Search for Sigma hypernuclear states

with the FINUDA spectrometer

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L'argomento di tesi di dottorato è la simulazione e la ricostruzione degli eventi che sono riconducibili a stati ipernucleari di tipo Sigma. La tesi consiste nello studio degli algoritmi per l'identificazione degli ipernuclei Sigma nell'ambito delle misure attuabili con lo spettrometro FINUDA. FINUDA è attualmente in allestimento nella fattoria mesonica DAΦNE, presso i Laboratori Nazionali di Frascati. La tesi tratta sia le problematiche teoriche che sperimentali della rivelazione degli ipernuclei Sigma, ovvero di stati nucleari legati nei quali un nucleone è sostituito da un iperone Sigma. In particolare, è stata affrontata la misura della risoluzione energetica e spaziale del rivelatore di vertice dell'esperimento FINUDA, misura cruciale per l'identificazione e la discriminazione dei livelli energetici degli ipernuclei Sigma.

In this thesis the simulation and reconstruction of events related to Sigma hypernuclear states have been developed. Algorithms capable of identifying Sigma hypernuclei by means of the FINUDA spectrometer have been proposed. FINUDA is currently under construction at the Φ-meson farm DAΦNE, Laboratori Nazionali di Frascati. Theoretical and experimental topics related to the detection of Sigma hypernuclei, namely bound nuclear states where a nucleon is substituted by a Sigma hyperon, have been discussed. Special attention has been paid to energy and spatial resolution of the vertex detector of FINUDA.
These topics are of crucial importance for the success of the experiment since they allow for the identification and discrimination of the energy levels of Sigma hypernuclei.
Chapter 1

The $\Sigma$ hypernuclei.

A hypernucleus is formed when a hyperon substitutes a nucleon into a nucleus, that is, when a nucleus acquires a unit of strangeness. In the case of $\Sigma$-hypernuclei only one bound state was detected, $^{\Lambda}H\theta$ [1] [2], and there is no evidence of any other bound state with atomic number higher than four. Data in medium and heavy nuclei are scarce. Analysis of $\Sigma$ hypernuclear system is fairly complex and a comprehensive picture on the interaction has not been established yet.

1.1 History.

A report on $\Sigma$ hypernuclear states with narrow widths ($\sim 8\,MeV$) was first given at CERN [3] for the $^9Be$ excitation energy spectrum $^9Be(k^-,:\pi^-)^9\Sigma\Lambda Be$, shown in fig. 1.1. Two peaks were reported, which appeared at the energies $E_{^\Lambda B\ell} + \Delta M$, where $\Delta M$ is the mass difference between $\Sigma$ and $\Lambda$ and $E_{^\Lambda B\ell}$ is the $\Lambda$ binding energy in $^9\Lambda\ell B\ell$. The observation of $\Sigma$ states as narrow as the $\Lambda$ states was unexpected. A $\Lambda$ particle can only decay weakly inside a nucleus, while a $\Sigma$ particle
Figure 1.1: The spectrum on the left panel is obtained from the \((k^-, \pi^-)\) reaction on \(^9\text{Be}\) at a kaon momentum of 720 MeV/c at CERN [3]. The spectrum is plotted as a function of \(M_{HY} - M_A\), which is the difference between the hypernuclear mass and the target mass. The \(M_{HY} - M_A\) distributions on the right panel are obtained from the \((k^-, \pi^\pm)\) reactions on \(^{12}\text{C}\) at 400 and 450 MeV/C [6]. Bumps corresponding to possible \(\Sigma\)-hypernuclear state are indicated with an dotted circle. Bump widths are ~ 5 MeV broad, they are narrow when considering the conversion reaction \((\Sigma N \to \Lambda N)\).

decays through a \(\Sigma N \to \Lambda N\) strong interaction and, therefore, with a large width ~ 25 MeV [4].

In 1981, \((k^-, \pi^+)\) spectra were taken at BNL on \(^6\text{Li}\) with a kaon beam of momentum 713 MeV/c. Several forward pion angles were measured [5]. The pion energy spectrum displayed two peaks, which disappeared at larger measured
1.1. HISTORY.

An experiment on $^{16}O$ revealed only a broad bump ($\Gamma \sim 19\,\text{MeV}$), from which it was not possible to extract any clear indication of a bound state. Such bump was later explained as the superposition of coherent and incoherent transitions to particle-hole states [5].

The CERN group [3] [6] [7] set up a new kaon beam line. They aimed at producing $\Sigma$ particles inside the nucleus with momentum close to zero (recoiless condition), for which a kaon momentum $p_k = 450\,\text{MeV}/c$ was chosen. Both $\pi^-$ and $\pi^+$ spectra on $^{12}C$ (see fig. 1.1) and $^{16}O$ were analyzed [6] [7]. For $^{12}C$, the $\pi^+$ spectrum shows a single peak in the continuum at $\Delta M = M_{HY} - M_A \approx 279\,\text{MeV}$, which corresponds to about 3 MeV $\Sigma^-$ binding energy. The width equals 4 MeV and to this state was assigned the orbital level number $p_{3/2}$. The $\pi^-$ spectrum displays two peaks at $\Delta M \approx 270\,\text{MeV}$ and $\approx 275\,\text{MeV}$. These two states are explained as substitutional states, where either a proton or a neutron in the $p_{3/2}$ shell is substituted by a $\Sigma^+$ or $\Sigma^0$. For $^{16}O$, the continuum of the $\pi^+$ spectra is populated by two peaks centered at 277.5 $\text{MeV}$ and 284 $\text{MeV}$ both with a width of $\sim 5\,\text{MeV}$. They were assigned to have $p_{3/2}$ and $p_{1/2}$ respectively. The $p_{1/2}$ peak appeared at about 6 MeV above the $p_{3/2}$ peak, which is opposite to what happens to $\Lambda$ hypernuclei. Therefore, it was deduced that the $\Sigma$ spin-orbit strength in the p-shell is about 12 MeV twice as much as that of the nucleon.

An experiment with stopped kaons on $^{12}C$ was performed at KEK [8] [9]. The purpose was to study the $\pi^+$ spectra by using a novel method capable of tagging $\Sigma^-$ events. It consists in detecting $\pi^0$'s from $\Lambda$ decays in coincidence with the $\pi^+$'s, where the $\Lambda$'s are produced by the conversion reaction $\Sigma N \rightarrow \Lambda N$. Among the various sources of $\pi^0$ emission, there is only one in coincidence with $\pi^+$'s: the free decays of $\Lambda \rightarrow n\pi^0$. Therefore, it is a study with a low level of contamination, which suppresses the continuum due to quasifree $\Sigma^-$'s. Two sub-
stitutional peaks on $^{12}C$ were detected and assigned to $\Sigma^-$ hypernuclear states: $\Delta M \approx 277\, MeV \, p_{3/2}$ and $\Delta M \approx 281\, MeV \, p_{1/2}$. The width of the $p_{3/2}$ orbital was measured to be $\sim 4\, MeV$ and the $p_{1/2}$ width slightly narrower. The measurement of the inclusive $\pi^+$ spectrum was later repeated at KEK, the experiment was performed to study the ground state of the $^{12}C\,\Sigma^-$ hypernucleus. This experiment did not observe any narrow peak [10], consequently the expected $\Sigma^-$ hypernuclear state was no longer claimed. In addition, the KEK experiment did not spot any enhancement in the cross section of the $^{12}C(\kappa^{-}_{\text{stop}}, \pi^{+}_{\text{prompt}})$ reaction (see fig. 1.2), thus denying the existence of the 279 MeV peak previously found by [6].

In the late eighties, the discovery at KEK of a bound state of the $^{4}\frac{3}{2}He$ hypernu-

![Figure 1.2: Inclusive energy distribution of the positive pions emitted after $k^{-}$ absorption for the $^{12}C(\kappa^{-}_{\text{stop}}, \pi^{+}_{\text{prompt}})$ reaction [10]. Superimposed to it is the spectrum predicted by Morimatsu and Yazaki’s calculation (solid line) [11].](image)

ucleus [1], also predicted by Harada and Akaishi [12], gave a lift to the research of the $\Sigma^-$ bound states. The $(k^{-}, \pi^{+})$ reaction on $^{4}He$ showed a bump in the pion spectrum, which was identified as a bound $\Sigma_{0^{+}}$ state with 3.2 MeV binding energy and a width of 4.6 MeV. A similar structure was not found in the $\pi^{+}$
1.1. HISTORY.

An experiment at BNL with $450 \div 720 \text{MeV/c}$ kaons [2] [13] confirmed the existence of the bound state $^4\Sigma H e$ with a clear peak below the $\Sigma^+$-production threshold for the $^4 H e(k^-, \pi^-)$ reaction. The measurement did not show any narrow structure on $^6 Li$ and $^9 Be$. An enhancement in the $\pi^-$ energy distribution was however noticed, which shifted with the nuclear mass number: it was $\sim 10 \text{MeV}$ for $^4 He$ and $\sim 14 \text{MeV}$ for $^9 Be$. Finally, the large differences between the pion energy distribution of the $(k^-, \pi^-)$ and $(k^-, \pi^+)$ reaction channels were simply accounted for as an evidence of isospin dependence.

The most recent experimental information on the $\Sigma$-nucleus potential has been proposed [14] for the inclusive $(\pi^-, k^+)$ reaction on Si, Ni, In and Bi at a pion spectrum.

Figure 1.3: (a) The $\pi^-$ momentum spectrum observed for the $^4 H e(k^-_{st.}, \pi^-)$ reaction [1]. The spectrum does not fall off smoothly at the quasi-free threshold, but reveals a peak-like structure, which is indicated with a dotted circle. (b) The $\pi^+$ spectrum observed for the $^4 H e(k^-_{st.}, \pi^+)$ reaction. In contrast to the $\pi^-$ spectrum, none peak-like structure is visible in the $\Sigma^-$ bound region, that is, between 260 and 280 MeV.
Figure 1.4: Left panel, evidence of a bound state of the $^{4}_{3}\Lambda He$ hypernucleus found at BNL. The excitation energy spectra of $^{4}_{3}\Lambda He(k^-,\pi^-)$ displays a peak structure below the energy threshold, whereas such a peak is not observed in the $^{4}_{3}\Lambda He(k^-,\pi^+)$ spectrum. On the right panel, the cross sections for the $(k^+,\pi^\pm)$ reactions on $^{4}_{3}\Lambda He,^6 Li$ and $^9 Be$ [13] are displayed. For the $(k^-,\pi^-)$ channel, a possible shift of the bump as a function of A can be observed.

beam momentum of 1.2 GeV/c. Data analysis is still under way, but preliminary results claim detectable yields in the bound region.

The experimental situation appears rather puzzling; except for $^{4}_{3}\Lambda He$, present data do not contribute to shed light on the existence of $\Sigma$-hypernuclei. Therefore, further information is required in order to clear up the situation on the $\Sigma$-nucleus interaction. A systematic investigation is also called for the sector of heavy nuclei, where theory has already anticipated the Coulomb assisted $\Sigma$-nuclear bound states [15].
1.2 The production of $\Lambda$ and $\Sigma$ hypernuclei.

In order to produce a hypernucleus, the hyperon from the reaction

$$a \ N \to Y \ b$$

has to remain inside the nucleus. The reaction (1.1) will henceforth be denoted by $N(a, b)Y$, or simply $(a, b)$, where N is a nucleon and Y a hyperon. The probability of producing a hypernucleus depends on the energy transferred during the process (1.1). If the momentum transferred to the hyperon ($q_Y$) is larger than the nuclear Fermi momentum ($k_F$), the hyperon has small probability to bind, and it leaves the nucleus right after the elementary process (1.1) has occurred. When $q_Y < k_F$, the hyperon may be produced in a bound state. The hyperon can also be produced with a non-negligible probability above its emission threshold, the process is called quasi-free Y production. Some hypernuclear states may lie in the continuum, they are defined quasi-bound states as they do not emit the hyperon.

The Y production is mediated by a strong process, which deposits into a nucleus a unit of strangeness. The following production reactions can be itemized as follows [16]:

1. Processes with strangeness exchange

$$k^- \ n \to \Lambda \, \pi^-$$

$$\Sigma^0 \, \pi^-$$

$$\Sigma^- \, \pi^0$$

$$k^- \ p \to \Sigma^- \, \pi^+$$

$$\Sigma^+ \, \pi^-$$

$$\Lambda \, \pi^0$$

$$\Sigma^0 \, \pi^0$$
In this case the incident kaon $k^-$ transforms the struck nucleon into a hyperon $\Lambda$ or $\Sigma$. The hyperon is accompanied by a pion whose energy is directly related to the hypernuclear levels. The reactions $n(k^-, \pi^-)\Lambda$ and $p(k^-, \pi^+)\Sigma^-$ are esoenergetic, then they can create the hyperons at rest. In this case, the transferred momentum is zero, $\vec{p}_K = \vec{p}_\pi = \vec{p}$, and the pion is emitted at $\theta = 0^\circ$ in the laboratory frame. From energy-momentum conservation:

$$\sqrt{\vec{p}^2 + m^2_\pi} + m_n = m_\Lambda + \sqrt{\vec{p}^2 + m^2_\pi}$$ (1.3)

one can derive the so-called magic momentum:

$$E_K = \sqrt{\vec{p}^2 + m^2_K} = \frac{m^2_K - m^2_\pi + (m_\Lambda - m_N)^2}{2(m_\Lambda - m_N)} \Rightarrow p \approx 530 \text{ MeV}/c$$ (1.4)

If the production reaction is $p(k^-, \pi^+)\Sigma^-$, the kaon magic momentum is $p \sim 280 \text{ MeV}/c$.

By using the strangeness exchange reaction at $\theta = 0^\circ$, the hyperon is predominantly produced in a state with the same quantum numbers as the struck nucleon; namely the neutron-hole state and the $\Lambda$ are coupled to $J^P = 0^+$ and $\Delta l = 0$ (substitutional reaction). By increasing $\theta$ the probability of transition with $\Delta l = 1, 2, \ldots$ increases, and hypernuclear states with higher spin are likely to be produced.

The $(k^-_{\text{stop}}, \pi^-)$ reaction was examined at KEK and in the near future it will be studied at DAΦNE. This process of strangeness production was the standard method to produce $\Lambda$-hypernuclei in emulsion and bubble chamber experiments during the 60’s. When a $k^-$ is stopped in the target, it is initially captured into an external atomic level. After falling into inner levels via electromagnetic interaction, the negative kaon is absorbed at the nuclear surface converting a nucleon into a $\Lambda$ or $\Sigma$. The momentum transferred is close to $k_F$; for instance, at $0^\circ$ scattering angle $q_\Lambda \approx 250 \text{ MeV}/c$. 

and \( q_{\Sigma} \approx 180 \text{ MeV/c} \). The \( \Lambda \)-hypernucleus yield following the \( k^- \) absorption at rest is about \( 10^{-3} \). Due to the different momentum transferred to prompt pions, the absorption permits a clear separation between the process of hypernucleus formation and the quasi-free hypernuclear production.

