. (12).

In SM4, one no longer has the relation Im $\lambda_{ct}^d = -\text{Im} \lambda_{ct}^s$ that makes Re $(\varepsilon'/\varepsilon)$ proportional to Im $(\lambda_{ct}^d)$. We take the $r_i^{(j)}$ values from Ref. [15] for $\Lambda_{MS}^4 = 310$ MeV, but reverse the sign of $r_i^{(j)}$ for above mentioned reason. Note that Re $(\varepsilon'/\varepsilon)$ depends linearly on $R_6$ and $R_8$. For fixed SD parameters $m_{t'}$ and $\lambda_{ct}^d = V_{td}^V V_{ts}^V$, one may adjust for solutions to $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ and $\varepsilon_K$.

For the “standard” parameter range of $R_6 = 1.23 \pm 0.16$ and $R_8 = 1.0 \pm 0.2$, we find $R_6 \sim 1.2$ and $R_8 \sim 1.0–1.2$ allows for solutions at $r_{ds} \sim (5–6) \times 10^{-4}$ with $\phi_{ds} \sim +(35^\circ–50^\circ)$, as illustrated by the elliptic rings on upper left part of Fig. 2. For $R_6 = 2.2 \pm 0.4$ found in 1/N_C expansion at next-to-leading order (and chiral perturbation theory at leading order), within SM3 one has trouble giving the correct Re $(\varepsilon'/\varepsilon)$ value. However, for SM4, solutions exist for $R_6 \sim 2.2$ and $R_8 = 0.8–1.1$, for $r_{ds} \sim (3.5–5) \times 10^{-4}$ and $\phi_{ds} \sim -45^\circ–60^\circ$, as illustrated by the elliptic rings on upper right part of Fig. 2. We will take

$$r_{ds} \sim 5 \times 10^{-4}, \quad \phi_{ds} \sim -60^\circ \text{ or } +35^\circ,$$

as our two nominal cases that satisfy all kaon constraints. The corresponding values for $R_6$ and $R_8$ can be roughly read off from Fig. 2. We stress again that these values should be taken as exemplary.

To illustrate in a different way, we plot $\varepsilon_K$, $B(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ and Re $(\varepsilon'/\varepsilon)$ vs $\phi_{ds}$ in Figs. 3(a), (b) and (c), respectively, for $r_{ds} = 4$ and $6 \times 10^{-4}$. The current 1\sigma experimental range is also illustrated. In Fig. 3(c), we have illustrated with $R_6 = 1.1$, $R_8 = 1.2$ [14] and $R_6 = 2.2$, $R_8 = 1.1$ [10]. For the former (latter) case, the variation is enhanced as $R_6$ (R_8) drops.

It is interesting to see what are the implications for the CPV decay $K_L \rightarrow \pi^0 \nu \bar{\nu}$. The formula for $B(K_L \rightarrow \pi^0 \nu \bar{\nu})$ is analogous to Eq. (4), except the change of $\kappa_L$ to $\kappa_L = (2.12 \pm 0.03) \times 10^{-10} \times (|V_{us}|/0.224)^8$, and taking only the imaginary part for the various CKM products. Since $\phi_{ds} \sim -60^\circ$ or $+35^\circ$ have large imaginary part, while $r_{ds} \approx |V_{td}^V V_{ts}^V| \sim 5 \times 10^{-4}$ is stronger than the SM3

FIG. 2: Allowed region from $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ (valley shaped shaded region), $\varepsilon_K$ (simulated dots) and $\varepsilon'/\varepsilon$ (elliptic rings) in $r_{ds}$ and $\phi_{ds}$ plane, as described in text, where $V_{td}^V V_{ts}^V \equiv r_{ds} e^{i \phi_{ds}}$. For $\varepsilon'/\varepsilon$, the rings on upper right correspond to $R_6 = 2.2$, and $R_8 = 0.8, 1.1$ (bottom to top), and on upper left, $R_6 = 1.0, 1.2$ (bottom to top), $R_8 = 1.2$.

FIG. 3: (a) $\varepsilon_K$, (b) $B(K^+ \rightarrow \pi^+ \nu \bar{\nu})$, (c) Re $(\varepsilon'/\varepsilon)$ and (d) $B(K_L \rightarrow \pi^0 \nu \bar{\nu})$ vs $\phi_{ds}$, for $r_{ds} = 4$ and $6 \times 10^{-4}$ and $m_{t'} = 300$ GeV. Larger $r_{ds}$ gives stronger variation, and horizontal bands are current (1\sigma) experimental range [8] (the bound for (d) is outside the plot). For (c), solid (dashed) lines are for $R_6 = 2.2$, $R_8 = 1.1$ ($R_6 = 1.1$, $R_8 = 1.2$).
the expectation of $\text{Im} V_{ub}^* V_{tb} \sim 10^{-4}$, we expect the CP decay rate of $K_L \to \pi^0 \nu \bar{\nu}$ to be much enhanced.

