Measurements of CP-Violating Asymmetries in $B^0 \rightarrow \alpha_1^2(1260) \pi^\mp$ Decays


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The angle \( \alpha \equiv \arg[-V_{td}V_{tb}^*/V_{ud}V_{ub}^*] \) of the unitarity triangle of the Cabibbo-Kobayashi-Maskawa (CKM) quark-mixing matrix \([1]\) has recently been measured by the BABAR and Belle Collaborations from time-dependent CP asymmetries in the \( B^0 \) decays to \( \pi^+\pi^- \) \([2]\), \( \rho^\pm\pi^\mp \) \([3]\), and \( \rho^+\rho^- \) \([4]\). The decay \( B^0 \) to \( a_1\pi \) \([5]\) proceeds dominantly through the \( b \to \bar{u}d\) process in the same way as the previously studied modes. However, due to the presence of additional loop contributions, these measurements determine an effective value \( \alpha_{\text{eff}} \), rather than \( \alpha \) itself. This obstacle can be overcome using isospin symmetry \([6]\), with bounds to \( \Delta \alpha = \alpha - \alpha_{\text{eff}} \) determined using either an isospin analysis \([7]\) or broken SU(3) flavor symmetry \([8]\). Because it has the smallest contribution from loop diagrams, the \( B^0 \to \rho^+\rho^- \) decay currently allows the most precise single determination of \( \alpha \) \([9]\). The BABAR collaboration recently reported the observation of \( B^0 \to a_1^+\pi^+ \) \([10]\), where the angle \( \alpha_{\text{eff}} \) can be determined by measuring time-dependent CP asymmetries \([11, 12]\). The state \( a_1^\pm \pi^\mp \), like \( \rho^\pm\pi^\mp \), is not a CP eigenstate and four flavor-charge configurations must be considered \( (B^0(B^0) \to a_1^+\pi^+) \). Theoretical bounds on \( \Delta \alpha \) in these decay modes based on SU(3) flavor symmetry have been derived in Ref. \([12]\).

In this Letter we report measurements of the CP parameters in the decay \( B^0 \to a_1^+\pi^+ \) with \( a_1^+ \to \pi^+\pi^+\pi^- \). The analysis is done in the quasi-two-body approximation \([13]\). The data were collected with the BABAR detector \([14]\) at the PEP-II asymmetric \( e^+e^- \) collider \([15]\). An integrated luminosity of 349 fb\(^{-1}\), corresponding to 384 ± 4 million \( B\bar{B} \) pairs, was recorded near the \( \Upsilon(4S) \) resonance (“on-resonance”) at a center-of-mass (CM) energy \( \sqrt{s} = 10.58 \) GeV. An additional 37 fb\(^{-1}\) were taken about 40 MeV below this energy (“off-resonance”) for the study of continuum background in which a charm or lighter quark pair is produced.

From a candidate \( B\bar{B} \) pair we reconstruct a \( B^0 \) decaying into the final state \( f = a_1\pi \) \( (B^0_{a_1\pi}) \). We also reconstruct the vertex of the other \( B \) meson \( (B^0_{\text{tag}}) \) and identify its flavor. The difference \( \Delta t \equiv t_{a_1\pi} - t_{\text{tag}} \) of the proper decay times of the reconstructed and tag \( B \) mesons, respectively, is obtained from the measured distance between the \( B^0_{a_1\pi} \) and \( B^0_{\text{tag}} \) decay vertices and from the boost \((\beta\gamma = 0.56)\) of the \( e^+e^- \) system. The \( \Delta t \) distributions are given \([12]\) by:

\[
F_{Q_{\text{tag}}}^{a_1^+\pi^+} (\Delta t) = (1 \pm \Delta C_{a_1\pi}) e^{-|\Delta t|/\tau} \left\{ 1 - Q_{\text{tag}} \Delta w^+ \right. \\
\left. (1 - 2w) \left[ (C_{a_1\pi} + \Delta S_{a_1\pi}) \sin(\Delta m_d \Delta t) - (C_{a_1\pi} + \Delta S_{a_1\pi}) \cos(\Delta m_d \Delta t) \right] \right\},
\]