2. Processes associated to the production of strange hadrons

\[
\begin{align*}
\pi^+ n & \rightarrow \Lambda k^+ \\
& \quad \Sigma^0 k^+ \\
& \quad \Sigma^+ k^0 \\
\pi^+ p & \rightarrow \Sigma^+ k^+ \\
\gamma p & \rightarrow \Lambda k^+ \\
e^- p & \rightarrow e^- \Lambda k^+ \\
p N & \rightarrow \Lambda k^+ N
\end{align*}
\] (1.5)

The production of a \( s\bar{s} \) quark pair following \( \pi^+ n \) interaction gives rise to the production of two strange hadrons in the final state: the \( s \)-quark becomes the constituent of a \( \Lambda \), and the \( \bar{s} \) is transferred to the positive pion, which becomes a \( k^+ \). The \( n(\pi^+, k^+)\Lambda \) reaction is complementary to the \( n(k^-, \pi^-)\Lambda \) one. However, the former reaction enables accurate studies of deeply bound states in medium and heavy hypernuclei to be carried on, since it produces almost background-free spectra. In addition, the \( n(\pi^+, k^+)\Lambda \) reaction has the advantage of using pion beams, which are of better quality and higher intensity with respect to kaon beams. Finally, the \( (k^+, \pi^\pm) \) and \( (\pi^+, k^+) \) reactions hardly produce hypernuclei laying in their ground states, because of the strong pion and kaon absorption in the nuclear medium.
CHAPTER 1. THE $\Sigma$ HYPERNUCLEI.

The electroproduction reaction $A \left( e^-, e'^+ k^+ \right)_A A$ is characterized by a large momentum transfer $\sim 350 \text{ MeV}/c$. Thus, the electroproduction cross section is small. The smallness of the reaction cross section is partially balanced by the high intensity and high momentum definition of electron beams $\Delta p/p \sim 10^{-3} \div 10^{-4}$.

3. Another promising reaction for hypernuclear studies is the $(\pi^-, k^+)$ reaction. This is a double-charge exchange process, which is free from the $\Lambda$-hypernuclear background.

4. Reactions of strangeness exchange and production. Production of $S = -2$, -3 hypernuclei.

\[
\begin{align*}
    k^- p & \rightarrow \Xi^- k^+ \\
    & \Xi^0 k^0 \\
    & k^+ k^0 \Omega^- \\
    k^- n & \rightarrow \Xi^- k^0 \\
    k^- p & \rightarrow \Lambda \pi^0 \\
    p p & \rightarrow \Xi^0 k^+ k^+ n
\end{align*}
\]

Some experiments have revealed the existence of $\Xi$-hypernuclei, which are produced through the $k^- p(n)$ reaction. When a $\Xi^-$-hypernucleus is formed, the hyperon strongly interacts with a nucleon of the nuclear medium and produces two $\Lambda$’s, by releasing only 28 MeV. This may produce a double-$\Lambda$ hypernucleus $\Xi^- +^{12}_{C} C \rightarrow^{12}_{\Lambda \Lambda} B + n$, which was firstly observed in the emulsion experiments during the 60’s. The study of $\Xi$ and $\Lambda \Lambda$-hypernuclei is related to the search for a stable $H$-particle, which is predicted to be a six quark state containing two $u$, two $d$ and two $s$ quarks. Nowadays, experiments looking for $H$’s are being performed. The current experimental status provides no clear evidence of di-baryon resonances in the strange
1.3 The $\Sigma$ width in nuclei.

The strong $\Sigma N \to \Lambda N$ conversion reaction plays a prominent role in the production of narrow $\Sigma$ states. The conversion reaction is predicted to be as broad as few tens of MeV in nuclear matter [15]. In order to detect a $\Sigma$-bound state, the $\Sigma$-width does not have to exceed the separation between two adjacent $\Sigma$-hypernuclear levels.

The conversion reaction can be represented via a Feynman diagram (fig. 1.5).

The cross section can be calculated by means of the S-matrix

$$\langle f \mid S - 1 \mid i \rangle = -\frac{i T}{(2\pi)^6} \sqrt{\frac{M_N}{E_{N_i}}} \sqrt{\frac{M_N}{E_{N_f}}} \sqrt{\frac{M_\Sigma}{E_\Sigma}} \sqrt{\frac{M_\Lambda}{E_\Lambda}} (2\pi)^4 \delta \left( p_\Sigma + p_{N_i} - p_\Lambda - p_{N_f} \right)$$

(1.7)

where $T$ is the transition amplitude. Since the transition probability between two states $\alpha$ and $\beta$ can be expressed as

$$\sigma_{\alpha\beta} = \frac{\pi (2l + 1)}{k^2} |S_{\alpha\beta} - \delta_{\alpha\beta}|$$

(1.8)

it can be derived that [17]

$$\sigma_{\Sigma N \to \Lambda N} = \frac{1}{v_{\text{rel}}} \int \frac{d^3k'}{(2\pi)^3} \int \frac{d^3l'}{(2\pi)^3} \frac{M_\Sigma}{E_\Sigma (k)} \frac{M_\Lambda}{E_\Lambda (k')} \frac{M_N}{E_N (p)} \frac{M_N}{E_N (p')} \times \frac{T^2}{(2\pi)^4} \delta \left( p_\Sigma + p_{N_i} - p_\Lambda - p_{N_f} \right)$$

(1.9)
where the $v_{\text{rel}}$ is the relative velocity of the $\Sigma N$ system and $M_N$ is the nucleon mass. In the case of $k_{\text{stop}}$, the non-relativistic approximation can be applied, that is, $M_i/E_i \simeq 1$ and $k' \rightarrow k - q$. By integrating over $p$, the conversion reaction cross section becomes

$$\sigma_{\Sigma N \rightarrow \Lambda N} = \frac{1}{v_{\text{rel}}} \int \frac{d^3 q}{(2\pi)^3} \times \delta \left( E_{\Sigma} (k) + E_N (p) - E_N (p + q) - E_{\Lambda} (k - q) \right)$$

where the sum over spins and isospins of $T^2$ is replaced by the $\Sigma \Sigma |T|^2$. In order to derive the $\Sigma$ width ($\Gamma_\Sigma$) in infinite nuclear matter, the quasi-particle method is used. It relates $\Gamma_\Sigma$ to the $\Sigma$ self-energy ($\Sigma^*$) by means of the equation

$$\Gamma_\Sigma = -2Im\Sigma^*$$

(1.11)

$\Sigma^*$ is depicted by the many-body diagram shown in fig. 1.6. The $Im\Sigma^*$ can be evaluated by following the standard Feynman rules. For the $\Sigma^-p \rightarrow \Lambda n$ reaction,

![Figure 1.6: Feynman many-body diagram for the $\Sigma$ self-energy, which incorporates the $\Sigma N \rightarrow \Lambda N$ transition. This transition is represented by the particles along the dotted line.](image)

it is shown [17] that the $\Sigma^-$ self-energy and the $\sigma_{\Sigma N \rightarrow \Lambda N}$ cross section are related by the equation

$$Im\Sigma^* = -\langle |v_{\text{rel}}| \rangle_{\text{av}} \rho_p / 2$$

(1.12)
where $\rho_p$ is the proton density, and the notation $\langle \sigma v_{\text{rel}} \rangle_{\text{av}}$ indicates that the quantity $\sigma v_{\text{rel}}$ is averaged over the Fermi sea, therefore

$$\Gamma_{\Sigma} = \rho_p \langle \sigma v_{\text{rel}} \rangle_{\text{av}} = \frac{1}{\rho_p} \int \frac{d^3 p}{(2\pi)^3} n(p) \int \frac{d^3 q}{(2\pi)^3} [1 - n(p + q)]$$

$$\times \sum_{k} |T|^2 (2\pi) \delta \left( E_{\Sigma}(k) + E_N(p) - E_N(p + q) - E_\Lambda(k - q) \right)$$  \hspace{1cm} (1.13)$$

The equation (1.13) provides the $\Sigma$ width as a function of the initial nucleon momentum averaged over the Fermi sea. The Pauli blocking is accounted for by the term $[1 - n(p + q)]$.

It is possible to estimate the effect of the Pauli principle by using the non-relativistic approximation and taking the $|T|^2$ independent from the angle; the integration over $p$ yields:

$$4 \int \frac{d^3 p}{(2\pi)^3} n(p) [1 - n(p + q)] = \rho P_F(q)$$ \hspace{1cm} (1.14)$$

where the Pauli blocking factor $P_F(q) = 1 - \theta(2 - \tilde{q}) \left(1 - \frac{3}{8} \tilde{q} + \frac{1}{16} \tilde{q}^3\right)$, $\tilde{q} = |q|/k_F$ and $\theta$ is the step function. In the non-relativistic limit $E_{\Sigma} - E_\Lambda \sim M_{\Sigma} - M_\Lambda$, then the final result is

$$\langle \sigma v_{\text{rel}} \rangle_{\text{av}} = \frac{1}{\pi} M\tilde{q} \sum_{k} |T|^2 P_F(\tilde{q}) \quad \tilde{q} = \sqrt{2\hat{M}(M_{\Sigma} - M_\Lambda)}$$ \hspace{1cm} (1.15)$$

where $\hat{M}$ is the reduced nucleon and lambda mass. Since $k_F = (3\pi^2 \rho/2)^{1/3} = 268 \text{ MeV}/c$, $P_F = 0.73$ for $\rho = \rho_0$.

In order to deal with finite nuclei, $\Gamma_{\Sigma}$ is evaluated in the framework of the local density approximation:

$$\Gamma_{\Sigma} = \int \Gamma(\rho(r)) |\phi_{\Sigma}(r)|^2 d^3 r$$ \hspace{1cm} (1.16)$$

where $\phi_{\Sigma}(r)$ is the $\Sigma$ wave function. By using a harmonic wave functions for the radial part $e^{-\alpha^2 r^2/2}$ ($\alpha^2 = 0.316 \text{ fm}^{-2}$), it is possible to evaluate $\Gamma_{\Sigma}$ for the 1s and 1p states of $^{16}O$ [17]:

$$\Gamma_{\Sigma}^{1s} = 15.1 \text{ MeV} \quad \Gamma_{1p} = 11.1 \text{ MeV}$$ \hspace{1cm} (1.17)$$
If the Pauli blocking factor is excluded, the result is:

\[ \Gamma_{\Sigma^-} = 19.4 \, MeV \quad \Gamma_{1p} = 13.6 \, MeV \]  \hspace{1cm} (1.18)

Finally, \( \Gamma_{\Sigma^-} = 29 \, MeV \) when omitting the local density approximation. The average over the nuclear density distribution reduces \( \Gamma \) to about one half. It is worthwhile noting that the Pauli blocking further reduces \( \Gamma_{\Sigma^-} \), even though not enough to explain the narrow widths measured \( \Gamma \sim 5 \, MeV \).

For the sake of completeness, it will be mentioned the approach brought up by the authors of Ref. [17], which may explain the narrow \( \Gamma_{\Sigma^-} \) so far measured. In the hypernuclei dynamics various mechanisms and selections rules can suppress the width substantially. Among the mechanisms introduced to suppress the calculated \( \Gamma_{\Sigma^-} \), the most important one is the Pauli blocking of the final nucleon in the \( \Sigma N \rightarrow \Lambda N \) conversion. They also consider the \( \Sigma \) width not as a constant parameter but as function of \( \rho \), \( \Gamma_{\Sigma^-}(\rho) \). \( \Gamma_{\Sigma^-} \) is then evaluated by making the same assumptions as in eq. (1.16) and by taking into account the induced reaction. The net effect is a reduction of \( \Gamma_{\Sigma^-} \) in nuclei, which for 1s and 1p are

\[ \Gamma_{1s} = 7.0 \, MeV \quad \Gamma_{1p} = 5.9 \, MeV \]  \hspace{1cm} (1.19)

In medium nuclei an unusual \( \Sigma^- \) bound state with narrow width may occur with the assistance of the attractive Coulomb potential, even in the case of a shallow absorptive nuclear potential. Such a bound state is called *Coulomb-assisted* bound state.


1.4 The $\Sigma$ bound states in nuclei.

In a nucleus the $\Sigma$ particle is subject to a complex potential, which is called optical potential\(^1\). The real part is referred to the elastic scattering and the imaginary part to inelastic processes. The potential is written

$$U (r) = - [V (r) + iW (r)] \quad (1.20)$$

where both the real and imaginary parts of the potential are energy dependent. At low energies the real part describes the motion of a single particle. Due to the short mean free path of the $\Sigma$ in a nucleus, the $\Sigma$ incident wave gets attenuated, which is accounted by the imaginary part of the potential $W (r)$. $V (r)$ is chosen so as to give the observed single-particle resonances in scattering lengths and in total cross sections, while $W (r)$ is chosen to reproduce mean free paths for different energies. The radial variation of the potential assumes the form:

$$\rho (r) = \frac{\rho_0}{1 + \exp [(r - R_0) / a]} \quad (1.21)$$

where $\rho_0 = 0.17 \text{ fm}^{-3}$ is nuclear density at saturation, $R_0$ and $a$ are free parameters, which are the radius and diffuseness, respectively. Considering a linear potential, other two multiplicative parameters are needed, one for the real part and the other for imaginary part. In the case of a $\Sigma$-hypernucleus, the optical potential parameters cannot be derived by $\Sigma$-nuclear data fitting because there are not enough data. The $\Sigma^*$ self-energy, which is equivalent to an optical potential, is treated in accordance with the procedure developed in Sec. 1.3, which permits to evaluate the imaginary part $W (r)$. The real part $V (r)$ is obtained by fitting the $\Sigma^-$-atom data [17]. With these assumptions, two kinds of optical po-

\(^1\)In the optical model the interaction of the incident particle wave function with the nucleus is treated in analogy to the passage of the light through an opaque medium.
potential were found:

\[
V (r) = 31 \frac{\rho (r)}{\rho_0} \, MeV, \quad W (r) = 15 \frac{\rho (r)}{\rho_0} \, MeV \quad (1.22)
\]

\[
V (r) = 31 \frac{\rho (r)}{\rho_0} \, MeV, \quad W (r) = \frac{15}{5.2} \arctan \left( 5.2 \frac{\rho (r)}{\rho_0} \right) \, MeV \quad (1.23)
\]

The former is a linear potential, which is calculated in absence of Pauli blocking and without the induced interaction. The latter is a saturating potential, which includes the effects of Pauli blocking and induced interaction, as explained in Sec. 1.3. With these potentials it is possible to evaluate the energies and the widths of \(\Sigma^-\)-hypernuclear states for different nuclei. The results obtained with the linear and saturating potential are reported in tab. 1.1. The binding energies of 1\(s\) and 2\(p\) states (in the atomic nomenclature) range from 14 MeV for \(^{12}C\) to 28 MeV for \(^{32}S\) for the 1\(s\) state, and from 4 MeV for \(^{16}O\) to 16 MeV for \(^{32}S\) for the 2\(p\) state. The widths range from 21 MeV for \(^{12}C\) to 27 MeV for \(^{32}S\) for the 1\(s\) state, and from 14 MeV for \(^{16}O\) to 23 MeV for \(^{32}S\) for the 2\(p\) state. Unfortunately, these states cannot be observed, since the widths are larger than the separation energies. The results yielded by the saturating potential are rather different. The widths are reduced from 3.0 to 3.5 times for light and medium nuclei, respectively. The width range from 4 to 8 MeV for 2\(p\) and 1\(s\) state, respectively. The binding energies increase slightly with respect to the linear potential because the absorptive part of the potential acts as a repulsive force and the saturation makes this repulsion less effective. As a consequence, the widths are smaller than the separation energies, therefore detectable.

