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The Field Localization of Yukawa Interaction in a Modified Randall-Sundrum Model

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Abstract. We study the localization properties of spinor field coupled to scalar field through Yukawa coupling in a Modified Randall-Sundrum (MRS) model. We derive general localization conditions and solve the solution of the field equation corresponding to the extra coordinate and match the solution to the localization conditions. We obtain that the spinor and scalar fields are localizable on the MRS brane for specific conditions with a decreasing warp factor.

1. Introduction

The formalism of interaction between two-component spinors with a scalar field was originally proposed by Yukawa representing the interaction between nucleons of fermions and scalar pions [1]. The Yukawa theory was denoted as a theoretical interpretation of nucleon-nucleon interaction. This idea was introduced in the beginning of nuclear theory when nucleons were regarded as elementary Dirac particles. Now it is known that nucleons are arranged from quarks.

Reference [2] suggested the mechanism of localization of spinor fields by introducing a Yukawa coupling to scalar field of the form $h\bar{\Psi}\Psi\Phi$. This has been studied in section 2 for the MRS model introduced in ref [3-6]. We analyze the localization properties of a spinor-scalar system interacting one another through Yukawa coupling. In this section we identify the localization conditions for this system. We also derive and solve the equations of motion corresponding to extra-dimensional component and check whether these solutions satisfy the conditions represented by finiteness of all integrals in the action over the fifth coordinate are finite. Section 3 is devoted to conclusions.

2. Field Localization

The MRS model [3,6] is specified by the metric

$$ds^2 = a^2(x^5)\left[\eta_{\mu\nu}dx^\mu dx^\nu - dx^5 dx^5\right], \tag{1}$$

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where $a(x^5)$ is a warp factor $a(x^5) = e^{-k|x^5|}, x^5 = r$ is the fifth coordinate, k is a constant and $\eta_{\mu\nu}$ is the Minkowski metric with signature $(+,-,-,-)$. The localization properties of matter fields that do not interact with other fields except with gravity in the MRS model are much better as compared to the RS model [3,4].

The five-dimensional action corresponding to the spinor field coupled to scalar field by a Yukawa coupling reads

$$S = \int d^5x \sqrt{g} \left[\overline{\Psi} i\Gamma^M D_M \Psi + h\overline{\Psi}\Psi\Phi + \frac{1}{2}\partial_M\Phi\partial^M\Phi\right], \tag{2}$$

where g is is the determinant of the MRS metric (1), h is a coupling constant, $\Gamma^M = e^M_{\overline{M}}\gamma^{\overline{M}}$ are gamma matrices in curved spacetime which are connected to the gamma matrices in flat spacetime, $\gamma^{\overline{M}}$ with $e^{\overline{M}}_M$ and $e^M_{\overline{M}}$ are funfbeins and their inverses respectively and $\gamma^{\overline{5}} = i\gamma^{\overline{0}}\gamma^{\overline{1}}\gamma^{\overline{2}}\gamma^{\overline{3}}$, Φ is a 5D scalar field $\Phi(x^M) = \varphi(x^\mu)\chi(x^5)$ and $\overline{\Psi} = \Psi^+ \, \Gamma^0 = (\Psi^*)^T e^0_{\overline{0}}\gamma^{\overline{0}}$. D_M is a covariant derivative, $D_M = \partial_M + \frac{1}{4}\omega_M^{\overline{M}\overline{N}}\sigma_{\overline{M}\overline{N}}$, where $\sigma_{\overline{M}\overline{N}} = \frac{1}{2}[\gamma_{\overline{M}}, \gamma_{\overline{N}}]$ is Dirac tensor and $\omega_M^{\overline{M}\overline{N}}$ is spin connection defined through the following expression

$$\omega_M^{\overline{M}\,\overline{N}} = \frac{1}{2}e^{N\overline{M}}\left(\partial_M e_N^{\overline{N}} - \partial_N e_M^{\overline{N}}\right) - \frac{1}{2}e^{N\overline{N}}\left(\partial_M e_N^{\overline{M}} - \partial_N e_M^{\overline{M}}\right) - \frac{1}{2}e^{P\overline{M}}e^{Q\overline{N}}\left(\partial_P e_{Q\bar{R}} - \partial_Q e_{P\bar{R}}\right)e_M^{\bar{R}} \,. \tag{3}$$