where \( Q_{\text{tag}} = 1(1) \) when the tagging meson \( B^0_{\text{tag}} \) is a \( B^0(\overline{B^0}) \), \( \tau \) is the mean \( B^0 \) lifetime, \( \Delta m_d \) is the mass difference between the two \( B^0 \) mass eigenstates, and the mistag parameters \( w \) and \( \Delta w \) are the average and difference, respectively, of the probabilities that a true \( B^0 \) is incorrectly tagged as a \( \overline{B^0} \) or vice versa. The time- and flavor-integrated charge asymmetry \( A_{C_{a_1\pi}}^{a_1^+\pi^+} \) measures direct CP violation. The quantities \( S_{a_1\pi} \) and \( C_{a_1\pi} \) parameterize the mixing-induced CP violation related to the angle \( \alpha \), and flavor-dependent direct CP violation, respectively. The parameter \( \Delta S_{a_1\pi} \) describes the asymmetry between the rates \( \Gamma(B^0 \to a_1^+\pi^-) + \Gamma(\overline{B^0} \to a_1^+\pi^+) \) and \( \Gamma(B^0 \to a_1^+\pi^+) + \Gamma(\overline{B^0} \to a_1^+\pi^-) \), while \( \Delta S_{a_1\pi} \) is related to the strong phase difference between the amplitudes contributing to \( B^0 \to a_1\pi \) decays. The parameters \( \Delta C_{a_1\pi} \) and \( \Delta S_{a_1\pi} \) are insensitive to CP violation. The flavor-tagging algorithm uses six mutually exclusive categories. Its analyzing power is measured to be \((30.4 \pm 0.3)\% \([16]\).

Charged particles are detected and their momenta measured by the combination of a silicon vertex tracker (SVT), consisting of five layers of double-sided silicon detectors, and a 40-layer central drift chamber, both operating in the 1.5-T magnetic field of a superconducting solenoid. Charged-particle identification (PID) is provided by the average energy loss (dE/dx) in the tracking devices and by an internally reflecting ring-imaging Cherenkov detector (DIRC) covering the central region. Separation between pions and kaons is achieved at the level of four standard deviations \( (\sigma) \) for momenta below 3 GeV, decreasing to 2.5 \( \sigma \) at 4 GeV.

Full Monte Carlo (MC) simulations \([17]\) of the signal decay modes, continuum, and \( B\bar{B} \) backgrounds are used to establish the event selection criteria. The MC signal events are simulated as \( B^0 \) decays to \( a_1\pi \) with \( a_1 \to \rho\pi \). For the \( a_1 \) meson parameters we take the mass \( m_0 = 1230 \) MeV and the width \( \Gamma_0 = 400 \) MeV \([18, 19]\).
We reconstruct the decay $a_1 \rightarrow 3\pi$ with the following requirement on the invariant mass: $0.87 < m_{a_1} < 1.8$ GeV. The intermediate dipion state is reconstructed with an invariant mass between 0.51 and 1.1 GeV. We impose several PID requirements to ensure the identity of the signal pions. For the decay pion coming from the $B$ meson we require the measured Cherenkov angle to be within $-2\sigma$ and $+5\sigma$ from the expected value for a pion. This requirement removes 98.6% of the background from $a_1 K$. A $B$ candidate is characterized kinematically by the energy-substituted mass $m_{ES} = \sqrt{(s/2 + p_t \cdot p_B)^2/E_s^2 - p_B^2}$ and energy difference $\Delta E = E_B^\oplus - E_\pi^\oplus$, where the subscripts 0 and B refer to the initial $Y(4S)$ and to the $B$ candidate in the laboratory frame, respectively, and the asterisk denotes the CM frame. The resolutions in $m_{ES}$ and in $\Delta E$ are about 3.0 MeV and 20 MeV respectively. We require $|\Delta E| < 0.1$ GeV and $5.25 \leq m_{ES} < 5.29$ GeV. To reduce the number of false $B$-meson candidates we require that the probability of the $B$ vertex fit be greater than 0.01. The absolute value of the cosine of the angle between the direction of the $\pi$ meson from $a_1 \rightarrow \pi\pi$ with respect to the flight direction of the $B$ in the $a_1$ meson rest frame is required to be less than 0.85 to suppress combinatorial background. The distribution of this variable is uniform for signal and peaks near unity for this background.

To reject continuum background, we use the angle $\theta_T$ between the thrust axis of the $B$ candidate and that of the rest of the tracks and neutral clusters in the event, calculated in the center-of-mass frame. The distribution of $\cos \theta_T$ is sharply peaked near $\pm 1$ for $q\bar{q}$ candidates, which have a jet-like topology, and is nearly uniform for the isotropic $B$-meson decays. We require $|\cos \theta_T| < 0.65$. We discriminate further against $q\bar{q}$ background with a Fisher discriminant $F$ that combines several variables that characterize the production dynamics and energy flow in the event [20]. The remaining continuum background is modeled from off-resonance data.