For the \(\Sigma^0\) and the \(\Sigma^+\), the widths are similar but the binding energies are smaller than those for the \(\Sigma^-\) states. The separation energies of the \(\Sigma^-\) states are still larger than the widths, therefore these states should also be observable.

As mentioned in Sec. 1.1, only the \(\frac{4}{3}H e\) bound system was found with a width equal to 4.6 MeV. The \(\Sigma\)-binding energy can be either 3.2 MeV for \(\Sigma^+\), or 6.2 MeV
for $\Sigma^0$. In the analysis of Ref. [12] and [18], a bound state at $E = 2.3 \text{ MeV}$ and

Figure 1.7: The $\Sigma$ triton potential calculated by [12]. $V$ is the real part and $W$ is the imaginary part of potential. Two characteristic features must be recalled: the short-range repulsive core in the real part, and the strong imaginary part due to $\Sigma - \Lambda$ conversion process. At short distances $< 0.6 \text{ fm}$ the potential becomes repulsive.

$\Gamma = 4.6 \text{ MeV}$ is found for the isospin $T = 1/2$, which is a combination of $\Sigma^+, \Sigma^0$ states. While a $T = 3/2$ $\Sigma^-$ state does not show any appreciable strength. This is due to the different nuclear potentials, which pertain to the two isospin channels. Fig. 1.7 depicts the real and the imaginary part of the optical potential for the $\Sigma$-triton hypernucleus [18]. The repulsive nature of $V(r)$ at short distance favours the $\Sigma$-interaction to take place at about $2 \text{ fm}$, where $V(r)$ has a dip. At this distance, $\Sigma$ can only probes the nuclear surface, then reducing the overlap with the nucleons, and consequently the $\Sigma$ width. This is the basic requirement for detecting $\Sigma$ bound states in light nuclei, along with the spacing between adjacent levels.

The repulsive behavior of $V(r)$ at short distance was extended to medium and heavy nuclei by phenomenological analyses of $\Sigma^-$ atomic data [19] [20]. How-
ever, the scarce amount of data for the $\Sigma - N$ interaction in nuclear matter renders difficult the assessment of both $W(r)$ and $V(r)$. In addition, $\Sigma^-$ atomic data can be employed to determine the potential only outside the nucleus around its surface, which implies that the short-range nuclear potential cannot be established by using atomic data.
1.4. **THE \( \Sigma \) BOUND STATES IN NUCLEI.**

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>Hyperon</th>
<th>Potential</th>
<th>2s</th>
<th>2p</th>
<th>1s</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>B</td>
<td>B</td>
<td>2( s )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>linear</td>
<td>14.213</td>
<td>14.861</td>
<td>6.873</td>
</tr>
<tr>
<td>( 12C )</td>
<td>( \Sigma^- )</td>
<td>saturating</td>
<td>1.670</td>
<td>14.646</td>
<td>6.781</td>
</tr>
<tr>
<td></td>
<td>( \Sigma^0 )</td>
<td>linear</td>
<td>6.695</td>
<td>19.864</td>
<td>6.666</td>
</tr>
<tr>
<td></td>
<td>( \Sigma^+ )</td>
<td>saturating</td>
<td>7.512</td>
<td>6.666</td>
<td></td>
</tr>
<tr>
<td>( 16O )</td>
<td>( \Sigma^- )</td>
<td>linear</td>
<td>3.900</td>
<td>16.572</td>
<td>20.724</td>
</tr>
<tr>
<td></td>
<td>( \Sigma^0 )</td>
<td>saturating</td>
<td>5.010</td>
<td>17.049</td>
<td>7.043</td>
</tr>
<tr>
<td></td>
<td>( \Sigma^+ )</td>
<td>linear</td>
<td>7.395</td>
<td>19.958</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>saturating</td>
<td>8.029</td>
<td>6.850</td>
<td></td>
</tr>
<tr>
<td>( 28Si )</td>
<td>( \Sigma^- )</td>
<td>linear</td>
<td>1.380</td>
<td>13.300</td>
<td>24.716</td>
</tr>
<tr>
<td></td>
<td>( \Sigma^0 )</td>
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<td>3.956</td>
<td>14.000</td>
<td>6.890</td>
</tr>
<tr>
<td></td>
<td>( \Sigma^+ )</td>
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<td>1.113</td>
<td>11.493</td>
<td>26.028</td>
</tr>
<tr>
<td></td>
<td></td>
<td>saturating</td>
<td>2.196</td>
<td>11.969</td>
<td>7.513</td>
</tr>
<tr>
<td>( 32S )</td>
<td>( \Sigma^- )</td>
<td>linear</td>
<td>3.622</td>
<td>1.270</td>
<td>22.700</td>
</tr>
<tr>
<td></td>
<td>( \Sigma^0 )</td>
<td>saturating</td>
<td>5.551</td>
<td>16.740</td>
<td>7.140</td>
</tr>
<tr>
<td></td>
<td>( \Sigma^+ )</td>
<td>linear</td>
<td>1.113</td>
<td>11.493</td>
<td>26.028</td>
</tr>
<tr>
<td></td>
<td></td>
<td>saturating</td>
<td>2.196</td>
<td>11.969</td>
<td>7.513</td>
</tr>
</tbody>
</table>

**Table 1.1:** Binding energies and widths of different \( \Sigma \) hypernuclear states calculated using the approach reported in Ref. [17]
Chapter 2

The FINUDA experiment.

FINUDA (FIsica NUcleare a DAΦNE) is an experiment of hypernuclear physics, which is carried out at the DAΦNE $e^+e^-$ collider by means of a large-acceptance and high-resolution magnetic spectrometer. The aim of FINUDA is the study of formation and decay of hypernuclei, which are induced by negative kaons at rest ($k_{stop}^-$) in a nucleus ($^AZ$) through the reaction:

$$k_{stop}^- + ^AZ \rightarrow ^AY Z + \pi^\pm_{prompt}$$  \hspace{1cm} (2.1)

where the elementary process of strangeness exchange transforms a nucleon into a Y hyperon (either $\Sigma$ or $\Lambda$), and a pion ($\pi^\pm_{prompt}$) promptly follows the reaction occurrence. The FINUDA spectrometer is designed to perform a high-resolution spectroscopy, to trace the particles from hypernuclear decay and to measure the lifetime of hypernuclei.

2.1 The DAΦNE collider.

The Istituto Nazionale di Fisica Nucleare (INFN) has built in the Laboratori Nazionali di Frascati a double annular ring accelerator of electron and positron beams. The $e^+e^-$ collisions produce $\phi(1020)$-mesons at rest in two interaction
regions. The intersecting directions form a horizontal angle of $\theta/2 = 10\degree\pm15\,\text{mrad}$.

**Figure 2.1:** The DAΦNE collider.

DAΦNE is a multi-bunch machine, with minimum and maximum beam energy of 0.250 and 0.750 GeV, respectively, and a design luminosity of $4 \times 10^{30}\,\text{cm}^{-2}\text{s}^{-1}$ per single bunch. With a maximum of 120 bunches, DAΦNE aims at reaching a luminosity of $5 \times 10^{32}\,\text{cm}^{-2}\text{s}^{-1}$. Presently, DAΦNE has a peak luminosity of $8 \times 10^{31}\,\text{cm}^{-2}\text{s}^{-1}$ (i.e., a daily luminosity of 4791 $\text{nbarn}^{-1}$), which corresponds to a mean value of $\sim 250\,\phi/s$.

There are three detectors operating at DAΦNE: the K Long Observation Experiment (KLOE) devoted to particle physics, FIsica NUcleare a DAΦNE (FINUDA) for nuclear physics experiments with kaons, and DAΦNE Exotic Atoms Research (DEAR), which examines the formation of kaonic atoms. $\phi$-mesons are produced by the annihilation of electrons and positrons. About 49% of $\phi$'s decay into a couple of charged k mesons. The $k^+k^-$ pairs are then used for the physics
2.2. HYPERNUCLEAR PHYSICS WITH FINUDA.

of FINUDA, which is also the name of the detector. The flux of slow $k^-$ is stopped in targets where negative kaons undergo nuclear capture. The advantages of such a kaon beam are the monochromatic low-momentum kaons, the lack of a hadronic background, the reduced straggling of the kaons in the target, and the possibility of tagging a $k_{stop}$ event through the associate $k^+$. Negative kaons can be stopped in a thin target $(0.3 \div 0.5 \text{ g/cm}^2)$ with minimal straggling. As a consequence, the prompt pions used for hypernuclear spectroscopy undergo minimal energy depletion, and the hypernuclear levels are measurable with high precision. In fact, the main limitation to a precise measurement is the thickness of target. FINUDA can achieve world-class results in hypernuclear physics with the present luminosity. As an example, the rate of $\Lambda$-hypernuclear production is about $10^{-3}/k_{stop}$, which yields about 40 reconstructed hypernuclear events per hour, when taking into account both the acceptance and efficiency of FINUDA. In detail, the acceptance of the spectrometer is 72%, the trigger efficiency 26%, the reconstruction efficiency 95%, and 50% of the prompt pions leave the target without further interaction with vertex region [21].

2.2 Hypernuclear physics with FINUDA.

FINUDA will pursue an extensive program of high-resolution spectroscopy of $\Lambda$ and $\Sigma$-hypernuclei. A $Y$-hypernucleus is a bound state of $Z$ protons, $(A - Z - 1)$ neutrons and one $Y = \Lambda, \Sigma$ hyperon. FINUDA will measure the momenta of pions following the reaction (2.1). In the case $Y = \Lambda$, the binding energy ($B_\Lambda$) of $^{A}_{\Lambda}Z$ can be calculated as follows

$$B_\Lambda = M_{A-1} + M_\Lambda - M_H$$  \hspace{1cm} (2.2)
Chapter 2. The FINUDA Experiment.

where $M_{A-1}$, $M_{A}$ and $M_{H}$ are the mass of the $^{A-1}Z$ residual nucleus, $\Lambda$ hyperon and $H$ hypernucleus ($^{A}Z$), respectively. From eq. (2.1), $M_{H}$ results

$$M_{H} = M_{A} + M_{k_{\text{slop}}} - M_{\pi_{\text{prompl}}} + T_{H} - T_{A} - T_{k_{\text{slop}}} - T_{\pi_{\text{prompl}}}$$

(2.3)

where $T_{H}, T_{A}, T_{k_{\text{slop}}}$ and $T_{\pi_{\text{prompl}}}$ are the kinetic energies of the hypernucleus, the target nucleus, the stop kaon and the prompt pion, respectively. In the case of the FINUDA experiment, $A$ is at rest, and $H$ recoils with negligible kinetic energy, therefore eq. (2.2) becomes

$$B_{\Lambda} = M_{A-1} + M_{A} - M_{A} - M_{k_{\text{slop}}} + M_{\pi_{\text{prompl}}} + T_{\pi_{\text{prompl}}}$$

(2.4)

Eq. (2.4) shows that the uncertainty of $B_{\Lambda}$ is directly related to $\Delta T_{\pi-}$ the pion kinetic energy uncertainty, $\Delta B_{\Lambda} = \Delta M_{H} = \Delta T_{\pi-}$. $\Delta T_{\pi-}$ can be expressed as a function of the prompt pion momentum

$$\frac{\Delta T_{\pi-}}{T_{\pi-}} = \frac{\sqrt{p_{\pi-}^{2} + m_{\pi-}^{2}}}{2} = f(p_{\pi-}) \frac{\Delta p_{\pi-}}{p_{\pi-}}$$

(2.5)

FINUDA can reach a momentum resolution $\Delta p_{\pi-}/p_{\pi-} \sim 0.3\%$, then for $p_{\pi-} = 250 \text{ MeV}/c$ ($T_{\pi-} \sim 147 \text{ MeV}$) the energy resolution is $\Delta T_{\pi-} \sim 0.6 \text{ MeV}$.

Hypernuclear spectroscopy offers important information on nuclear and particle physics. Nuclear physics: when $\Lambda$ is embedded in a nucleus, it behaves like a distinguishable baryon, which can probe deeply bound states and highly excited states without disrupting the shell structure of the nucleus, since the $\Lambda - N$ interaction is appreciably weaker than the $N - N$ one. Particle physics: the $\Lambda - N$ interaction can be studied in the nuclear medium, on the contrary, low-energy $\Lambda - N$ scattering can hardly be performed due to the short mean-life of lambdas. In order to improve the knowledge of the free $\Lambda - N$ interaction, theorists need precise data. A detailed spectroscopic study of hypernuclei with high-resolution experiments offers the possibility of increasing the experimental information on the $\Lambda - N$ interaction. The possibility to observe effects due
to the partial deconfinement of \( \Lambda \) in nuclear matter is also very interesting. In order to improve the knowledge of \( \Lambda - N \) interaction, theory needs precise data on the free \( \Lambda - N \) interaction, which are difficult to obtain due to low-intensity hyperon beam and short lifetime of hyperons. The spin dependent parts of the \( \Lambda - N \) interaction are related to the modeling of the interaction short-range part. The meson exchange theories predict spin-orbit splitting for the doublet in \( ^9\Lambda Be \), and 390 \( \pm 780 \) \( KeV \) for the doublet in \( ^{13}\Lambda C \), whereas the quark model based theories predict 30 \( \pm 40 \) \( KeV \) and 150 \( \pm 200 \) \( KeV \), respectively. Recent measurements of the splitting were carried away by using \( \gamma \) ray spectroscopy [22] [23], the spacing of the two levels was measured to be 31 \( \pm 2 \) \( KeV \) for the doublet in \( ^9\Lambda Be \) and 152 \( \pm 54 \) (stat) \( \pm 36 \) (syst) \( KeV \) for the doublet in \( ^{13}\Lambda C \). FINUDA cannot reach such an energy definition; nevertheless, it is capable of studying the heavy hypernuclei spin-orbit level splittings, since they are found to be larger: the E369 experiment at KEK measured a orbit splitting of \( \sim 2 \) \( MeV \) in \( ^{89}\Lambda Y \).

FINUDA is designed to reconstruct hypernuclear events; therefore, it is capable of investigating the decay products of \( \Lambda \)-hypernuclei. A free \( \Lambda \) decay via final mesons writes

\[
\Lambda \rightarrow p + \pi^- \quad (\Gamma_{p\pi^-} \sim 64\%) \quad \Lambda \rightarrow n + \pi^0 \quad (\Gamma_{n\pi^0} \sim 36\%) \quad (2.6)
\]

The decay may in principle occur with an isospin transition \( \Delta I = 1/2 \) or \( 3/2 \), but it is experimentally proven that the component \( \Delta I = 1/2 \) dominates. When \( \Lambda \) is embedded in a medium (or heavy) nucleus, the mesonic decay is inhibited since the nucleon from the mesonic decay carries away a momentum \( p \sim 100 \) \( MeV/c \), which is below the Fermi momentum \( k_f \sim 280 \) \( MeV/c \). Thus, the pionic decay modes eq. (2.6) are blocked by the Pauli principle, and non-mesonic decays
become the prevailing decay modes

\[ \Lambda + p \rightarrow n + p \ (\Gamma_{np}) \quad \Lambda + n \rightarrow n + n \ (\Gamma_{nn}) \]  

(2.7)

The study of the non-mesonic weak decay provides a primary means of exploring the baryon weak interaction \( \Lambda N \rightarrow NN \). Precise measurements of the branching ratio \( \Gamma_{nn}/\Gamma_{np} \) may shed some light on the \( \Delta I = 1/2 \) rule, and provide information about the structure of the weak Hamiltonian.