In the MRS metric, the non-zero spin connections are

$$\omega_\nu^{\bar{\mu}\,\bar{5}} = \delta_\nu^\mu\left(\frac{-a'}{a}\right). \tag{4}$$

where the prime represents the derivative with respect to r. Thus the covariant derivative take the form

$$D_\mu = \partial_\mu + \frac{ik}{2}\gamma_{\bar{\mu}}\gamma_{\bar{5}}, \;\; D_r = \partial_r. \tag{5}$$

Using Weyl spinor representation and decomposing the five-dimensional spinor as follows

$$\Psi(x^\mu, r) = \begin{pmatrix} \psi_R^{(1)}(x_\mu)P_R^{(1)}(r) \\ \psi_R^{(2)}(x_\mu)P_R^{(2)}(r) \\ \psi_L^{(1)}(x_\mu)P_L^{(1)}(r) \\ \psi_L^{(2)}(x_\mu)P_L^{(2)}(r) \end{pmatrix}, \tag{6}$$

the action becomes

$$\begin{aligned}
S = &\int d^5x \frac{\sqrt{g}}{a^2(r)} \Big[\psi_R^{(1)*}P_R^{(1)*}(i\partial_0)\psi_R^{(1)}P_R^{(1)} + \psi_R^{(2)*}P_R^{(2)*}(i\partial_0)\psi_R^{(2)}P_R^{(2)} \\
&+ \psi_L^{(1)*}P_L^{(1)*}(i\partial_0)\psi_L^{(1)}P_L^{(1)} + \psi_L^{(2)*}P_L^{(2)*}(i\partial_0)\psi_L^{(2)}P_L^{(2)}\Big] \\
&+ \int d^5x \frac{\sqrt{g}}{a^2(r)} \Big[\psi_R^{(2)*}P_R^{(2)*}(i\partial_1)\psi_R^{(1)}P_R^{(1)} + \psi_R^{(1)*}P_R^{(1)*}(i\partial_1)\psi_R^{(2)}P_R^{(2)} \\
&- \psi_L^{(2)*}P_L^{(2)*}(i\partial_1)\psi_L^{(1)}P_L^{(1)} - \psi_L^{(1)*}P_L^{(1)*}(i\partial_1)\psi_L^{(2)}P_L^{(2)}\Big] \\
&+ \int d^5x \frac{\sqrt{g}}{a^2(r)} \Big[\psi_R^{(2)*}P_R^{(2)*}i(i\partial_2)\psi_R^{(1)}P_R^{(1)} - \psi_R^{(1)*}P_R^{(1)*}i(i\partial_2)\psi_R^{(2)}P_R^{(2)} \\
&- \psi_L^{(2)*}P_L^{(2)*}i(i\partial_2)\psi_L^{(1)}P_L^{(1)} + \psi_L^{(1)*}P_L^{(1)*}i(i\partial_2)\psi_L^{(2)}P_L^{(2)}\Big] \\
&+ \int d^5x \frac{\sqrt{g}}{a^2(r)} \Big[\psi_R^{(1)*}P_R^{(1)*}(i\partial_3)\psi_R^{(1)}P_R^{(1)} - \psi_R^{(2)*}P_R^{(2)*}(i\partial_3)\psi_R^{(2)}P_R^{(2)} \\
&- \psi_L^{(1)*}P_L^{(1)*}(i\partial_3)\psi_L^{(1)}P_L^{(1)} + \psi_L^{(2)*}P_L^{(2)*}(i\partial_3)\psi_L^{(2)}P_L^{(2)}\Big] \\
&- 2\int d^5x \frac{\sqrt{g}}{a^2(r)} \Big[\psi_L^{(1)*}P_L^{(1)*}k\psi_R^{(1)}P_R^{(1)} + \psi_L^{(2)*}P_L^{(2)*}k\psi_R^{(2)}P_R^{(2)} \\
&- \psi_R^{(1)*}P_R^{(1)*}k\psi_L^{(1)}P_L^{(1)} - \psi_R^{(2)*}P_R^{(2)*}ka\psi_L^{(2)}P_L^{(2)}\Big] \\
&+ \int d^5x \frac{\sqrt{g}}{a^2(r)} \Big[\psi_L^{(1)*}P_L^{(1)*}\partial_r\psi_R^{(1)}P_R^{(1)} + \psi_L^{(2)*}P_L^{(2)*}\partial_r\psi_R^{(2)}P_R^{(2)} \\
&- \psi_R^{(1)*}P_R^{(1)*}\partial_r\psi_L^{(1)}P_L^{(1)} - \psi_R^{(2)*}P_R^{(2)*}\partial_r\psi_L^{(2)}P_L^{(2)}\Big] \\
&+ h\int d^5x \frac{\sqrt{g}}{a^2(r)} a(r) \Big[\psi_L^{(1)*}P_L^{(1)*}\psi_R^{(1)}P_R^{(1)} + \psi_L^{(2)*}P_L^{(2)*}\psi_R^{(2)}P_R^{(2)} \\
&+ \psi_R^{(1)*}P_R^{(1)*}\psi_L^{(1)}P_L^{(1)} + P_R^{(2)*}\psi_L^{(2)}P_L^{(2)}\Big]\,\varphi(x^v)\chi(r) \\
&+ \int_0^\infty dr\, a^3(r)\chi\chi\ \eta^{\mu\nu}\frac{1}{2}\int d^4x\ \partial_\mu\varphi\,\partial_\nu\varphi - \int_0^\infty dr\, a^3(r)\,\partial_r\chi\,\partial_r\chi\ \frac{1}{2}\int d^4x\varphi\,\varphi
\end{aligned} \tag{7}$$