We plot $B(K_L \to \pi^0 \nu \bar{\nu})$ vs $\phi_{ds}$ in Fig. 3(d), for $r_{ds} = 4$ and $6 \times 10^{-4}$. Reading off from the figure, we see that the $K_L \to \pi^0 \nu \bar{\nu}$ rate can reach above $10^{-9}$, almost two orders of magnitude above SM3 expectation of $0.3 \times 10^{-10}$. It is likely above $5 \times 10^{-10}$, and in general larger than $K^+ \to \pi^+ \nu \bar{\nu}$. Specifically, for our nominal value of $r_{ds} \sim 5 \times 10^{-3}$ and $\phi_{ds} \sim +35^\circ$, $B(K_L \to \pi^0 \nu \bar{\nu})$ and $B(K^+ \to \pi^+ \nu \bar{\nu})$ are 6.5 and $2 \times 10^{-10}$, respectively, while for the $\phi_{ds} \sim -60^\circ$ case, they are 12 and $3 \times 10^{-10}$, respectively. The latter is closer to the Grossman-Nir bound $^{17}$, i.e. $B(K_L \to \pi^0 \nu \bar{\nu})/B(K^+ \to \pi^+ \nu \bar{\nu}) \sim \tau_{K_L}/\tau_{K^+} \sim 4.2$, because $V_{td} V_{ts}^*$ is more imaginary. Thus, both $K^+ \to \pi^+ \nu \bar{\nu}$ and $K_L \to \pi^0 \nu \bar{\nu}$ should be very interesting at the next round of experiments. We note that the ongoing E391A experiment could $^{13}$ attain single event sensitivity with the Grossman-Nir bound based on the current $B(K^+ \to \pi^+ \nu \bar{\nu})$ measurement. However, for $r_{ds} \sim 5 \times 10^{-3}$ and $\phi_{ds} \sim -45^\circ$, which is still a solution for $R_6 \sim 2.2$, one has $B(K_L \to \pi^0 \nu \bar{\nu}) \sim 4 \times 10^{-10}$ with $B(K^+ \to \pi^+ \nu \bar{\nu})$ at lower end of current range.

With $\phi_{ab} \sim 70^\circ$ and $\phi_{ds} \sim -60^\circ$ (and $+35^\circ$) both sizable while the associated CKM product is larger than the corresponding SM3 top contribution, there is large impact on $b \to s$ and $s \to d$ transitions from $Z$ penguin and box diagrams. It is therefore imperative to check that one does not run into difficulty with $b \to d$ transitions. Remarkably, we find that the impact on $b \to d$ is mild. From Eqs. (1), (3), (10) and (13), we infer

$$r_{db} \sim 1 \times 10^{-3}, \quad \phi_{db} \sim 10^\circ (105^\circ).$$

Since $r_{db}$ is much smaller than $|V_{td}^* V_{tb}| \sim \lambda^3 \sim 0.01$ in SM3, the impact on $b \to d$ is expected to be milder, i.e. we are not far from the $V_{td} \to 0$ limit. We stress that this is nontrivial since there is a large effect in $b \to s$; it is a consequence of imposing $s \to d$ and $Z \to b \bar{b}$ constraints. We illustrate in Fig. 1(b) the unitarity quadrangle

$$V_{td}^* V_{tb} + V_{td} V_{tb}^* + V_{td}^* V_{tb} + V_{td} V_{tb} = 0.$$  (17)

In contrast to Fig. 1(a), $(V_{td}^* V_{tb} + V_{td} V_{tb}^*)_{SM3}$ and $(V_{td} V_{tb}^*)_{SM3}$ can hardly be distinguished.

The $B_{d}^0 - \bar{B}_{d}^0$ mass difference and CP violation phase in mixing are respectively given by $\Delta m_{B_d} \equiv 2 |M_{12}|$ and sin $2 \Phi_{B_d} = \text{Im} (M_{12}/|M_{12}|)$, where

$$M_{12} = \lambda_{bd}^2 \eta_t S(x_t) + (\lambda_{db}^2 \eta_t^* S(x_{t^*})) + 2 \lambda_{bd} \lambda_{db}^* \eta_t \eta_t^* S(x_t, x_{t^*}),$$  (18)