Hypernuclear lifetimes will precisely be measured because of the FINUDA timing performance, and the statistics that might be reached. FINUDA will measure the hypernucleus lifetime by means of the time difference between prompt pions and protons or neutrons from eq. (2.7). Assuming an overall time resolution of 700 ps at FWHM simulations predict that the mean-life of a hypernucleus can be determined with a statistical error below 5% for about ~ 4000 events in the time-of-flight spectrum (see fig. 2.2 and Appendix C for a detailed discussion).

It is worthwhile recalling that other physics topics can be studied along with hypernuclear physics. The charge-exchange reaction

\[ k^- + p \rightarrow k^0 + n \]  

(2.8)
which has never been measured at momenta below 100 MeV/c. This is an important measurement since it allows chiral symmetry theories to be tested. Furthermore, the branching ratio \( k^+ \rightarrow e^+\nu_e \) to \( k^+ \rightarrow \mu^+\nu_\mu \) will be measured with an improved statistical precision of a factor of 3.

### 2.3 The FINUDA spectrometer.

FINUDA is a non-focusing magnetic spectrometer, which detects \( k^+k^- \) pairs with an acceptance of about 90\%. The apparatus is shown in fig. 2.3. FINUDA was designed to:

1. achieve a momentum resolution \( \Delta p/p \leq 0.3\% \) FWHM;

2. have a versatile fast trigger, which can select events with multiplicity \( \geq 1 \);
3. trace and discriminate the charge particles involved in hypernuclear reactions;

4. detect the neutrons such as those from non-mesonic decays.

In order to obtain $\Delta p/p \leq 0.3\%$ FWHM, the magnetic field is set at $B_z = 1.1\, T$, $B_x = B_y = 0\, T$, with an uniformity better than 5\% along $z$ and 1\% along $x$ and $y$ over a tracking volume of about $3.3\, m^3$.

The apparatus cylindrical geometry indicates three natural coordinates $\rho$, $\phi$, $z$. The $z$ axis is defined by the direction of the $e^+e^-$ beam and oriented towards the $e^+$ beam, as shown in fig. 2.4. The $(\Phi \to) k^+k^-$ pairs are uniformly distributed over $\phi$; the $k^+k^-$ $\theta$-angle distribution follows a $\sin^2 \theta$ law, where $\theta$ is the angle formed by the $k^+k^-$-direction with respect to the $z$-axis. The $e^+-e^-$ bunch volume determines the interaction region extension. Their nominal dimensions are equal to $\sigma_x \simeq 2\, mm$, $\sigma_y \simeq 0.02\, mm$ and $\sigma_z \simeq 3\, cm$.

**2.3.1 The spectrometer framework.**

FINUDA is a detector designed for nuclear physics experiments. Its arrangement reflects the dispersed location of the kaon beam. The spectrometer consists of 3 main components: the vertex region, the tracking system, which sur-
2.3. THE FINUDA SPECTROMETER.

rounds the vertex region, and the outer cylindrical scintillator array.

The vertex region.

The vertex region (see fig. 2.5) deals with $k^+k^-$ pairs. The detectors of this region are designed to deliver a fast trigger signal when a hypernuclear event takes place, and define the hypernuclear production vertex, thus the target involved. The inner detector TOFINO (see description below) initiates a fast-triggering operation when a back-to-back event hits its segmented structure. Such an event topology is assumed to belong to $k^+k^-$ pair. The operation is successfully completed if a particle is detected by TOFONE within a time delay of $\sim 10 ns$. This last event is attributed to the prompt pion associated to the $k^-$ absorption. The whole operation defines the first level trigger for a hypernuclear event, which enables the FINUDA electronics to acquire the event itself. Other reactions require a different triggering patterns, see for example eq. (2.8).

The inner element of the vertex region is the beam pipe, which is composed by a cylinder of beryllium 500 $\mu m$ thick. The beam pipe is surrounded by TOFINO, an array of 12 plastic scintillator slabs arranged like the staves of a barrel, whose inner diameter is 112 mm. Each slab 2.3 mm thick and 20 cm long is viewed at both ends by hybrid photodiodes (HPD), which are capable of working inside a magnetic field. TOFINO is used to trigger events with a back-to-back topology and to generate the START signal for FINUDA time-of-flight measurements. TOFINO has an intrinsic time resolution of 400 $ps$ at FWHM. The total length (17 cm) of the detector does not limit the geometrical acceptance of FINUDA. Moving outward, there is a first array of double-sided silicon microstrip modules (ISIM), mounted according to the faces of an octagonal prism. ISIM measures the particle trajectories with a resolution better than 30 $\mu m$, which determines
the $k^{-}$ stop position in a target with a precision of 600 $\mu$m [24]. In addition, ISIM measures the energy deposited by a crossing particle (via $\Delta E/\Delta x$, see section 2.4), which is used for particle identification PID. Targets are arranged in a ladder whose average distance from ISIM is about 2 $mm$. Targets, up to a maximum of 8, are self-supporting slabs, which may range from Li to Pb. The target thickness is chosen to stop negative kaons as close as possible to the external surface. The thickness varies between 200 and 300 $mg/cm^2$. A second set of 10 modules (OSIM) surrounds the target ladder and constitutes the first element of the tracking system. Both ISIM and OSIM form the so called FINUDA silicon vertex detector (I/OSIM), which will be described in detail in Sec. 2.4.

The tracking region.

The tracking device measures the particle momenta, for prompt pions the overall resolution is $\Delta p/p \leq 0.3\%$ at FWHM. It consists of three different types of de-
tectors (see fig. 2.6), which are immersed in a helium atmosphere to minimize the multiple scattering, which would otherwise severely affect the momentum resolution in a non-focusing spectrometer like FINUDA. Prompt pions initially encounter OSIM, the outer array of silicon microstrip de-

Figure 2.6: Front view of the FINUDA spectrometer (left) and side view (right) with a simulated $\Lambda$-hypernuclear event.

The last array of the tracking system is a composite layer of aluminized mylar
straw tubes (ST) 2.5 m long, and each tube has a diameter of 15 mm. This layer consists of six sub-layers arranged in three groups of two sub-layers each one. The first group is placed along the beam axial direction and the other two are $\pm 12^\circ$ tilted. The spatial resolution is $100 \mu m \ (\sigma)$ in the x-y plane and the information from the stereo ST gives a resolution along the z direction of $220 \mu m \ (\sigma)$. The choice of this kind of detector was dictated by the fact that particles hit the last array of detectors with a large angle $\geq 50^\circ$, and the ST cost is comparatively low for $8 m^2$ detection area.

The outer cylindrical scintillator array.

The outermost layer consists of a plastic scintillator barrel (TOFONE), located at a distance of 127 cm from the beam axis, and is made up of 72 slabs. Each slab is 255 cm long and 10 cm thick, the section is shaped like a trapezoid with the outer and inner bases 12 cm and 11 cm long, respectively. The slabs are coupled to XP2020 phototubes at each end. TOFONE has an intrinsic time-of-flight resolution of 500 ps at FWHM, which brings the overall time resolution to 700 ps at FWHM (see Appendix C). TOFONE allows for the detection of neutrons, which follow the non-mesonic decay of hypernuclei (2.7). The neutron energies range over an interval $40 \div 150 \ MeV$ [25], and are detected with an average efficiency of $\sim 15\%$.

Although none of the sub-detectors use a fully innovative technology, some performances obtained by each of them are advanced: the use of silicon microstrip for $\Delta E/\Delta x$ measurements, the excellent time resolution for HPDs, the large area LMDC’s operated with He-based mixtures, the high number of long straw tubes (ST), and the large volume of plastic scintillators for neutron detection. The combination of these performances allows FINUDA to follow up the hyper-
nuclear formation and decay in all its steps.

2.4 The FINUDA silicon vertex detector.

The FINUDA vertex detector comprises two layers of double-sided silicon microstrip modules. The internal silicon detector layer (ISIM) is at a radius of about 6.5 cm far from the beam axis, and the outer layer (OSIM) at 8.5 cm (fig. 2.7). IS/OSIM aims at:

1. identifying the $k^+k^-$ pairs. Such events are initially defined by the detection of two co-linear particles in TOFINO. Kaons are later mass identified through their energy deposition in the silicon modules; in this case, the energy released by kaons is about twenty times larger than a minimum ionizing particle;
2. determining the position of the interaction vertex within the target;

3. delivering the first hit for the outer tracking system;

4. discriminating in mass prompt pions from stopped kaons and from protons, which follows the non-mesonic decays \([26]\) \([27]\).

The choice of such a vertex detector was indicated by the working environment and by the physics to be done. As far as the last issue is concerned, the spatial resolution required is \(35 \mu m \ (\sigma)\), a better resolution is not necessary due to the multiple scattering that a charged particle undergoes in the volume of FINUDA. In addition, the detector was required to be insensitive to magnetic fields and be a low-mass device. I/OSIM is equipped with 18 modules (see fig. 2.7). A picture of the silicon vertex detector is displayed in fig. 2.8, and the supporting structure in fig. 2.9. Fig. 2.10 shows a spot of ISIM, target and OSIM and, finally, figs. 2.11 and 2.12 a module. The active elements of each module (see fig. 2.12) are three double-sided silicon microstrip detectors, each with an area of \(52.6 \times 65.4 \ mm^2\) and a thickness of \(300 \mu m\). They are glued head-to-head in order to achieve an active area of \(196.2 \times 52.6 \ mm^2\) for each module. Quartz spacers, \(300 \mu m\) thick and \(5 \ mm\) long, are glued to each end to increase the thermal resistance between hybrids and silicon detectors. Two plates of Beryllium ceramic \(300 \mu m\) thick are glued to the quartz spacers and they support one or two hybrids. These plates direct the hybrid heat away from a module. Three hybrids equip a module: two on the n-side and one on the p-side, and are DC-decoupled from the hosting module by means of capacitors. A hybrid is populated by 8 VIKING VA1 chips interconnected to the read-out electronics via a network of electrical lines. Through these lines, the VA1 output analog pulses are multiplexed before being sent to the readout electronics. A VA1 integrates and shapes the charge collected by the strips of the silicon detector, and deliv-
2.4. THE FINUDA SILICON VERTEX DETECTOR.

Figure 2.8: The outer layer of the FINUDA silicon vertex detector (OSIM) during the installation at the Laboratori Nazionali di Frascati.
Figure 2.9: The supporting structure of the FINUDA silicon vertex detector. The structure is designed to have maximum clearance to charged particles. The two pairs of holding handles are removed after insertion in FINUDA.
2.4. THE FINUDA SILICON VERTEX DETECTOR.

Figure 2.10: Detail of the vertex detector. The lower module is followed by a carbon target and, in turn, by another module which belongs to OSIM, the outer layer of the vertex detector.
Figure 2.11: View of a module of the silicon vertex detector. The sensitive part is represented by three double sided silicon microstrip detectors glued head to head. At the extremities appear two hybrids each populated by 8 VA1 chips, which are followed by 8 charge-decoupling capacitors (larger chips).
Figure 2.12: $\phi$- and z-side of a module. Bottom, side view.
ers it upon the arrival of an external trigger. Each VA1 is capable of reading 128 strips of a module, and the analog signals are digitized by charge-sensitive flash ADC’s.

The physical pitch of the p-side side strips is $25 \, \mu m$ but only every second strip is connected, therefore the resulting readout pitch is $50 \, \mu m$. Each single silicon detector has 1021 readout strips on the p-side side, which are daisy-chained to a hybrid. The strips on the n-side side are orthogonal to those on the p-side side. The n-type strips have a pitch of $50 \, \mu m$, and as for the p-side side, only every second strip is physically connected to an amplifier and therefore the readout pitch is 100 microns, for a total of 1280 strips.

The connection between the strips and the readout electronics is made via either a upilex or a glass fanout glued directly to the n-side side of the detector. The p-side side is also called φ-side because its strips measure the FINUDA φ cylindrical coordinate. Similarly, the n-side side strips measure the z-coordinate and it is called the z-side.

Each hybrid is connected to a “repeater” card that tunes the supply voltages for the hybrid and provides buffers for both the incoming hybrid digital control signals and the outgoing analogue readout signal. The repeater card is connected via ten meters long cables to a “decoupler” card, where both analogue and digital signals are sent to VME flash-ADC cards. The digital control signals necessary to readout the hybrids are provided by a VME “sequencer” card, which generates readout control sequences (multiplexing) in response to an external trigger. A VME CPU enables to read ADC’s via a VME DMA bus. Finally, the CPU sends the data to the FINUDA run control FRC and, in turn, to the storage system through an optical fiber device PVIC. The data acquisition system of I/OSIM is depicted in fig. 2.13.

A GEANT Monte Carlo code is used to simulate a module behavior when charged
particles cross it. In order to calculate the charge induced on each strip, the code simulates the drift and lateral diffusion of electrons and holes created. In addition, the code parameters are chosen to explicitly generate delta-rays production. The charge is distributed to preamplifiers by taking into account the different effects of the capacitive charge division for the floating and non-floating strips. The capacitive coupling simulation takes into account the 8 neighbor strips with their coupling constant measured by [26] and [27]. The preamplifiers noise is simulated using a simple gaussian distribution, without taking into account the correlation between adjacent channels.

By simulating the energy deposition of ionizing particles in a module-like medium,
it is possible to directly study the discrimination between $\pi^-$ and $p$ (from non mesonic decay) and $k$. Fig. 2.14 shows the results of the PID simulations. Between the proton and kaon distributions the separation is so large that misidentifications are found to be negligible. Between the pion and proton distributions there is some overlap due to the Landau tail of the pion energy distribution. Fig. 2.15 shows the particle identification reached by combining the intrinsic PID of $I/O$SIM with the particle momentum determined by the FINUDA tracking system. The diffusion plot further indicates that the particles involved in the hypernuclear formation and decay can selectively be separated.

Figure 2.14: Simulated charge distributions for $\pi$, $p$ and $k$ involved in the hypernuclear reaction. Particles are requested to traverse a module-like medium.

<table>
<thead>
<tr>
<th>Particle Type</th>
<th>RMS Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pions</td>
<td>30.1</td>
</tr>
<tr>
<td>Protons</td>
<td>66.0</td>
</tr>
<tr>
<td>Kaons</td>
<td>73.4</td>
</tr>
</tbody>
</table>
2.4. THE FINUDA SILICON VERTEX DETECTOR.

Figure 2.15: Simulated particle energy loss in I/OSIM versus particle momenta.
Chapter 3

Alignment of the FINUDA vertex detector.

The search for $\Sigma$-hypernuclei requires an experimental device capable of high spatial resolution. In FINUDA, this task is accomplished by the silicon vertex detector, which is placed close to the $e^+e^-$ interaction point. The vertex detector measures the spatial coordinates of each charged particle traversing it, both in the plane orthogonal to the beam direction and along the beam direction. However, the high intrinsic resolution (about few tens of microns at FWHM) is depleted if the location of the detector is poorly known with respect to the other tracking devices of FINUDA. This chapter deals with the alignment strategy adopted for the vertex detector of FINUDA (also called I/OSIM).