The fields in the system are localizable on the brane if the action integrals over the fifth coordinate are finite (i=1,2):

$$\int_0^\infty dr\, a^3(r)\, P_L^{(i)*}P_L^{(i)} = N_1\ ; \int_0^\infty dr\, a^3(r)P_R^{(i)*}P_R^{(i)} = N_2, \tag{8a}$$

$$\int_0^\infty dr\, a^3(r)P_L^{(1)*}P_L^{(2)} = N_3; \int_0^\infty dr\, a^3(r)P_L^{(2)*}P_L^{(1)} = N_4, \tag{8b}$$

$$\int_0^\infty dr\, a^3(r)P_R^{(1)*}P_R^{(2)} = N_5; \int_0^\infty dr\, a^3(r)P_R^{(2)*}P_R^{(1)} = N_6, \tag{8c}$$

$$\int_0^\infty dr\, a^3(r)(-2k)P_L^{(i)*}(r)P_R^{(i)}(r) + \int_0^\infty dr\, a^3(r)P_L^{(i)*}(r)\partial_r P_R^{(i)}(r) = -m, \tag{8d}$$

$$\int_0^\infty dr\, a^3(r)(2k)P_R^{(i)*}(r)P_L^{(i)}(r) - \int_0^\infty dr\, a^3(r)P_R^{(i)*}(r)\partial_r P_L^{(i)}(r) = -m, \tag{8e}$$

$$\int_0^\infty dr\, a^4(r)P_L^{(1)*}P_R^{(1)}\chi(r) = N_7; \int_0^\infty dr\, a^4(r)P_L^{(2)*}P_R^{(2)}\chi(r) = N_8, \tag{8f}$$

$$\int_0^\infty dr\, a^4(r)P_R^{(1)*}P_L^{(1)}\chi(r) = N_9; \int_0^\infty dr\, a^4(r)P_R^{(2)*}P_L^{(2)}\chi(r) = N_{10}, \tag{8g}$$

$$\int_0^\infty dr\sqrt{g}\ g^{\mu\nu}\chi\,\chi = \int_0^\infty dr\, a^3(r)\eta^{\mu\nu}\chi\chi\ = N_{11}\eta^{\mu\nu}, \tag{8h}$$

$$\int_0^\infty dr\, \sqrt{g}g^{rr}\partial_r\chi\partial_r\chi = -\int_0^\infty dr\, a^3(r)\partial_r\chi\partial_r\chi\ = -m_\varphi{}^2. \tag{8i}$$

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All N_i are finite constants, m and m_φ represent the mass of spinor and scalar fields respectively in four-dimensional Minkowski space.