We use MC simulations of $B^+\bar{B}^0$ and $B^+B^-$ decays to look for $B\bar{B}$ backgrounds, which can come from $B$ decays with or without charmed particles in the final state. Neutral and charged $D$ mesons may contribute to background through particle mis-identification or mis-reconstruction. We remove any combinations of the decay products, including possible additional $\pi^0$, with invariant mass consistent with nominal mass values for $D^{\pm \pm} \rightarrow K^{\mp \pm} \pi^{\pm \pm}$ or $K_S^{0} \pi^{\pm \pm}$ and $D^0 \rightarrow K^{\mp \pm} \pi^{\pm \pm}$ or $K^{\mp} \pi^{\pm} \pi^0$. The decay mode $B^0 \rightarrow a_2^{\pm}(1320) \pi^\mp$ has the same final-state particles as the signal. We suppress this decay with the angular variable $H$, defined as the cosine of the angle between the normal to the plane of the $3\pi$ resonance and the flight direction of the primary pion from $B$ meson evaluated in the $3\pi$ resonance rest frame. Since the $a_1$ and $a_2(1320)$ mesons have spins of 1 and 2 respectively, the distributions of the variable $H$ for these two resonances differ. We require $|H| < 0.62$.

The average number of candidates found per selected event is 1.32. In the case of events with multiple candidates we choose the candidate with the best $B$-vertex fit probability. From simulated signal events we find that this algorithm selects the correct candidate in about 92% of the events containing multiple candidates, and introduces negligible bias.

We obtain the $CP$ parameters and signal yield from an unbinned extended maximum likelihood (ML) fit with the input observables $\Delta E$, $m_{ES}$, $F$, $m_{a_1}$, $H$, and $\Delta t$. We have six fit components in the likelihood: signal, charm and charmless $B\bar{B}$ background, $B^0 \rightarrow a_2^\pm(1320) \pi^\mp$, continuum $q\bar{q}$ background, and non-resonant $\rho\pi\pi$. The charmless component also includes candidates that were incorrectly reconstructed from particles in events that contain a true signal candidate. Based on measurements of branching fractions for similar charmless decays, we assume $B(B^0 \rightarrow \rho^0 \pi^+\pi^-) = (2 \pm 2) \times 10^{-6}$, which corresponds to 19 expected events in the ML fit sample. This yield is fixed in the fit and a systematic uncertainty is assigned to the final results.

The total probability density function (PDF) for the component $j$ and tagging category $c$ in the event $i$, $P_{j,c}^i$, is written as a product of the PDFs of the discriminating variables used in the fit. The factored form of the PDF is a good approximation since linear correlations among observables are below 10%. The systematic uncertainty from residual correlations is taken into account in the fit bias. We write the extended likelihood function for all events as

$$\mathcal{L} = \prod_c \exp\left(-n_c\right) \prod_i P_{j,c}^i \sum_{N_j} n_j f_{j,c} P_{j,c}^i,$$

where $n_j$ is the yield of events of component $j$, $f_{j,c}$ is the fraction of events of component $j$ for each category $c$, $n_c = \sum_j f_{j,c} n_j$ is the number of events found by the fitter for category $c$, and $N_j$ is the number of events of category $c$ in the sample. We fix $f_{j,c}$ to values obtained with MC events for the charmless and charm fit components and allow it to vary for the $q\bar{q}$ component.

The PDF $P_{\text{sig}}(\Delta t, \sigma_{\Delta t}; c)$, for each category $c$, is the convolution of $F(\Delta t; c)$ (Eq. 1) with the signal resolution function (sum of three Gaussians) determined from the $B_{\text{had}}$ sample. The $\Delta t$ resolution functions for all the other fit components are also modeled with the sum of three Gaussians. For charmless, $B^0 \rightarrow a_2^\pm(1320) \pi^\mp$, and $\rho\pi\pi$ components in the nominal fit to the data we assume $S = 0$, $C = 0$, $\Delta S = 0$, and $\Delta C = 0$ and we vary these parameters when evaluating systematic uncertainties on final results. We use an effective $B$ lifetime for the charmless component as obtained from a fit to MC signal...
events. The continuum (charm) $\Delta t$ distributions are parameterized as sums of three Gaussians with parameters determined from a fit to off-resonance (MC) events.

The PDF of the invariant mass of the $a_1$ meson in signal events is parameterized as a relativistic Breit-Wigner line-shape with a mass-dependent width that takes into account the effect of the mass-dependent $\rho$ width [22]. The PDF of the invariant mass of the $a_2(1230)$ meson is parameterized by a sum of three Gaussian function distributions. The $m_{ES}$ and $\Delta E$ distributions for signal are parameterized as a sum of two Gaussian distributions. The $\Delta E$ distribution for continuum background is parameterized by a linear function, and the combinatorial background in $m_{ES}$ is described by a phase-space-motivated empirical function [23]. We model the Fisher distribution $F$ using a Gaussian function with different widths above and below the mean. The $A$ distributions are modeled using polynomials.