3.1 The alignment of I/OSIM.

Any deformation of the mechanical structure of the I/OSIM modules or their mislocation from the nominal position implies a depletion of the particle po-
sition determination. Therefore, mapping and recovering the position of the vertex detector is of central importance to preserve the intrinsic spatial resolution of the detector during the long period of data collection.

In order to pursue this goal a procedure has been worked out, which distinguishes two different aspects:

- **Mapping of I/OSIM.** A full mapping of the surface is initially done, by a 3D metrological machine, which measures the supporting structure of I/OSIM. Then, it is looked for any disagreement between the nominal and actual design.

- **Alignment of I/OSIM.** After the insertion of I/OSIM into the supporting structure of the FINUDA spectrometer, the localization of I/OSIM and the monitoring of its position is performed by using a statistical method based on particle tracks.

The alignment procedure must be flexible to account for a wide range of possible occurrences, which arise during the elapsed time of the measurement. These are variations of the environmental temperature and mechanical deformations of the supporting structure, all of them displace the modules of I/OSIM. A method based on statistics was developed to monitor configuration changes and to recover the actual position of the detector. The method relies on the reconstructed tracks of detected charged particles. When a trajectory is reconstructed, it is extrapolated to its reference surface. The residual between the extrapolated point and the reconstructed point has a functional dependence on both the geometrical misalignment of the module and the track parameters. An analysis of the residual distribution allows the actual module position to be obtained with an accuracy that is only limited by the available statistics and the detector intrinsic resolution.
3.2 **The alignment method.**

In 3D track fitting, the vector defined by the closest distance between the module measured position and the fitted track is called 3D residual vector [28], see fig. 3.1. The 3D residual vector does not usually lie in the plane of the module.

![Figure 3.1](image)

**Figure 3.1:** Definition of the 3D residual vector \( \vec{r}, \vec{r} = \vec{t} - \vec{s} \). The vector \( \vec{t} \) points to the track, while the vector \( \vec{s} \) identifies the measured point on the module. The vector \( \vec{r} \) satisfies the minimum distance requirement.

The 3D residual vector can further be related to the error matrix, which contains information on both the precision of the measurement and of the fitted track position. The advantage of using residual vectors with error matrices is that they contain all the information needed for a 3D alignment.

A 2D residual vector method could not be applied to align the I/OSIM modules, since a 2D approach is blind to displacements along the track direction. Along this direction, the modules suffer of both sagitta and distortions. The former is caused by the slim profile of the modules (0.3 \( mm \) thick \( \times \) 196.2 \( mm \) long),
while the lightness of the module supporting structure (designed to maximize its transparency to particles) causes distortions on the modules. Both defects must constantly be monitored and corrected. A full treatment is given in Appendix A.

Let \( \vec{r} \) be the residual vector, as displayed in fig. 3.1. The vector \( \vec{r} \) is chosen so that \( \vec{r} = \vec{t} - \vec{s} \), which is the vector along the closest distance of approach between the module measured position and the fitted track. The vector \( \vec{t} \) points to the track, while vector \( \vec{s} \) identifies the measured point on the module. In virtue of this definition, the vector \( \vec{r} \) satisfies the minimum distance requirement. The uncertainties in the measured and fitted track points determine the overall uncertainty of \( \vec{r} \). The sign convention is that residual vectors point from measured points to fitted tracks, so that residual vectors and their positions on the detector indicate the magnitude and direction of the steps needed to align the modules.

The uncertainties of the three components of \( \vec{r} \) are given in the error matrix \( (E) \), whose diagonal elements are the squares of the uncertainties in \( r_x, r_y \) and \( r_z \), while the off-diagonal elements are the correlated uncertainties. For a given residual vector \( \vec{r}_i \) and \( E_i \), where \( i \) is related to the track number, the \( \chi^2 \) is defined as:

\[
\chi^2 = \sum_{i=0}^{n} \left( \vec{r}_i^T E^{-1}_i \vec{r}_i \right) = \sum_{i=0}^{n} \vec{r}_i^T E^{-1}_i \vec{r}_i \quad (\vec{r}_i = \vec{t}_i - \vec{s}_i) \tag{3.1}
\]

where “\( i \)” runs over all measured tracks. To obtain the best values of the alignment parameters, the \( \chi^2 \) is minimized with respect to these parameters; that is, the translations and rotations of the detector components.

When a module is rotated by \( R \) and then translated by \( \vec{h} \), the measured positions \( \vec{s}_i \) transform as:

\[
\vec{s}'_i = R \vec{s}_i + \vec{h} \tag{3.2}
\]
3.2. THE ALIGNMENT METHOD.

The fitted track positions \( \bar{r}_i \) remain stationary so that the residuals transform as:

\[
\bar{r}'_i = \bar{r}_i - \bar{s}'_i \\
\bar{r}'_i = \bar{r}_i + \bar{s}_i - R\bar{s}_i - \bar{h} \\
\bar{r}'_i = \bar{r}_i - Q\bar{s}_i - \bar{h} \tag{3.3}
\]

where \( Q = R - 1 \). The vectors \( \bar{r}'_i \) are linear in the values of the \( \bar{h} \) components, since \( \bar{h} = h_i u_i \) and the \( u_i \) are unit vectors along the axes. When using the cartesian coordinates with the angles \( \theta_i, i = 1, 2, 3 \) around the axes, \( R \) is represented by the matrix:

\[
R = \begin{pmatrix}
\cos \theta_2 \cos \theta_3 & -\sin \theta_2 \cos \theta_3 & \sin \theta_2 \\
\cos \theta_3 \sin \theta_1 \sin \theta_2 + \sin \theta_3 \cos \theta_1 & -\sin \theta_3 \sin \theta_1 \sin \theta_2 + \cos \theta_1 \cos \theta_3 & -\sin \theta_1 \cos \theta_2 \\
\sin \theta_3 \sin \theta_1 - \sin \theta_2 \cos \theta_1 \cos \theta_3 & \sin \theta_3 \sin \theta_2 \cos \theta_1 + \sin \theta_1 \cos \theta_3 & \cos \theta_1 \cos \theta_2 \\
\end{pmatrix}
\]

For small rotation around the coordinate axes, \( R \) can be approximated by:

\[
R = \begin{pmatrix}
1 & -\theta_3 & \theta_2 \\
\theta_3 & 1 & -\theta_1 \\
-\theta_2 & \theta_1 & 1 \\
\end{pmatrix} \tag{3.4}
\]

and \( Q \) can be linearized by using the expression \( Q = \theta_j Q_j \), where \( Q_j \) is represented by the set of matrices:

\[
Q_1 = \begin{pmatrix}
0 & 0 & 0 \\
0 & 0 & -1 \\
0 & 1 & 0 \\
\end{pmatrix} \\
Q_2 = \begin{pmatrix}
0 & 0 & 1 \\
0 & 0 & 0 \\
-1 & 0 & 0 \\
\end{pmatrix} \\
Q_3 = \begin{pmatrix}
0 & -1 & 0 \\
1 & 0 & 0 \\
0 & 0 & 0 \\
\end{pmatrix} \tag{3.5}
\]

Since \( \bar{h} = h_j u_j \), \( \bar{r}' \) can be fully linearized:

\[
\bar{r}'_i = \bar{r}_i - \sum_{j=1}^{3} \theta_j Q_j \bar{s}_i - \sum_{j=1}^{3} h_j u_j \tag{3.6}
\]

the rotation \( Q \) and translation \( \bar{h} \) enter the definition of \( \chi^2 \):

\[
\chi^2 = \sum_{i=0}^{n} \bar{r}'_i^T E_i^{-1} \bar{r}'_i \quad (\bar{r}'_i = \bar{r}_i - Q\bar{s}_i - \bar{h}) \tag{3.7}
\]
Finally, the minimization of $\chi^2$ with respect to the $\theta_m$ and $h_n$ parameters leads to the following equations:

$$
\sum_{m=1}^{3} A_{jm} \theta_m + \sum_{n=1}^{3} B_{jn} h_n = c_j \quad j = 1, 3
$$

$$
\sum_{m=1}^{3} D_{jm} \theta_m + \sum_{n=1}^{3} F_{jn} h_n = g_j \quad j = 1, 3
$$

where the constants $A_{jm}, B_{jn}, c_j, D_{jm}, F_{jn}$ and $g_j$ are defined by the equations:

$$
A_{jm} = \sum_{i=0}^{n} \bar{\xi}_i^T Q_j^T E_i^{-1} Q_m \bar{\xi}_i
$$

$$
B_{jn} = \sum_{i=0}^{n} \bar{\mu}_i^T Q_j^T E_i^{-1} \bar{\mu}_n
$$

$$
c_j = \sum_{i=0}^{n} \bar{\xi}_i^T Q_j^T E_i^{-1} \bar{\mu}_i
$$

$$
D_{jm} = \sum_{i=0}^{n} \bar{\mu}_j^T E_i^{-1} Q_m \bar{\xi}_i
$$

$$
F_{jn} = \sum_{i=0}^{n} \bar{\mu}_j^T E_i^{-1} \bar{\mu}_n
$$

$$
g_j = \sum_{i=0}^{n} \bar{\mu}_j^T E_i^{-1} \bar{\mu}_i
$$

Furthermore, these six equations can be written as a single six-dimensional matrix equation:

$$
M \bar{p} = \bar{k}
$$

where the six-by-six matrix $M$, the six-vectors $\bar{p}$ (parameters) and $\bar{k}$ (constants) are defined by:

$$
M = \begin{pmatrix} A & B \\ D & F \end{pmatrix}, \quad \bar{p} = \begin{pmatrix} \bar{\theta} \\ \bar{h} \end{pmatrix}, \quad \bar{k} = \begin{pmatrix} \bar{c} \\ \bar{g} \end{pmatrix}
$$

Finally, the minimum for $\chi^2$ is reached when the roto-translation six-vector $\bar{p}$ is related to $M$ via the equation:

$$
\bar{p} = M^{-1} \bar{k}
$$
3.3. THE ALIGNMENT STRATEGY.

with $M$ and $\bar{k}$ given by the equations (3.11).

The error matrix $V$ of $\bar{p}$ is defined by the equation:

$$V_{jk}^{-1} = \frac{1}{2} \frac{\partial^2 \chi^2}{\partial p_j \partial p_k}$$

(3.13)

which is related to $M$ via the equation $M^{-1} = V$, see demonstration in Appendix B. Therefore, $\bar{p}$ can be written as:

$$\bar{p} = V_{k\bar{k}}$$

(3.14)

which expresses the parameters $\bar{p}$ in terms of their error matrix $V$ times the constant vector $\bar{k}$.

In the present formulation, the detector alignment does not account for the physical detector: the detector elements are rigid plates that can be aligned by rotation and translation. In order to embody non-rigid transformations such as sagitta, distortion and varying strip pitch, the alignment theory must be further developed to include the detector characteristics.

3.3 The alignment strategy.

The alignment of finely-segmented detectors is a complex process when using reconstructed particle tracks. The detector geometry must be accurately known, since the volume parameters describing the detector location enter directly the event reconstruction. These parameters are varied to steadily improve the alignment, thus the parameter best-set must be given by a fast converging analytical method (i.e., the method discussed in Sec. 3.2).

I/OSIM consists of 18 modules, therefore a total of 108 parameters must be determined to have a fully aligned vertex detector. In addition, the I/OSIM modules are locally displaced, which further increases the number of parameters to
be determined. The large number of degrees of freedom called for a sequential and iterative code also capable of monitoring the alignment and diagnosing problems related to parameter convergence. The general approach used for the alignment of I/OSIM can be lately iterated over the other tracking layers of the FINUDA spectrometer. The outward moving of the iteration is dictated by the higher intrinsic resolution of the vertex detector.

The alignment of I/OSIM requires particle tracks crossing four modules (4-hit tracks). Event with 4-hit tracks are provided either by $\Phi$ decays ($\Phi \rightarrow e^+e^-, \mu^+\mu^-, \pi^+\pi^-$), or from Bhabha scattering ($e^+e^- \rightarrow e^+e^-$), or from cosmic rays (mainly $\mu$'s). However, dilepton $\Phi$ decays are rare ($\sim 10^{-4}$), and electrons from Bhabha events undergo multiple scattering, which degrades their position definition. Therefore, both events are useless for alignment. Cosmic rays do not have the same geometrical distribution as central collisions, they are uniformly distributed in position and follow an angular distribution which is peaked toward the vertical direction. Their momentum distribution span over a wide range from 0.1 to 100 GeV/c. In order to minimize the effects of the multiple scattering, the low momentum tail of the distribution is rejected. However, this is only possible with the FINUDA magnetic field on.

In order to achieve an initial first-order alignment, a stack of four modules is aligned with a sequential and iterative code, which is stopped when the requested precision is reached. Then, the rigid body of four modules serves to align an adjacent module. In turn, this module is taken to form a new stack and the next adjacent module is aligned. This procedure is repeated for all modules as far as I/OSIM is fully aligned. Among all possible stacks, the initial stack is selected by the higher statistics of good events crossing it.

If multiple scattering and the second order misalignment can be considered negligible, the error in the alignment of a module ($\sigma_{tot}$) is the sum in quadra-
3.4. **THE COSMIC RAYS TEST.**

The number of tracks to be collected is determined by the condition:

\[
\left( \frac{\sigma_{\text{align}}}{\sigma_{\text{intr}}} \right)^2 \ll 1
\]  

(3.16)

Once a suitable statistics of tracks is reached, the parameters of alignment can be estimated by minimizing the sum in quadrature of the residuals given by eq. (3.12).

### 3.4 The cosmic rays test.

![Figure 3.2: View of the FINUDA vertex detector. The inner layer (ISIM) consists of 8 modules, the outer layer (OSIM) consists of 10 modules.](image)
In order to work out an alignment strategy for the 18 double-sided silicon microstrip detectors and to test their performances, a cosmic rays test was done at Laboratori Nazionali di Frascati (LNF) during 2001. The modules of I/OSIM were supported by the final detector frame. I/OSIM is organized in two layers of 8 (ISIM) and 10 (OSIM) modules, respectively, as shown in fig. 3.2. A plastic scintillator placed along the horizontal plane of the vertex detector delivered a trigger signal when crossed by cosmic rays. I/OSIM worked continuously for almost 3 months, and over a million of “good” events were collected (see fig. 3.3). The mechanical structure was water-cooled and the water temperature was kept constant at $18^\circ \pm 2^\circ C$ during the data taking to ensure a good mechanical stability of the vertex detector. A temperature change of $1^\circ C$ makes the vertex detector expand by 5 $\mu m$.

3.4.1 Charge analysis

An initial check on the I/OSIM modules is performed by studying the distribution of the charge released by a minimum ionizing particle. When a charged particle passes through a module, the charge generated by ionization is collected by one or more adjacent strips on both sides of the module. The raw ADC reading includes the charge released by a traversing particle, a common baseline for the VA1 128 channels (common-mode noise), and the strip standard noise. Both the strip common-mode and standard noise are initially subtracted to the overall collected charge. A cluster charge, which sums up the charge collected by a cluster of strips, is analyzed when passing a double threshold algorithm. The algorithm was steadily improved by minimizing the random electronic noise and optimizing the pattern recognition of signals. Fig. 3.4 shows the charge distribution of a module. It follows a Landau distribution with a
3.4. THE COSMIC RAYS TEST.