The equation of motion for spinor field corresponding to the action (2) reads

$$\sqrt{g}\mathrm{i}\Gamma^M D_M \Psi + \sqrt{g}\mathrm{h}\Psi\Phi = 0. \tag{9}$$

Feeding the covariant derivative (5), gamma matrices, decomposing $\Psi(x^M)$ as (6) and finally using the 4D Lagrangian of spinor field coupled to scalar field $\mathcal{L} = i\bar{\psi}\gamma^\mu\partial_\mu\psi - m\bar{\psi}\psi + h\bar{\psi}\psi\varphi + \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - \frac{1}{2}m_\varphi{}^2\varphi\varphi$, the equation of motion (9) can be written as follows (*i=1,2*)

$$\begin{aligned} mP_R^{(i)} + hP_R^{(i)}\varphi\chi(a(r)-1) - 2kP_R^{(i)} + \partial_r P_R^{(i)} &= 0, \\ mP_L^{(i)} + hP_L^{(i)}\varphi\chi(a(r)-1) + 2kP_L^{(i)} - \partial_r P_L^{(i)} &= 0. \end{aligned} \tag{10}$$

The equation of motion for scalar field corresponding to the action (2) is

$$\partial_M\left(\sqrt{g}g^{MN}\partial_N\Phi\right) - \sqrt{g}\mathrm{h}\bar{\Psi}\Psi = 0. \tag{11}$$

Recalling the MRS metric $\sqrt{g} = a^5(r), g^{\mu\nu} = a^{-2}(r)\eta^{\mu\nu}, g^{55} = -a^{-2}(r)$ and decomposing Φ and Ψ into 4D and extra-dimensional components, the above equation becomes

$$\begin{aligned} &\chi\eta^{\mu\nu}\partial_\mu\partial_\nu\varphi + 3k\varphi\,\partial_5\chi - \varphi\,\partial_5\partial_5\chi \\ &\quad - ah\Big(\psi_L^{(1)*}P_L^{(1)*}\psi_R^{(1)}P_R^{(1)} + \psi_L^{(2)*}P_L^{(2)*}\psi_R^{(2)}P_R^{(2)} + \psi_R^{(1)*}P_R^{(1)*}\psi_L^{(1)}P_L^{(1)} \\ &\quad + \psi_R^{(2)*}P_R^{(2)*}\psi_L^{(2)}P_L^{(2)}\Big) = 0. \end{aligned} \tag{12}$$

Next, we should get the solutions of equation (10) and (12) for extra-dimensional component and check whether the solutions satisfy the localization conditions. The simplest choice to solve eq. (12) is to take $P_L^{(1)} = P_L^{(2)} = 0$ and then the eq. (12) becomes

$$\frac{1}{\varphi}\eta^{\mu\nu}\partial_\mu\partial_\nu\varphi + \frac{1}{\chi}3k\,\partial_5\chi - \frac{1}{\chi}\partial_5\partial_5\chi = 0. \tag{13}$$

Since x^μ- and *r*-coordinates are independent one another thus we can choose $\frac{1}{\varphi}\eta^{\mu\nu}\partial_\mu\partial_\nu\varphi = c$ as a constant. Thus we get the solution of equation (13)

$$\chi(r) = c_0 e^{\frac{3}{2}kr}e^{-\frac{1}{2}r\sqrt{9k^2+4c}} + b_0 e^{\frac{3}{2}kr}e^{\frac{1}{2}r\sqrt{9k^2+4c}}. \tag{14}$$