The PDF parameters are determined from MC simulated events with the exception of the continuum background, where we use off-resonance data, and of the signal resolution function, where we use the $B_{\text{flav}}$ sample. Large data control samples of $B$ decays to charmed final states of similar topology are used to verify the simulated resolutions in $m_{ES}$ and $\Delta E$. Where the control samples reveal differences between data and MC in mass and energy resolution, we shift or scale the resolution used in the likelihood fits.

We test and calibrate the fitting procedure by applying it to ensembles of simulated $q\bar{q}$ experiments drawn from the PDF, into which we have embedded the expected number of signal, charmless, $B^0 \rightarrow a_2^+(1320)\pi^+$, the charm, and the $\rho\pi\pi$ events randomly extracted from the fully simulated MC samples. The measured quantities $S_{a_1}, C_{a_1}, \Delta S_{a_1}, \Delta C_{a_1}$, and $A_{CP}^{a_1}$ have been corrected for the fit biases and a systematic uncertainty equal to half of the bias found in MC simulations is assigned on the final results.

In the fit there are 35 free parameters, including $S_{a_1}, C_{a_1}, \Delta S_{a_1}, \Delta C_{a_1}$, the charge asymmetries for signal and continuum background, five yields, the signal $a_1$ width, eleven parameters determining the shape of the combinatorial background, and 12 tagging efficiencies for the continuum. The main contributions to the systematic error on the signal parameters are summarized in Table I. We have studied systematic uncertainties arising from several sources: variation of the signal PDF shape parameters within their errors; modeling of the signal $\Delta t$ distribution; tagging efficiency and mistag rates determined from the $B_{\text{flav}}$ sample [21]; uncertainties in $\Delta m_{B_d}$ and $\tau$ [18]; uncertainty in fit bias; uncertainty due to $CP$ violation present in the $B\bar{B}$ background, the $a_2^+(1320)\pi^+ CP$ violation; uncertainty due to the interference between $B^0 \rightarrow a_1^+\pi^-$ and other 4$\pi$ final states have been estimated with MC simulations; doubly-Cabibbo-suppressed (DCS) $b \rightarrow \bar{u}cd$ amplitude for some tag-side $B$ decays [24]; SVT alignment; and the particle identification algorithm. We allow for a $CP$ asymmetry up to 20% in $B$ decays to charmless final states, and up to 50% in $B$ decays to $a_2(1230)\pi$.

From the fit to a sample of 29300 events, we obtain a signal yield of 608 $\pm$ 53, of which 461 $\pm$ 46 have their flavor identified and are used to measure the following additional parameters: $S_{a_1} = 0.37 \pm 0.21 \pm 0.07, \Delta S_{a_1} = -0.14 \pm 0.21 \pm 0.06, C_{a_1} = -0.10 \pm 0.15 \pm 0.09, \Delta C_{a_1} = 0.26 \pm 0.15 \pm 0.07, A_{CP}^{a_1} = -0.07 \pm 0.07 \pm 0.02$. Linear correlations between these fit parameters are small.

The angle $\alpha_{\text{eff}}$ can be defined [12] as:

$$\alpha_{\text{eff}} = \frac{1}{4} \left[ \arcsin \left( \frac{S_{a_1} + \Delta S_{a_1}}{\sqrt{1 - (C_{a_1} + \Delta C_{a_1})^2}} \right) \right]$$

Using the measured CP parameters, we determine the angle $\alpha_{\text{eff}}$ and one of the four solutions, $\alpha_{\text{ext}} = 78.6^\circ \pm 7.3^\circ$, is compatible with the result of SM-based fits. Using the published branching fraction [10] and adding statistical and systematic errors in quadrature, we obtain also the following values for the flavor-charge branching fractions [25] (in units of $10^{-6}$): $B(B^0 \rightarrow a_1^+\pi^-) = 17.9 \pm 4.8, B(B^0 \rightarrow a_1^-\pi^+) = 11.4 \pm 4.7, B(B^0 \rightarrow a_1^+\pi^-) = 13.0 \pm 4.3, B(B^0 \rightarrow a_1^-\pi^+) = 24.2 \pm 5.8$.

Figure 1 shows distributions of $m_{ES}$ and $\Delta E$, enhanced in signal content by requirements on the signal-to-continuum likelihood ratios using all discriminating variables other than the one plotted. Figure 2 gives the $\Delta t$ projections and asymmetry for flavor tagged events.

In summary, we have measured the $CP$-violating asymmetries in $B^0 \rightarrow a_1^+(1260)\pi^+$ decays and determined the angle $\alpha_{\text{eff}}$. We do not find evidence for direct or mixing-induced $CP$ violation in these decays. Once measurements of branching fractions for SU(3)-related decays become available, quantitative bounds on $\Delta \alpha$ obtained with the method of Ref. [12] will provide significant con-

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[5] For the $a_1(1260)$ meson we use the short notation $a_1$.

* Deceased