Figure 3.3: Strip occupancy of a module for about 200000 cosmic rays. The shape along the two axes is due to the angular dependence of cosmic rays and the solid angle subtended by I/OSIM.

mean value of $\sim 33$ ($\sim 35$) ADC channels on p-side (n-side). The mean value of charge distribution divided by mean noise is commonly taken as a definition of the module signal-to-noise ratio. For the module under consideration, the signal-to-noise ratio is $\sim 20$ for both sides. The module is a good module since it satisfies the required prescriptions: 1) the charge distribution must follow a Landau law, and 2) the signal/noise $\geq 15$. When 2 or more particles hit a single module, they may be mislocated. To solve or to reduce multi-hits ambiguities, it is requested that the charge produced into a module by single-particle events is the same on the p- and n-side. Fig. 3.5 shows the linear correlation existing between the two sides of a module for single-particle events. The band width reflects the resolution of the detector and the strip noise.

At the end of preliminary analysis, the particle impact position is calculated by
Figure 3.4: Charge distributions of the p- and n-side of a module due to cosmic rays (gray filled diagram). The distributions are fitted to a Landau law (red line).

means of the center-of-gravity weighted for the overall cluster charge on both sides of the silicon module. The charge sharing between two strips is a non-linear function of the distance between the impact point and a strip. This is measured by the $\eta$ function $\eta = \frac{Q_r}{Q_r + Q_l}$, where $Q_r$ ($Q_l$) is the charge signal on the right-(left-)hand side of a strip. The knowledge of $\eta$ as function of the impact point position is used to improve the algorithm of particle position reconstruction.

The physics of FINUDA involves low and medium energy particles emitted at all angles. For this scenario, the flexibility of the center-of-gravity approach is preferred over the precision offered by the $\eta$ function method [29]. Instead for cosmic rays, the $\eta$ algorithm is more appropriate, since the mean number of strips involved in cluster formation is equal to two, as shown in fig. 3.6.
3.4. THE COSMIC RAYS TEST.

Figure 3.5: Cluster charge scatter plot of strips at work for single-particle events.

3.4.2 First order alignment

The initial setup of the I/OSIM modules is known with a precision of $\approx 200 \mu m$, which is the mechanical tolerance of the module holding devices. Therefore, modules need a finer alignment in order to aim at their intrinsic resolution $\sim 20 \mu m$. To obtain a first order alignment of I/OSIM, the residual method discussed in Sec. 3.2 is employed. It is repeatedly iterated over the 18 modules by following the approach described below.

1. An initial stack of 4 modules is selected; for instance 12/3/7/17 (see fig. 3.7). The procedure of alignment begins for the module 12 with respect to the stack 3/7/17. When the convergence of alignment is reached, the iteration stops. Fig. 3.8 shows the results of the first iteration: the convergence for all the parameters (i.e., 6 for each module) is reached but $\bar{p}$ is
large\(^1\). The same procedure is carried on for the remaining stack modules.

2. The procedure described in 1. is iterated. Fig. 3.9 shows the results of iteration 2: \(\vec{p}\) has improved but it is still large.

3. After about 130 iterations of point 2., the parameters still converge and \(\vec{p} \simeq 0\), as reported in fig. 3.10. The stack 12/3/7/17 is aligned to the first order.

4. The stack now selected is 11/3/7/17, and the module 11 is aligned with respect the reference stack 3/7/17. The procedure is then iterated over all the modules of I/OSIM.

\(^1\)The vector \(\vec{p}\) is defined by eq. (3.11) as a general six-vector of the alignment parameters, but it is also possibile to redefine \(\vec{p}\) as \((\Delta \alpha, \Delta \beta, \Delta \gamma, \Delta r, \Delta \phi, \Delta z)\) by using the local reference system shown in fig. 3.7.
3.4. THE COSMIC RAYS TEST.

In order to check the correctness of the I/OSIM alignment, the residual distribution of reconstructed cosmic rays is studied for a module. If the distributions along $\phi$ and $z$ denote a $\sigma$-value comparable to the module intrinsic resolution (see fig. 3.11), the first order I/OSIM alignment is achieved.

Fig. 3.12 depicts the behavior of the $\phi$-residual mean-value ($\sigma$-residual) along $\phi$, left panel (right panel). Fig. 3.13 shows the same observables but along $z$. The substantial flatness of spectra ensures that the modules 12,3,7 and 17 do not suffer of misalignments. However, there are fluctuations around the average value of distributions, which exceed the data error bars. This indicates that higher order corrections are still needed.

Figure 3.7: Layout of the FINUDA central region. The stack of 4 modules used in the initial analysis is colored in red.
3.4.3 Multiple scattering.

The quality of particle tracks is of prime importance to achieve a good alignment. The low-energy component of cosmic rays $< 1 \text{GeV}$ is strongly affected by multiple scattering, therefore it is not suitable for position calibration. Since a magnetic field was not available to reject low-energy cosmic rays, a sub-group of prime tracks was selected by applying a cut to $\chi^2$.

It is worthwhile reminding that the track fitting is done by using 3 out of 4 modules, and the residual vector is calculated on the fourth module. With this ap-
3.4. THE COSMIC RAYS TEST.

Figure 3.9: Same caption as in fig. 3.8. Second step of iteration. $\Delta \alpha \approx -0.2 \text{ mrad}, \Delta \beta \approx -0.2 \text{ mrad}, \Delta \gamma \approx -0.1 \text{ mrad}, \Delta r \approx 105 \mu\text{m}, \Delta \phi \approx 2.6 \mu\text{m} \text{ and } \Delta z \approx -1.4 \mu\text{m}$.

...approach, the residual vector and the alignment of the fourth module are not affected by the $\chi^2$ cut. Fig. 3.14 and 3.15 display the results of simulations, which were made to evaluate the effect of multiple scattering on residuals. The GEANT Monte Carlo was employed to this purpose. In addition, the Molière theory is chosen to evaluate the polar angle deflection. The detector intrinsic resolution is simulated by Gaussian distribution, which is further smeared out to account for the diffusion of holes and electrons, and the capacitive coupling. The capacitive coupling simulation takes into account the 8 neighbour strips with their coupling constant measured by [26] and [27]. The delta-rays production and
Figure 3.10: Final step of iteration, the module is aligned. All the alignment parameters are next to 0. Ndf denotes the degrees-of-freedom, which (nearly) equals the number of reconstructed events.

pair conversion in the silicon microstrip detector are also taken into account. The I/OSIM simulation regards only the 18 modules geometrically organized as shown in fig. 3.2.

### 3.4.4 Second order alignment: sagitta correction

As earlier explained, the I/OSIM modules are only constrained at their extremities in order to have the higher clearance possible to particles. This sort of mechanical bound makes raise a noticeable sagitta in the horizontal modules of
3.4. THE COSMIC RAYS TEST.

Figure 3.11: Left panel, the distribution of residuals along $\phi$, and the distribution of residuals along $z$ (right panel).

Figure 3.12: Left panel, the mean value distribution of residuals along $\phi$ and right panel, the sigma distribution of residuals along $\phi$.

Figure 3.13: Same caption as in fig. 3.12, but the residuals are taken along $Z$. 
Figure 3.14: Results of the cosmic rays simulations. From left to right: a) residual without multiple scattering along $\phi$, b) residual with multiple scattering along $\phi$, c) residual with multiple scattering along $\phi$, but selecting the $\chi^2$ track fitting.

Figure 3.15: Same caption as fig. 3.14 but along z.

I/OSIM. If the module weight is assumed to be continuously distributed along the module length, it is possible to calculate the maximum vertical displacement, also called sagitta ($\Delta_{sag}$):

$$\Delta_{sag} = \frac{1}{384} \frac{WL^4}{EI}$$  \hspace{1cm} (3.17)

where $\Delta_{sag}$ is the maximum vertical displacement in meters, $W$ is the load expressed in Newton per meter, $L$ is the total length in meters, $E$ is the modulus of elasticity of the material in Pascal, and $I$ is the momentum of inertia in $m^4$. For a horizontal module the sagitta is $\sim 100 \mu m$, thus requiring the correction for deflection. A method was developed to correct such an effect; it consists of dividing the module into four parts along the length ($z$ axis) and calculating the roto-translation parameters for each part. The value of radial parameters determines the sagitta, the other parameters are used only to check the consistence among adjacent parts of the same module.
3.4. THE COSMIC RAYS TEST.

Figure 3.16: Measure and correction of sagitta. The continuous line in the low-right panel represents the final profile of the sagitta-free module.

The radial points were fitted along the z axis by means of a catenary function with two additional parameters in order to allow the curve to translate along the radial and z directions:

\[ r = p_0 \cosh \frac{z - p_1}{p_0} + p_2 \]  

(3.18)

As in the first order corrections, the sagitta calculation needs to be iterated few times before straightening up the module, see fig. 3.16 from step 1 to step 4.

Fig. 3.17 reports the mean-values of the residual vector as a function of the z-coordinate. The comparison of the two figures renders the sagging effect visible, which appears to be remarkably large at the extremities of a horizontal module (left panel). When the correction is applied the sagging is reduced to \( \pm 10 \mu m \).
Figure 3.17: On the left panel it is reported the mean-value distribution of the residuals along z. The presence of a deflection disappears after the sagitta correction (right panel). The effect of corrections is significant along the z axis 6 \( \mu m \) (see right panel). The distribution after the correction; sigma becomes 6 \( \mu m \) smaller.

Figure 3.18: Left panel, the residual distribution along z before sagitta correction. Right panel, the distribution after the correction; sigma becomes 6 \( \mu m \) smaller.

### 3.4.5 Spatial resolution.

The expected resolutions for a detector with a strip distance (pitch) of 50 \( \mu m \) on p-side and 100 \( \mu m \) on n-side are:

\[
\sigma_{int} = \frac{\text{pitch}}{\sqrt{12}} \Rightarrow \sigma_{int_\phi} \approx 15 \mu m \quad \sigma_{int_z} \approx 30 \mu m
\]  

(3.19)
3.4. THE COSMIC RAYS TEST.

These must be compared with the overall resolutions of I/OSIM which are $\sigma_{\text{tot,} \phi} \approx 22 \, \mu m$ and $\sigma_{\text{tot,} z} \approx 34 \, \mu m$ (see figs. 3.18 and 3.19, right panel). They are somewhat higher than those expected, because the residuals are the superimposition of the module intrinsic resolution $\sigma_{\text{int}}$, the systematic uncertainty $\sigma_{\text{ms}}$ (from multiple scattering), and the module torsions.

By means of the simulation (in Sec. 3.4.3, figs. 3.14 and 3.15) it is possible to evaluate the multiple scattering contributions for the two coordinates:

$$
\sigma_{\text{ms}} \approx \sqrt{\sigma_{\text{tot,} \phi}^2 - \sigma_{\text{int}}^2} \Rightarrow \sigma_{\text{ms,} \phi} \approx 14 \, \mu m \quad \sigma_{\text{ms,} z} \approx 23 \, \mu m
$$

Finally, the intrinsic resolutions are $\sigma_{\text{int,} \phi} \approx 18 \, \mu m, \sigma_{\text{int,} z} \approx 25 \, \mu m$, which are comparable with the expected resolution (i.e, eq. (3.19)).

### 3.4.6 The assessed time scale.

The fully alignment of I/OSIM requires about 20000 tracks. This fixes the time scale of cosmic rays data taking to 2 weeks. When I/OSIM is seated in FINUDA a complication arises, which is due to the presence of the beam-pipe, the TOFINO scintillator slabs and the targets. However, it is possible to account for these masses by simulating the effect of multiple scattering for them. Fig. 3.20 shows
that the multiple scattering degrades the resolution on the p-side from 9 \( \mu m \) to 25 \( \mu m \), and on the n-side from 24 \( \mu m \) to 41 \( \mu m \). This calls for high momentum particles. Fortunately, the magnetic field of FINUDA permits to select the momentum of cosmic rays. For instance, the choice nearly restores the original resolution, but doubles the period of data taking, i.e. \( \sim 4 \) weeks. A compromise between the overall I/OSIM spatial resolution and the length of cosmic rays data collection is therefore required.

![Figure 3.20: The distribution of residuals along \( \phi \) and \( z \) given by a cosmic ray simulation with the FINUDA final setup.](image)
Chapter 4

Identifying $\Sigma$-hypernuclei with FINUDA.

A tagging method to suppress the $\pi^{\pm}$ background affecting the $\Sigma$-hypernuclear spectroscopy is proposed. The method, which is based on Monte Carlo simulations, investigates the capabilities of $\Sigma$ to form a hyperon-nucleus bound system. The hypernuclear system can be found either in a ground state or in a low-lying state as well as in a resonance state. The tracking capabilities of the FINUDA spectrometer to detect $\Sigma$-hypernuclei are used.

4.1 $\Sigma$-hypernuclei in FINUDA.

The FINUDA spectrometer [31] is designed for systematic studies of the $\Lambda$-hypernuclei production and decay with high momentum resolution and large detection acceptance. As described in Sec. 2.3, the $\Lambda$-hypernucleus studies with FINUDA are done with $B = 1.1\ T$. Negative kaons are brought at rest in the target to be studied. They will then be caught by an external atomic orbital and finally fall into a nucleus. Kaons will interact with a nucleon and form a hyperon
either Λ or Σ, which will be followed by a prompt emission of a pion \( (\pi^\pm_{prompt}) \).

When a Σ-hypernuclear bound state is formed \( (\Sigma; A) \) then the following reaction occurs:

\[
k_{\text{stop}}^- + A \rightarrow \Sigma \ A + \pi^\pm_{prompt}
\]  

(4.1)

In a nucleus the Σ-hyperon interacts strongly with a nucleon via a process that is customarily called \textit{conversion reaction}:

\[
\Sigma + N \rightarrow \Lambda + N
\]  

(4.2)

The Σ-hyperon acquires energy due to the Σ - Λ mass difference, which allows Λ to move out from the nucleus and to decay into the free space. A simulated Σ-hypernuclear event in FINUDA is displayed in fig. 4.1. Two vertices can be distinguished: the first describes the topology of the \( ^{12}_\Sigma \) \( Be \) production and decay, while the second shows the Λ decay following the \( \Sigma \rightarrow \Lambda \) conversion reaction. The two different vertices can be separated from one to another, since the vertex detector is capable of high spatial resolution \( (\Delta r < 35 \mu m \text{ at FWHM}) \), thus the Σ-hypernucleus is unambiguously identified.

The topology of the Σ-hypernucleus production and decay indicates a possible strategy for tagging Σ events, which is simply based on the detection of prompt pions in coincidence with \( \pi^- \ p \) pairs. In order to make sure that a \( \pi^- \ p \) pair originates from the free-Λ decay, the \( \pi^- \ p \) invariant mass must be reconstructed when the separation between two vertices exceeds \( \Delta r \).
4.2 Monte Carlo simulations of $\Sigma$-hypernuclei.