with $\lambda_{bd} = \frac{G_F^2}{12\pi} m_b^2 n_{B_d} B_{B_d} f_{B_d}$. The functions $S(x)$ and $S(x, y)$ can be found in $^{10}$. We take $\eta_t = 0.55$, $\eta_{t^*} = 0.58$ and $\eta_t^* = 0.50$, and plot in Fig. 4(a) $\Delta m_{B_d}$ vs $\phi_{db}$ for $r_{db} = 8$ and $12 \times 10^{-4}$ (corresponding to $r_{ds} = 4$ and $6 \times 10^{-4}$). We have taken the experimental value of $\Delta m_{B_d} = (0.505 \pm 0.005) \text{ ps}^{-1}$ from PDG 2005 $^8$, and illustrated with the lower range of $f_{B_d} \sqrt{B_{B_d}} = (246 \pm 38) \text{ MeV} ^{21}$. We have scaled up the error for the latter by 1.4, since it comes from the new result on $f_{B_d}$ with unquenched lattice QCD $^{21}$, but $B_{B_d}$ is not yet updated. We see from Fig. 4(a) that $\Delta m_{B_d}$ does not rule out the parameter space around Eq. (16) (equivalent to Eq. (15)). The overall dependence on $r_{db}$ and $\phi_{db}$ is mild, and error on $f_{B_d} \sqrt{B_{B_d}}$ dominates. Seemingly, a lower value of $f_{B_d} \sqrt{B_{B_d}} \sim 215 \text{ MeV}$ is preferred. SM3 would give $\Delta m_{B_d} = 0.44 - 0.62 \text{ ps}^{-1}$ for $f_{B_d} \sqrt{B_{B_d}} = 208 \text{ MeV} - 246 \text{ MeV}$, so the problem is not with SM4.

We plot sin $2 \Phi_{B_d}$ vs $\phi_{db}$ in Fig. 4(b), for $r_{db} = 8$ and $12 \times 10^{-4}$. One can see that sin $2 \Phi_{B_d}$, which is not sensitive to hadronic parameters such as $f_{B_d} \sqrt{B_{B_d}}$, is well within experimental range of “sin $2\phi_1 = 0.73 \pm 0.04$ from PDG 2005 $^8$ for the $\phi_{db} \sim 10^\circ$ case. However, for $\phi_{db} \sim 105^\circ$ case, which is much more imaginary, sin $2 \Phi_{B_d}$ is on the high side $^{22}$, and it seems that CP in $B$ physics prefers $R_6 \sim 2.2$ over $R_6 \sim 1$. As another check, we find the semileptonic asymmetry $A_{SL} = -0.7 \times 10^{-3}$ ($-0.2 \times 10^{-3}$) for $\phi_{db} \sim 10^\circ (105^\circ)$, which is also well within range of $A_{SL} = -1.1 \pm 7.9 (7.0) \times 10^{-3}$ $^{23}$. With Eqs. (1), (10) and (13), together with standard (SM3) values for $V_{cb}$ and $V_{tb}$, we can get a glimpse of the typical 4 $\times$ 4 CKM matrix, which appears like

$$\begin{pmatrix}
0.9745 & 0.2225 & 0.0038 e^{-1.60^\circ} & 0.0281 e^{1.61^\circ} \\
-0.2241 & 0.9667 & 0.0415 & 0.1164 e^{-1.66^\circ} \\
0.0073 e^{-1.25^\circ} & -0.0555 e^{-1.25^\circ} & 0.9746 & 0.2168 e^{-1.47^\circ} \\
-0.0044 e^{-1.10^\circ} & -0.1136 e^{-1.70^\circ} & -0.2200 & 0.9688
\end{pmatrix}$$  (19)