Feeding the above solution to the localization condition (8h), the finiteness is obtained if $b_0 = 0$. From (10) since $\varphi(x^\mu)$ is the only function depending on x^μ-coordinates thus this function can be considered as a constant. Inserting (14) to the equation (10) and using a condition $\sqrt{9k^2+4c} = 2k$ gives the following solution

$$P_R^{(1)} = P_R^{(2)} = P_R = b_{1/2}exp\left[-mr + \frac{4}{k}h\varphi c_0 cosh\left(\frac{k}{2}r\right) + 2kr\right], \tag{15}$$

where $b_{1/2}$ is an integration constant. From the localization conditions (8d) and (8e), since no mixes between the right- and left-handed spinor result *m = 0*, the corresponding solution of equation of motion (15) is $P_R = b_{1/2} exp\left[\frac{4}{k} h\varphi c_0 cosh\left(\frac{k}{2}r\right) + 2kr\right]$. Feeding this solution to the condition (8c) gives

$$b_{1/2}{}^2 \int_0^\infty dr\, e^{kr} e^{2\left(\frac{4}{k}h\varphi c_0 cosh\left(\frac{k}{2}r\right)\right)} \tag{16}$$

The above integral is finite if $2\frac{4}{k}h\varphi c_0 cosh\left(\frac{k}{2}r\right) + kr < 0$. This can be fulfilled if *k<0* (an increasing warp factor).

Similarly, we can choose $P_R^{(1)} = P_R^{(2)} = 0$. Under this choice, the corresponding equation of motion for scalar field (12) gives the solution for extra-dimensional component as (14) with $b_0 = 0$ to get the finiteness. From (8d) and (8e), since no mixes between the right- and left-handed spinors give *m* = 0 thus the corresponding solution of equation (10) is

$$P_L^{(1)} = P_L^{(2)} = P_L = d_{1/2} exp\left[-\frac{4}{k}h\varphi c_0 cosh\left(\frac{k}{2}r\right) + 2kr\right]. \tag{17}$$

Feeding the above equation to localization condition (8c), we get

$$d_{1/2}{}^2 \int_0^\infty dr\, e^{kr} e^{2\left(-\frac{4}{k}h\varphi c_0 cosh\left(\frac{k}{2}r\right)\right)}. \tag{18}$$

The finiteness of the above integral can be obtained if $2\frac{4}{k}h\varphi c_0 cosh\left(\frac{k}{2}r\right) > kr$. This requires *k>0* (a decreasing warp factor).

Next check whether the integral over the fifth coordinate in the condition (8i) is finite. Inserting the solution (14) with $b_0 = 0$ and $\sqrt{9k^2 + 4c} = 2k$ to (8i) then recalling in the MRS model $a = e^{-kr}$, $\sqrt{g} = a^5(r), g^{\mu\nu} = a^{-2}(r)\eta^{\mu\nu}, g^{55} = -a^{-2}(r)$ give the finiteness for *k>0* (a decreasing warp factor). In conclusion the only possible localization for spinor and scalar felds is that $P_R = 0$ and a decreasing warp factor. This does not mean that $\psi_R = 0$, i.e we still have a complete pair of spinors in a 4D spacetime (ψ_R, ψ_L).

3. Conclusions

We have analyzed the localization properties of the spinor field coupled to scalar field by Yukawa coupling. We get the localization conditions (8a)-(8i) corresponding to the action (7). The fields are localizable if all integrals over the fifth coordinate (8a)-(8i) are finite. From the equation of motion for scalar field (12), we introduce $P_L = 0$ giving the solution (14). The solution of equation of motion for spinor field is given (15). From the conditions (8d) and (8e), the choice $P_L = 0$ gives the massless spinor field. From the obtained solution (15) gives the finiteness of condition (8c) for *k*<0. While the solution (14) gives the finiteness of condition (8i) for *k>0*. This conflicts with the previous result thus the spinor-vector fields are not localizable for $P_L = 0$. From the equation of motion for scalar field (12) if we choose $P_R = 0$ we also get the solution (14). The corresponding solution for spinor field is given in (17). Feeding (17) to condition (8c) gives finiteness for *k*>0. This is in agreement with the conditions (8i) that the finiteness is obtained for *k*>0. Thus the spinor-vector fields are localizable on the MRS brane for $P_R = 0$ and a decreasing warp factor.

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