The performances of FINUDA are initially tested with respect to $^{12}C$ by considering the following reactions:

\[
\begin{align*}
\text{k}_\text{stop} + ^{12}C &\rightarrow \\
&\left\{ \\
^{12}_{\Sigma^-}Be + \pi^{+}_{\text{prompt}} &\rightarrow ^{12}_{\Sigma^-}Be \rightarrow ^{10}Be + \Lambda + n \\
^{12}_{\Sigma^+}C + \pi^{-}_{\text{prompt}} &\rightarrow ^{12}_{\Sigma^+}C \rightarrow ^{10}B + \Lambda + p \\
^{12}_{\Sigma^0}C + \pi^{-}_{\text{prompt}} &\rightarrow ^{12}_{\Sigma^0}C \rightarrow ^{10}C + \Lambda + n \\
&\rightarrow ^{10}B + \Lambda + p
\end{align*}
\]
In the simulations the binding energies and widths of the $\Sigma$-hyperfine states are taken from ref. [17], and reported in table 1.1 of Sec. 1.4. Bound nucleons have the Fermi momentum distribution with $k_F$ of 221 MeV/c [33]. The $\Sigma$ momentum ($k$) distribution corresponds to the probability density function $(k^2 \Psi(k)^2)$ [32] where $\Psi(k)$ is the 1s (and 1p) wave function in momentum space. The wave functions are obtained by solving the Schröedinger equation with a Wood-Saxon potential and a Coulomb term, and are depicted in fig. 4.2.

The center-of-mass energy of final state particles is checked to be larger than the sum of final state particles masses, otherwise the event is rejected. The final state particles are tested in order to assure that the final distribution in phase space is in agreement with the Pauli principle, which rules out the possibility of finding more than one fermion in a single quantum state. In a Fermi-gas model, all states below the Fermi momentum are already occupied, therefore all secondary nucleons must be above $k_F$. If this does not occur, the interaction is suppressed.

The kinematics of final state particles feeds the GEANT3 transport particle
Figure 4.3: Some results of simulations for the $k_{sl}^{+\pi} + {^{12}C} \rightarrow ^{10}_{\Sigma^-} Be + \pi^+_{\text{prompt}}$ reaction: momentum distributions of positive prompt pions (left), and range ($c\tau$) distributions of free $\Lambda$'s (right).

Figure 4.4: Results of simulations for $^{12}_{\Sigma^-} Be$: momentum distributions of negative pions (left) and protons (right) from $\Lambda \rightarrow \pi^- p$ decay.

framework. The geometrical structure and the different layers of FINUDA are embedded in the GEANT3 Monte Carlo, which also accounts for the energy loss of particles crossing the layers of FINUDA. Therefore, the particle trajectories in FINUDA are properly reconstructed. Results of these simulations are displayed in figs. 4.3, 4.4 and 4.5 for the $\Sigma$-hypernucleus production reaction and the sub-
identifying $\Sigma$-hypernuclei with FINUDA.

Figure 4.5: Further results of simulations: angular distributions between negative pion and proton coming from $\Lambda \rightarrow \pi^- p$ decay (left), the $\Lambda$-decay path distributions (right).

sequent decay products.

\[
k_{\text{stop}}^- + ^{12}\text{C} \rightarrow \frac{12}{\Sigma^-} \text{Be} + \pi^+_{\text{prompt}}
\]

\[
\frac{12}{\Sigma^-} \text{Be} \rightarrow ^{10} \text{Be} + \Lambda + n
\]

(4.4)

$\Lambda \rightarrow p + \pi^-$

The present analysis emphasizes that: a) the accuracy in the vertex reconstruction for stopped kaons $\sim 0.6\, mm$ [21] [24] is $20 \div 30$ times better than the mean value of the $\Lambda$ range ($c\tau$) distribution $16\, mm$; and b) the particle momenta from $\Lambda$ decay can be analyzed, although with lower efficiency, by the FINUDA spectrometer even with a magnetic field set at $B = 1.1\, T$. A lower magnetic field $B \sim 0.6\, T$ would be preferable.

4.3 Monte Carlo simulations of background reactions.

The analysis continues considering all the reactions that do not form hypernuclei but involve $k_{\text{stop}}^-$'s and $N$'s such as to form $\pi^\pm$ in the final state. Such reactions
4.3. MONTE CARLO SIMULATIONS OF BACKGROUND REACTIONS.

and their branching ratios in $^{12}$C ([9] [34] [35] [36]) are listed in the following$^1$:

- Production of quasi-free $\Lambda$:

  \[ k^- + p \rightarrow \Lambda + \pi^0 \quad (4\%) \quad (4.5) \]

  \[ \Lambda \rightarrow p + \pi^- \quad (2.5\%) \]

  \[ k^- + n \rightarrow \Lambda + \pi^- \quad (7\%) \quad (4.6) \]

  \[ \Lambda \rightarrow p + \pi^- \quad (4.5\%) \]

- Production of quasi-free $\Sigma^+$ and $\Sigma^0$ with reaction involving two bodies:

  \[ k^- + p \rightarrow \Sigma^+ + \pi^- \quad (31\%) \quad (4.7) \]

  \[ \Sigma^+ \rightarrow n + \pi^+ \quad (14.9\%) \]

  \[ k^- + p \rightarrow \Sigma^0 + \pi^0 \quad (21\%) \quad (4.8) \]

  \[ \Sigma^0 \rightarrow \Lambda + \gamma \]

  \[ \Lambda \rightarrow p + \pi^- \quad (13.4\%) \]

  \[ k^- + n \rightarrow \Sigma^0 + \pi^- \quad (3\%) \quad (4.9) \]

  \[ \Sigma^0 \rightarrow \Lambda + \gamma \]

  \[ \Lambda \rightarrow p + \pi^- \quad (1.9\%) \]

- Production of quasi-free $\Sigma^-$ with reaction involving two bodies:

  \[ k^- + p \rightarrow \Sigma^- + \pi^+ \quad (14\%) \quad (4.10) \]

  \[ \Sigma^- \rightarrow n + \pi^- \quad (\sim 14\%) \]

  \[ k^- + n \rightarrow \Sigma^- + \pi^0 \quad (3\%) \quad (4.11) \]

  \[ \Sigma^- \rightarrow n + \pi^- \quad (\sim 3\%) \]

- Production of quasi-free $\Sigma^- \Sigma^+ \Sigma^0$ with a reaction involving three bodies:

  \[ k^- + (NN) \rightarrow \Sigma^- + N \quad (3.4\%) \quad (4.12) \]

  \[ \Sigma^- \rightarrow n + \pi^- \quad (\sim 3.4\%) \]

$^1$only decays producing a $\pi^\pm$ have been listed as background reactions
$k^-(NN) \rightarrow \Sigma^+ + N \quad (5.7\%) \quad (4.13)$
$\Sigma^+ \rightarrow n + \pi^+ \quad (2.7\%)$

$k^-(NN) \rightarrow \Sigma^0 + N \quad (7.9\%)$  \hspace{1cm} (4.14)
$\Sigma^0 \rightarrow \Lambda + \gamma$  \hspace{1cm}$\Lambda \rightarrow p + \pi^- \quad (5\%)$

- $\alpha$-cluster absorption and $^4\Lambda H$ mesonic decay (hyperfragment production) [37]:

$k^- + ^4He \rightarrow ^4\Lambda H + \pi^0 \quad (1.0\%)$ \hspace{1cm} (4.15)
$^4\Lambda H \rightarrow ^4He + \pi^- \quad (0.5\%)$

In order to analyze the contribution of all these reactions to the final $\pi^\pm$ momentum spectra, main reactions and secondary decays were accounted for phase space simulations. Both $k^-$ and $^{12}C$ have been considered at rest in the laboratory system; the center-of-mass system and the laboratory system coincide, and the total energy available for the system sums up the masses of $k^-$ and $^{12}C$. For each reaction, the residual nucleus is also considered, which enters the calculation of final momenta with the following scheme: $^{11}C$ for reactions involving a neutron, $^{11}B$ for reactions involving a proton, $^{10}B$ for reactions involving two nucleons. In the case of quasi-free reactions, the recoil momentum of the residual nucleus equals the nucleon momentum.

The results of Monte Carlo simulations of background reactions are shown in fig. 4.6 for negative pions and in fig. 4.7 for positive pions. The contribution of reaction (4.15) is not considered since it is negligible. The background in the $\pi^-$ spectrum consists in three components. The lower momentum regions is dominated by $\pi^-$’s from the decay of quasi-free $\Lambda$’s. The middle region is filled by pions associated to the production of sigmas ($\Sigma^+$ and $\Sigma^0$). The higher momentum region is formed by pions from the quasi-free production of lambdas and by the
4.3. MONTE CARLO SIMULATIONS OF BACKGROUND REACTIONS.

Figure 4.6: Monte Carlo simulation for $\pi^-$ background spectra after $k^-$ absorption in $^{12}C$. The arrows indicate the momentum range of the prompt pion of the $k_{n,\text{stop}}^-$ $^{12}C \rightarrow ^{12}_{\Sigma^+,\Lambda} C + \pi^-$ reaction.

in-flight decay of negative sigmas. When a $\Sigma^-$ is produced via a $k^-$ + $(NN)$ absorption, the $\pi^-$ momenta extend broadly above the threshold of $\Lambda$, that is, the region of $\Lambda$-hypernucleus production.

The interpretation of the $\pi^+$ spectrum is simpler, and consists in two components. Pions deriving from quasi-free production of $\Sigma^-$ populate the lower momenta region. The high-momentum region is due to pions from $\Sigma^+$ decays. Finally, the decay of $\Sigma^+$ from $k^-$ + $(NN)$ absorption leads to a broad momentum distribution.

The ability of FINUDA to closely follow a hypernuclear event is used to separate the signal of hypernuclear formation, from the overwhelming background signals, which are due to $\Sigma$ and $\Lambda$ quasi-free production and decay. A test on the
Σ-hypernuclear topology is required to free the Σ-hypernuclear π-signal from competitive background signals. To clarify this point, let us consider the reaction $k^- + ^{12}\text{C} \rightarrow ^{12}\text{Be} + \Sigma^- \rightarrow \Lambda + n$ and the free Λ-decay $\Lambda \rightarrow \pi^- p$. A free Λ can selectively be identified by 1) observing a common reaction vertex for the $\pi^- p$ pair nearby the target layer, and 2) examining the $\pi^- p$ invariant mass. The further request for the event to have a + - + polarity (i.e., $\Sigma^+ \rightarrow \pi^+ p$) ensures uniqueness to the Σ-event. Finally, the same consideration applies to $\Sigma^+$ and $\Sigma^0$-hypernuclei. The event-polarity for these hypernuclei is - - -, which competes with the polarity of reactions (4.6) and (4.9); thus impairing the uniqueness test.

Another source of background is the rescattering of outgoing pions; in fact,
these mesons ($\pi^{0,+,\cdot\cdot\cdot}$) may undergo secondary interactions with the nucleons of the residual nucleus. For instance, a $\pi^0$ from quasi-free $\Lambda$ production may be rescattered and produce a $\pi^+$ [38]:

$$k^- + p \rightarrow \pi^0 + \Lambda$$

$$\pi^0 + p \rightarrow \pi^+ + n$$  \hspace{1cm} (4.16)

In this case, the final state particles are $\pi^+ p \pi^-$ (the $p \pi^-$ pair is from $\Lambda$ decay), which may contaminate the $\Sigma^+$ identification. The rescattering (4.16) occurs in p-wave, while the reaction conversion of $\Sigma^-$ hypernucleus occurs in s-wave, which indicates a way to discriminate background reactions from $\Sigma$ hypernuclear events: precisely using angular correlation among $\pi^0$, $\pi^+$ and $n$. Such an approach to the discrimination goes to the expenses of statistics since it involves the detection of neutrons, and the efficiency of FINUDA to neutron detection is about 15%. When the $\pi^0$ rescattering cannot be ignored the angular correlation studies can be performed by using the tracking capabilities of FINUDA: the condition $\cos \theta \sim \pm \pi/2$, where $\theta$ is the scattering angle, reduces the contribution of the rescattering reaction (4.16).

### 4.4 The FINUDA trigger and magnetic field.

A charged particle impinging upon TOFONE in coincidence with a $k^+k^-$ pair triggers FINUDA. The threshold momentum for such a particle $\sim$ 200 $MeV/c$ is determined by both the magnetic field and the geometry of the spectrometer, which are 1.1 $T$ and 127 cm in radius, respectively. With such a field only a fraction of $\Sigma$-hypernuclei can be examined; in fact, protons from $\Lambda$-decay stop into
Figure 4.8: A $^{12}_{\Sigma}Be$ event in FINUDA at $1.1\,T$. The trigger in the outer scintillator is given by positive muon which follows the $k^+$ decay.

the FINUDA skin$^2$, and negative pions are not energetic enough to reach the outer scintillator. Only positive muons from $k^+$ decays are capable of triggering FINUDA, a full event is depicted in fig. 4.8. The trigger rate derived by simulations is $14\%$ per $(k^+k^-)$ pair stopped. In addition, events triggered by muons may not be related to $\Sigma$-hypernucleus formation, which further contributes to lower the trigger rate related to $\Sigma$-hypernuclei. In order to increase the triggering efficiency, the FINUDA magnetic field must be lowered to at least $0.6\,T$. At

$^2$the FINUDA skin is a cylinder of aluminum $8\,mm$ thick, which ensures further mechanical stability to the holding frame of FINUDA (see fig. 4.9).
4.4. THE FINUDA TRIGGER AND MAGNETIC FIELD.

Figure 4.9: A \( ^{12}_{\Lambda} Be \) event in FINUDA for the magnetic field set at 0.6 \( T \). TOFONE is triggered by a prompt pion. The proton and pion from \( \Lambda \)-decay (\( \Lambda \rightarrow \pi^- p \)) cross almost the entire tracking volume.

At this field, a \( (k^+, k^-) \times \pi^\text{prompt} \) trigger identifies \( ^{12}_{\Lambda} Be \) with a percentage of \( \sim 50\% \). At this field, both the event tagging and vertex fitting remarkably improve, see for example fig. 4.9. The proton and the pion following the \( \Lambda \) decay traverse almost the entire volume of FINUDA, thus allowing an unambiguous identification and reconstruction.
4.5 The reconstruction of $\Sigma$-hypernuclei events.

The analysis of an event, which starts from the decoding of detector signals and ends by identifying a $\Sigma^-$-hypernucleus, is organized in three steps:

1. cluster finding in each sub-detectors of the FINUDA spectrometer;

2. track finding; and

3. track fitting.

The cluster finding aims at defining one or more points on the sub-detectors crossed by one or more particles, and determining the energy loss by the same particles. The points, called hits, are grouped such as to represent a track. This step of analysis is called track finding or pattern recognition, and the group of point is called set. By means of a set fitting, it is possible to calculate the particle momentum at any point of its trajectory that is a helix, and at the point of interaction, called vertex. The pattern recognition of FINUDA aims at the identification of useful events within the events selected by the trigger. This is given by a stop $k^-$ followed by a prompt $\pi^\pm$, which crosses at least three layers of the spectrometer. For hypernuclear events the pattern recognition follows two steps:

1. identification of a $k^+ k^-$ pair which crosses the interaction region and releases hits in two opposite TOFINO slabs and in the related ISIM modules. The reconstruction of the $k^+ k^-$ trajectories permits to estimate their stopping points;

2. identification of the helical trajectories, which starts from one hit in OSIM and move outward through the detector layers. When the helical trajectories correspond to physical tracks, they are connected to the points where
the $k^+$ and $k^-$ have come to rest. For backward tracks that cross the interaction region before entering into the spectrometer, the additional hits in the internal detectors are placed near the helical trajectory.

