Available on CMS information server

CMS NOTE 2006/037

The Compact Muon Solenoid Experiment

CMS Note

Mailing address: CMS CERN, CH-1211 GENEVA 23, Switzerland

6^{th} February 2006

Reconstruction of the signal amplitude of the CMS electromagnetic calorimeter

Renaud Brunelière
CERN, Geneva, Switzerland

Alexandre Zabi
Imperial College, London, UK

Abstract

The amplitude of the signal collected from the $PbWO_4$ crystals of the CMS electromagnetic calorimeter is reconstructed by a digital filtering technique. The amplitude reconstruction has been studied with test beam data recorded from a fully equipped barrel supermodule. Results on the performance of the method are given, and test beam specific issues are investigated, together with conclusions about implementation of the method for CMS data taking.

1 Introduction

The scintillation signal from the CMS electromagnetic calorimeter (ECAL), is detected and amplified by photodetectors: avalanche photodiodes (APD) in the barrel and vacuum phototriodes (VPT) in the endcaps [1]. The electrical signal output from the photodetectors is amplified and shaped by a multi-gain preamplifier (MGPA) before digitization by ADCs at a frequency of 40 MHz [2]. For each channel three signals, resulting from amplification with three different gains, are simultaneously digitized in three ADCs. Further logic chooses the highest non-saturated digital value, allowing a dynamic range of about 5×10^4 from the least significant bit of about 35 MeV to saturation at 1.7 TeV in the barrel.

The data read out consists of a series of consecutive digitizations, corresponding to a sequence of samplings of the signal at 40 MHz. It is envisaged that a time frame of 10 consecutive samplings will be read out in LHC operation. The signal amplitude must be reconstructed using these samplings.

The complete process of signal amplification followed by signal digitization and amplitude reconstruction should not degrade the energy resolution of the calorimeter other than by the inevitable introduction of noise. The simplest method of reconstructing the amplitude is to take a sampling on the signal pulse maximum as the measurement. Reading out a larger number of samples allows identification of out of time (other bunch crossing) pileup and an event by event subtraction of the pedestal. It also allows more sophisticated digital processing of the signal to improve the signal to noise, and a measurement of the signal timing. We report here on the performance of a method which implements a digital filter. The digital filter is optimized by minimizing the noise contribution, although the potential gain is relatively small. At the same time, it is essential to keep to a negligible level any effect which would contribute to the constant term of the energy resolution. Such effects present quite a challenge in test beam data.

Test beam data taken in the H4 beam line at CERN in October and November 2004 using electron beams with a range of momenta between 20 and 250 GeV/c, have been used to investigate the method of amplitude reconstruction. The tests were performed with a complete supermodule, installed on a movable table which allowed the beam to be directed into any part of the calorimeter. Electrons were incident at an angle of $\simeq 3^\circ$ to the direction of the crystal axis in both transverse directions reproducing the average incident angle of particles emerging from the collision region in LHC running. Plastic scintillator counters were used to trigger the readout. The position of the incident electrons in the transverse directions was determined by four planes of scintillating fibre hodoscopes.

The test beam provides data for the verification of the amplitude reconstruction method, but these data differ in an important way with respect to those which will be taken in running at the LHC. In the test beam the scintillation signals have a random timing with respect to the ADC clock, while during LHC operation the ADC clock will be synchronous with the bunch crossing. It is necessary to identify and investigate effects which are specific to asynchronous running before being able to achieve full performance with test beam data. The coefficients, or weights, of the digital filter used to reconstruct the signal amplitude are optimized to minimize the noise contribution, but reconstruction of the amplitude from the time frames read out in asynchronous running imposes additional constraints. If these are not met an error proportional to the signal amplitude is introduced, which becomes a constant term in the energy resolution function (σ_E/E).