for $\phi_{db} \sim 10^\circ$ case ($V_{cd}$ and $V_{cs}$ pick up tiny imaginary parts, which are too small to show in angles). For the $\phi_{db} \sim 105^\circ$ case, the appearance is almost the same, except $V_{cd} \simeq 0.0082 e^{-1.17^\circ}$ and $V_{ub} \simeq 0.029 e^{-1.74^\circ}$. Note the “double Cabibbo” nature, i.e. the 12 and 34 diagonal 2 $\times$ 2 submatrices appear almost the same. This is a consequence of our choice of Eq. (10). To keep Eq. (1) intact, however, weakening $s_{34}$ would result in even larger $V_{t's}$, but it would still be close to imaginary. Since $V_{t'0}^*(d) V_{t'0}^*(s)$ are tiny compared to $V_{t'd}^* V_{t's} \simeq -V_{t'd} V_{t's}$, the unitarity quadrangle for $s \to d$ cannot be plotted as in Fig. 1.
However, note that $V_{cb}^*V_{ts}$ is almost real, and CPV in $s \to d$ comes mostly from $t'$. The entries for $V_{ib'}$, $i = u, c, t$ are all sizable. $|V_{ub'}| \sim 0.03$ satisfies the unitarity constraint $|V_{ub'}| < 0.08$ from the first row, but it is almost as large as $V_{cb}$. However, the long standing puzzle of unitarity of the first row could be taken as a hint for finite $|V_{ub'}| \sim 0.03$. The element $V_{tb'} \simeq -V_{tb}^*$ is even larger than $V_{cb}$ and close to imaginary. Together with finite $V_{ub'}$, $V_{cb}V_{tb'} \simeq 0.0033 e^{-i\theta^D} \pm 0.0034 e^{i\phi^D}$ is not negligible, and one may worry about $D^0-D^0$ mixing. Fortunately the $D$ decay rate is fully Cabibbo allowed. Using $f_D \sqrt{B_D} = 200$ MeV, we find $\Delta m_{D^0} < 0.05$ ps$^{-1}$ for $m_{b'} < 280$ GeV, for both nominal cases of Eq. (19). Thus, the current bound of $\Delta m_{D^0} < 0.07$ ps$^{-1}$ is satisfied, and the search for $D^0$ mixing is of great interest. This bound weakens by factor of 2 if one allows for strong phase between $D^0 \to K^+\pi^-$ and $K^+\pi^-$. If $m_{b'} < m_{b'}$, as slightly preferred by $D^0-D^0$ mixing constraint, the direct search for $b'$ just above $200$ GeV at the Tevatron Run II could be rather interesting. Since $V_{cb}$ is not suppressed, the $b'$ quark would decay via charged current. Both $b'$ and $t'$, regardless of which one is lighter, with $m_{b'} \sim 300$ GeV and $|m_{b'}-m_{b'}| \lesssim 85$ GeV [5], can be easily discovered at the LHC.

The large and main imaginary element $V_{t's} \simeq -V_{cb}^*$ in Eq. (19), being larger than $V_{ts}$ and $V_{cb}$, may appear unnatural (likewise for $V_{ub'}$ vs $V_{ub}$). However, it is allowed, since the main frontier that we are just starting to explore is in fact $b \to s$ transitions. The current situation that $A_{K\to s} \sim -0.12$ while $A_{K\to s} \gtrsim 0$ in $B \to K\pi$ decays may actually be hinting at the need for such large $b \to s$ CPV effects. The litmus test would be finding $\Delta m_{B_s}$ not far above current bound, but with sizable $\sin 2\Phi_{B_s} < 0$, which may even emerge at Tevatron II. Our results studied here are for illustration purpose, but the main result, that $K_L \to \pi^0\bar{\nu}\nu$ may be rather enhanced, is a generic consequence of Eq. (1), which is a possible solution to the $B^+ \to K^+\pi^0$ CPV puzzle.

In summary, the deviation of direct CPV measurements between neutral and charged $B$ decays, $A_{K^+\pi^0} - A_{K^+\pi^-} \simeq 0.16$ while $A_{K^+\pi^0} \simeq -0.12$, is a puzzle that could be hinting at New Physics. A plausible solution is the existence of a 4th generation with $m_{t'} \sim 300$ GeV and $V_{t's}V_{t'b} \sim 0.025 e^{i\theta^D}$. If so, we find special solution space is carved out by stringent kaon constraints, and the $4 \times 4$ CKM matrix is almost fully determined. $K^+ \to \pi^0\bar{\nu}\nu$ may well be of order $(1-2) \times 10^{-10}$, while $K_L \to \pi^0\bar{\nu}\nu \sim (4-12) \times 10^{-10}$ is greatly enhanced by the large phase in $V_{t's}V_{t'b}$. With kaon constraints satisfied, $B_d$ mixing and $\sin 2\Phi_{B_d}$ are consistent with experiment. Our results are generic. If the effect weakens in $b \to s$ transitions, the effect on $K \to \pi\nu\bar{\nu}$ would also weaken. But a large CPV effect in electroweak $b \to s$ penguins would translate into an enhanced $K_L \to \pi^0\bar{\nu}\nu$ (and $\sin 2\Phi_{B_s} < 0$).

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[6] Our results do not change drastically as $\phi_{ab}$ is varied by $\pm 10^5$.
[21] A. Gray et al. [HPQCD Collab.], hep-lat/0507015.
[22] The summer 2005 result by K. Abe et al. [Belle Collab.], hep-ex/0507037 reports a low value of $\sin 2\Phi_{B_s} \sim 0.652 \pm 0.039 \pm 0.020$, but this is for $B^0 \to J/\psi K^0$ mode only, and it is too early to draw any conclusion.
[23] E. Nakano et al. [Belle Collab.], hep-ex/0505017. This new result is in agreement with PDG 2005 with slightly better errors.
Recent kaon decay results imply a more stringent bound of $1 - |V_{ud}|^2 - |V_{us}|^2 = 0.0004 \pm 0.0011$ ($|V_{ub}|^2$ is negligible), or $|V_{ub}| < 0.047$ at 90% C.L., which is still satisfied by our value.