The low energy of stopping kaons leads to relevant multiple scattering effects. Due to the short path from the ISIM detector to the target, such an effect is moderate, and the $k^-$ stop point can be determined with a spatial resolution of about $600 \mu m$ at FWHM along both the target length and width, and $250 \mu m$ at FWHM along the target depth [24]. In addition, by using the information of the kaon stop point and the extrapolation of the $\pi^\pm$ trajectories at the target surface, the measured momenta of pions can be corrected for the energy released in the target medium.

As far as the track fitting procedure is concerned, two main algorithms were developed:

1. a numerical integration of the track trajectory based on the Runge-Kutta method, which is implemented in the reconstruction software;

2. an iterative method of track fitting based on the Wind algorithm, which couples the minimization of the stereo tubes residuals to the inner tracking volume.

The first method exploits the information from each detector and accounts for the energy loss along the trajectory, but it is unstable for an uncertain initial hit of the fitting procedure and, furthermore, it needs more computing time than the second method, which is stable, accurate and faster than the first method.

The results of reconstruction of simulated $\Sigma$-hypernuclear events are summarized in tab. 4.1. The trigger rate with $B = 1.1 T$ is due to $\mu^+$ coming from $k^+$ decay, while at $0.6 T$ the trigger is given by $\pi^\pm_{prompt}$. The different efficiency to positive and negative prompt pions for the same magnetic field is mainly due to
the fact that FINUDA was optimized to the $\bar{\pi}_{\text{prompt}}$ detection.

Table 4.1: Results of reconstruction of simulated $\Sigma$-hypernuclear events.

<table>
<thead>
<tr>
<th>Event Type</th>
<th>Magnetic Field</th>
<th>$\pi_{\text{prompt}}^\pm$ triggered</th>
<th>$\pi_{\text{prompt}}^\pm$ reconstructed</th>
<th>tot. eff.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{12}<em>{\Sigma^-} Be; \pi</em>{\text{prompt}}^+$</td>
<td>1.1 T</td>
<td>14 %</td>
<td>10 %</td>
<td>1.4 %</td>
</tr>
<tr>
<td>$^{12}<em>{\Sigma^-} Be; \pi</em>{\text{prompt}}^+$</td>
<td>0.6 T</td>
<td>52 %</td>
<td>33 %</td>
<td>17 %</td>
</tr>
<tr>
<td>$^{12}<em>{\Sigma^+} C; \pi</em>{\text{prompt}}^-$</td>
<td>1.1 T</td>
<td>14 %</td>
<td>12 %</td>
<td>1.7 %</td>
</tr>
<tr>
<td>$^{12}<em>{\Sigma^+} C; \pi</em>{\text{prompt}}^-</td>
<td>0.6 T</td>
<td>54 %</td>
<td>52 %</td>
<td>28 %</td>
</tr>
</tbody>
</table>

Fig. 4.10 shows the ability of FINUDA to resolve neighbour peaks of prompt pions coming from $\Sigma^-$ hypernuclei. For $B = 1.1 \, T$, the reconstruction of particle trajectories (helix) requires only 3 hits to resolve two adjacent peaks, for particle momenta $< 200 \, M eV/c$. For $B = 0.6 \, T$, both efficiency and resolution improve (see fig. 4.11), an $\Sigma-A$ event is triggered by $\pi_{\text{prompt}}^-$ and the helix fitting requires 4 hits. In addition, the whole event of $\Sigma-A$ formation and decay can be detected and analyzed.
4.5. **THE RECONSTRUCTION OF $\Sigma$-HYPERNUCLEI EVENTS.**

![Graph](image)

**Figure 4.10:** Reconstruction of $\Sigma^-$ hypernucleus at 1.1 T: distribution of simulated momenta of pions fed into reconstruction program (top) and distribution of reconstructed momenta (bottom).
Figure 4.11: Reconstruction of $\Sigma^-$ hypernuclei at 0.6 T: distribution of simulated momenta of pions (top) and distribution of reconstructed momenta (bottom).
Conclusion.

The dynamics of $\Sigma$-hypernuclear formation and decay is far from being understood. The whole picture is further complicated by the lack of knowledge about the $\Sigma$-$N$ interaction, and by the shortage of experimental data. Only one bound state is known, $^4_2\Lambda He$, and there is no evidence of any other bound state. In general, data in medium and heavy nuclear systems are scarce, although the existence of heavy $\Sigma$-hypernucleus bound states are expected [39]. Their formation is certainly favored by the Coulomb attraction of the nucleus. There are many open questions about $\Sigma$-hypernuclei: the width of a bound state, the role of the Coulomb field, the quasi-free spectrum competition and the decay channels. Chap. 4 shows that all these studies can be done with the FINUDA spectrometer at DAΦNE.

The influence of the continuum on the $\Sigma$-hypernuclear spectra is discussed in Sec. 4.3. Since the $\Sigma$-hypernuclear single-particle well is considered to be shallower than that of the $\Lambda$ one, a minor number of bound states should be encountered in the $\Sigma$ excitation spectra. Consequently, the quasi-free part of spectra are more pronounced and peaks less visible. The high spatial and momentum resolution are the keywords for an unambiguous identification of $\Sigma$-hypernucleus signature. The FINUDA silicon vertex detector allows for a clear $\Sigma$-hypernucleus identification through the vertex reconstruction of the $\Lambda$-decay $\Lambda \to \pi^- p$, which follows the conversion reaction $\Sigma N \to \Lambda N$. The FINUDA vertex
detector has a nominal intrinsic resolution of about few tens of microns, which may be impaired if the detector location is poorly known with respect to the other tracking devices of FINUDA. The alignment of the tracking system is the most important operation, which is needed for extracting physics results from the FINUDA spectrometer. The alignment procedure is developed in Chap. 3, it allows the detector location to be determined with an accuracy that is limited by the available statistics and the detector intrinsic resolution. The achieved spatial resolution allows FINUDA to identify unambiguously the $\Sigma$-hypernuclear states; therefore, it will be possible to determine the $\Sigma$-hypernuclear state width, which is expected to be narrow ($\sim 5\, \text{MeV}$) despite the large ($\sim 25\, \text{MeV}$) $\Sigma \to \Lambda$ conversion width.
Appendix A

Explicit calculation of 2D residual vector.

In Sec. 3.2 the roto-translation parameters have been calculated as a function of the 3D residual vectors. In the present appendix, the 2D residual vectors are calculated as a function of roto-translation parameters, in order to clarify the contribution of misalignments to the determination of residual vectors [40] [41]. In this way, it is possible to understand the misalignment effects on the residual distributions, which are obtained by incrementing the histogram bins with the residual coordinates. In order to extract the dependence of residual vector on both the roto-translation and the track inclination, 2D residual vectors are used.

Let the cosmic rays be considered as a straight line \( t \) passing through the point \((t_1, t_2, t_3)\) with direction \((p_1, p_2, p_3)\). Such a straight line can be represented on cartesian reference system by the following equations:

\[
\begin{align*}
  x_2 - t_2 &= \frac{p_2}{p_1} (x_1 - t_1) \\
  x_1 - t_1 &= \frac{p_1}{p_3} (x_3 - t_3)
\end{align*}
\]

(A.1)
where \((x_1, x_2, x_3)\) are the variables. The microstrip module can be described on the same reference system by a generic plane equation \(\pi\):

\[
\vec{n} \cdot (\vec{x} - \vec{h}) = 0
\]  

(A.2)

where \(\vec{h}\) is the translation parameter vector and \(\vec{n}\) is the versor normal to the module plane. \(\vec{n}\) is defined by the equations:

\[
\begin{align*}
    n_1 &= -\cos \theta_2 \sin \theta_3 \\
    n_2 &= -\sin \theta_1 \sin \theta_2 \sin \theta_3 + \cos \theta_1 \cos \theta_3 \\
    n_3 &= \sin \theta_1 \cos \theta_3 + \cos \theta_1 \sin \theta_2 \sin \theta_3
\end{align*}
\]  

(A.3)

where \(\vec{\theta} = (\theta_1, \theta_2, \theta_3)\) are the rotation angles around the coordinate axis.

Let the cartesian reference system be defined by the nominal position of a microstrip module: two axes are defined along the reading strip directions and one axis along the normal to the module plane. This cartesian reference system can be translated and rotated by the module misalignment values, so that a new reference system is defined (see fig. A.1). In this way, the translation \((\vec{h})\) and rotation \((\vec{\theta})\) parameters of \(\pi\) in eq. A.2 are equal to the roto-translation misalignment.

The intersection between \(\pi\) and \(t\) gives the intersection point at the nominal reference system. The intersection point coordinates depend on the 6 misalignment parameters, which can be linearized by means of the first order Taylor expansion. Furthermore, the intersection point coordinates can be transformed to the reference system, which is defined by the actual position of the microstrip module. In this way the intersection coordinates can be compared to the measured coordinates. In this way, the intersection coordinate results:

\[
x_{1}^{int} = \left( t_1 - \frac{p_1}{p_2} t_2 \right) + \left( h_1 + \frac{p_1}{p_2} h_2 + 0 \cdot h_3 \right) + \\
+ \left( + \frac{p_1}{p_2} \left( \frac{p_3}{p_2} t_2 - t_3 \right) \theta_1 - \left( t_3 - \frac{p_3}{p_2} t_2 \right) \theta_2 - \left( \frac{p_1}{p_2} \right)^2 \left( t_2 - \frac{p_2}{p_1} t_1 \right) \theta_3 \right)
\]  

(A.4)
Finally, the residual vector $x_1^{res}$ coordinate can be written as:

$$x_1^{res} = x_1^{int} - s_1$$

where $s_1$ is the first coordinate of the measured point on the microstrip module. Eq. (A.4) shows that the residual coordinate depends strongly on the track selected. From these considerations it is possible to draw some conclusions about the effects of misalignment on the residual coordinate distributions:

1. A $h_1$ translation changes the residual coordinate, which is irrespective of the track direction. A $h_1$ displacement strongly determines a shift of the residual coordinate mean value but it does not affect the distribution width;

2. $h_2$ is multiplied by the ratio $\frac{n_1}{p_2}$. Therefore, the displacement $h_2$ changes the residual coordinate $x_1^{res}$ less than the displacement $h_1$;

3. There is no first order effects caused by $h_3$, since $x_3$ is the strip direction and the transverse coordinate is the measured one;
4. As far as rotations are concerned, the $\frac{p_1}{p_2}$ factor determines the amount of displacement caused by $\bar{\theta}$ to $x_1$. The contributions are given by $\theta_2$, $\theta_1$ and $\theta_3$, in descending order of strength;

5. The average contribution of $\theta_1$ and $\theta_2$ to $x_1^{res}$ is null, so both do not contribute to the shift of the residual coordinate mean value, while both affect the distribution width.
Appendix B

The error matrix of roto-translation parameters.

The error matrix $V$ of the six-vector of parameters $\bar{p}$ is given by the equation

$$V_{jk}^{-1} = \frac{1}{2} \frac{\partial^2 \chi^2}{\partial p_j \partial p_k} \quad (B.1)$$

Let us consider eq. (3.7), $\chi^2 = \sum_{i=0}^{n} \hat{r}_i^T E_i^{-1} \hat{r}_j$, where $\hat{r}_i = \bar{r}_i - Q \bar{s}_i - \bar{h}$, the process of minimization of $\chi^2$ with respect to the parameters $\theta_j$ and $h_k$ leads to the equations

$$\frac{\partial \chi^2}{\partial \theta_j} = \sum_{i=0}^{n} \left( \bar{s}_i^T Q_j T E_i^{-1} \hat{r}_i + \bar{r}_i^T E_i^{-1} Q_j \bar{s}_i \right) \quad j = 1, 3$$

$$\frac{\partial \chi^2}{\partial h_j} = \sum_{i=0}^{n} \left( \bar{u}_j^T E_i^{-1} \hat{r}_i + \bar{r}_i^T E_i^{-1} \bar{u}_j \right) \quad j = 1, 3. \quad (B.2)$$
and the second derivatives are

\[
\frac{1}{2} \frac{\partial^2 \chi^2}{\partial \theta_j \partial \theta_k} = \sum_{i=0}^{n} \bar{s}_i^T Q_j^T E_i^{-1} Q_k \bar{s}_i = A_{jk} \quad j, k = 1, 3
\]

\[
\frac{1}{2} \frac{\partial^2 \chi^2}{\partial h_j \partial h_k} = \sum_{i=0}^{n} \bar{u}_j^T E_i^{-1} \bar{u}_k = F_{jk} \quad j, k = 1, 3
\]

\[
\frac{1}{2} \frac{\partial^2 \chi^2}{\partial h_j \partial \theta_k} = \sum_{i=0}^{n} \bar{u}_j^T E_i^{-1} Q_k \bar{s}_i = D_{jk} \quad j, k = 1, 3
\]

\[
\frac{1}{2} \frac{\partial^2 \chi^2}{\partial \theta_j \partial \theta_k} = \sum_{i=0}^{n} \bar{s}_i^T Q_j^T E_i^{-1} \bar{u}_k = B_{jk} \quad j, k = 1, 3
\]

(B.3)

They are related to the constants \( A_{jm}, B_{jn}, D_{jm} \) and \( F_{jn} \), which are defined by eq. (3.9). \( V^{-1} \) is therefore equal to \( M \), the latter being the six-by-six matrix defined by eq. (3.11).
Appendix C

Lifetime measurements of \(\Lambda\)-hypernuclei with FINUDA.

FINUDA determines the \(\Lambda\)-hypernuclear lifetime by measuring the time difference between the arrival of the prompt pion at TOFONE and the nucleon from the non-mesonic decay. The overall time resolution is \(700\, \text{ps}\) at FWHM \((\sigma = 300\, \text{ps})\), which also includes the time uncertainties inherent in the corrections of the particle flight paths. The hypernuclear decay rate \(P(t)\) follows the probability distribution

\[
P(t) = \frac{1}{\tau} e^{-\frac{t}{\tau}}
\]

where the \(\tau\) is the mean life of the hypernucleus. Since the apparatus has a finite resolution, which is represented by a gaussian distribution \(G(t)\) with standard deviation \(\sigma\), the actual time distribution \(S(t)\) is the convolution of \(P(t)\) with the response function \(G(t)\) of the detector

\[
S(t) = \int_0^\infty P(t') G(t - t') \, dt'.
\]
The convolution of an exponential and a gaussian resolution distribution between 0 and $\infty$ yields

$$A \frac{e^{-\frac{x^2}{4\tau^2}}}{2\tau} \left[ 1 + Erf \left( \frac{\tau t - \sigma^2}{\sqrt{2}\sigma^2\tau} \right) \right]$$  \hspace{1cm} (C.3)

where $A$ is a normalization constant.

Fig. C.1 shows a simulated event distribution of a hypernucleus that decays

```

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Figure C.1: Fitting of simulated hypernuclear lifetime of 200 ps for a resolution of 700 ps at FWHM.

with a lifetime of 200 ps (histogram). The distribution is fitted by eq. (C.3) (red curve) with $\sigma = 300$ ps, which indicates that the hypernuclear mean-life can be determined with an accuracy of 5\% for about 4000 events populating the time-of-flight spectrum.
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