In order to determine the phase between the signal peak and the sampling time, a TDC was used to measure the delay between the trigger and the 40 MHz ADC digitization clock. Verification of the performance of amplitude reconstruction as it is envisaged to be used in synchronous data taking at the LHC, has been made by selecting test beam events within a narrow window around a chosen phase.

In this note, the principle of the digital filter method is described first. The derivation of weights is described, as well as an investigation of the impact of pileup events. The algorithm intended for use in CMS data taking is discussed, and then issues specific to asynchronous operation in the test beam are identified and discussed. Fuller details of the mathematical formalism used to determine the weights of the digital filter can be found in an appendix.

2 The weights method

The method used to reconstruct the amplitude from the digitized samples is based on a digital filtering technique. The signal amplitude, $\hat{\mathcal{A}}$, is computed from a linear combination of discrete time samples.

$$\hat{\mathcal{A}} = \sum_{i=0}^{i=N} w_i \times S_i \tag{2.1}$$

where w_i are the weights, S_i the time sample values in ADC counts and N is the number of samples used in the filtering, with the index, i, running over the time samples. The weights w_i are obtained by minimizing the variance of $\hat{\mathcal{A}}$ (see Appendix A). Requiring that the estimator of the amplitude be not just proportional, but equal to the amplitude, $E[\hat{\mathcal{A}}] = A$, implies that:

$$\sum_{i}^{N} w_i f_i = 1 \tag{2.2}$$

where $f_i = f(t = t_i)$ and $f(t)$ is a function which corresponds to the time development of the signal pulse, normalized to have an amplitude of 1.

The determination of an optimal set of weights requires a representation of the signal pulse ($f(t)$). The question of how precisely the function needs to be matched to the shape and timing of each channel to enable the derivation of a set of weights giving satisfactory amplitude reconstruction is one that we attempt to answer in this paper.

The function f that best represents the electronics signal is a digital representation (profile histogram) directly built from the test beam data. An example of such a representation, obtained using an electron beam of 120 GeV, is shown in Fig. 1. The rise time is about 50 ns, which corresponds to the 10 ns decay time of the crystal and the 40 ns shaping time of the MGPA. In the test beam the data was read out so that at least three samples, which we refer to as presamples, were taken before the start of the signal rise.

The weights are extracted by minimization of the χ^2, which is given by:

$$\chi^2 = \sum_{i,j} (S_i - G_i) \times \mathrm{C}^{-1}_{i,j} \times (S_j - G_j) \tag{2.3}$$

where

- S_i is the sample magnitude in ADC counts recorded at the time t_i.
- The signal pulse is described by G_i which is a function that depends on different parameters: $G_i = G_i(A, P, T_{\mathrm{Max}})$ where A is the true amplitude, P is the pedestal and T_{Max} is the peaking time (see Fig. 1).
- C is the covariance matrix representing the correlation between time samples.

If there is no noise correlation between time samples ($\mathrm{C} = \mathbb{1}\sigma^2$, where σ is the single sample noise defined below) and the pedestal and peaking time are known (so that $G = G(A) = Af(t)$), the optimal weights are given by the formula:

$$w_i = f_i / \sum_{j}^{N} f_j^2 \tag{2.4}$$

The derivation of this formula is given in Appendix A.

These weights give the best estimation of the amplitude A. Since the samples contain information about the peaking time and the pedestal also, two further sets of weights can be derived to measure these parameters.

3 Optimization

3.1 The samples considered

The reconstruction of the signal amplitude could use a single sample taken at the signal peak. In synchronous running the pulse maximum time, represented by the parameter T_{Max}, can be adjusted so that the peak coincides

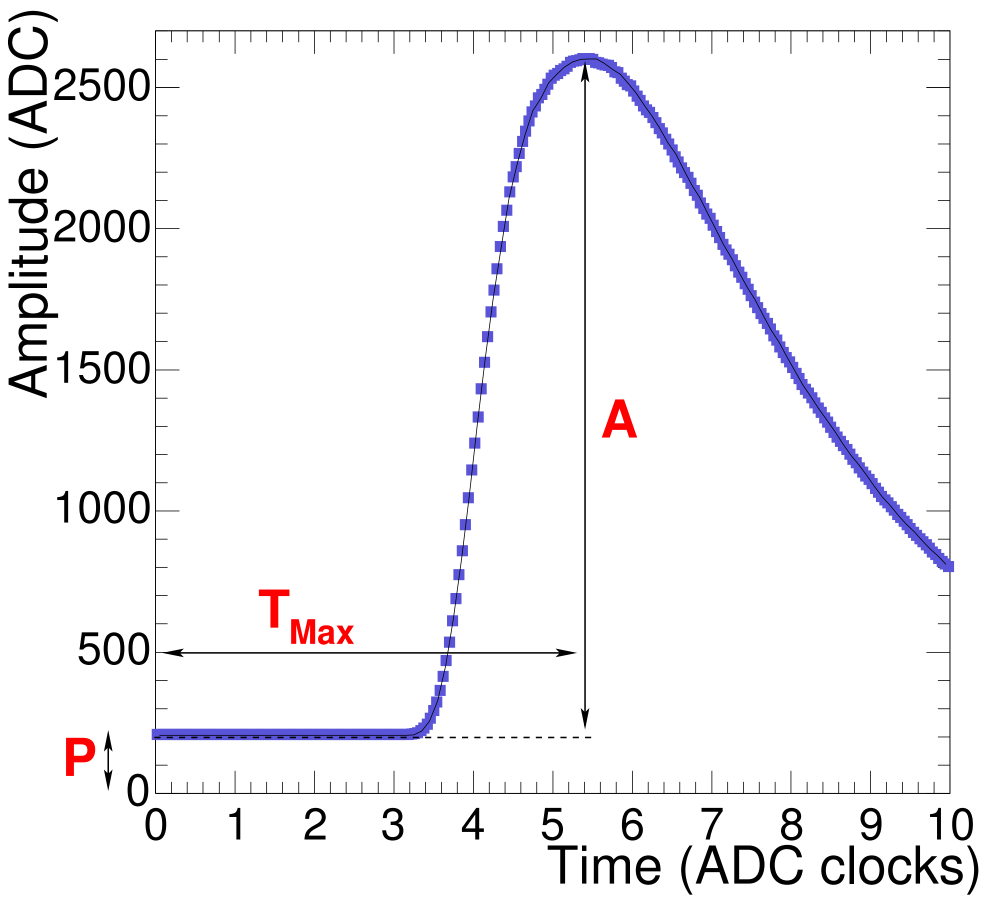


Figure 1: Profile of the signal pulse from a crystal of the supermodule using an electron beam of 120 GeV. The peaking time T_{Max}, the pedestal P and the amplitude of the signal A are shown.

with one of the samplings. If the pedestal is determined independently and subtracted from the maximum, this one sample gives the pulse amplitude. This technique has the advantage of being relatively insensitive to any possible jitter on T_{Max} (this will be discussed in Section 4). However using a larger number of samples allows some reduction of the noise contribution.

When considering the weights obtained assuming no noise correlations between time samples, given by Equation 2.4, the square root of the variance is:

$$\sigma_{\hat{A}} = \sigma / \sqrt{\sum f_i^2} \tag{3.1}$$

where σ is the noise present on a single digitization, the single sampling noise. Thus, an increase of the number of samples will reduce the value of the noise in the reconstructed amplitude. Furthermore, better noise reduction is achieved if samples near the peak, containing more signal, are used. The noise on the reconstructed amplitude should be reduced from that of a single sampling by a factor $\sqrt{\sum w_i^2}$. Calculating weights from the functional representation shown in Fig. 1, assuming no noise correlation between samples, it is found that the use of 5 samples should give a noise reduction of $\simeq$0.6 (Fig. 2) with little improvement when more samples are used. The 1-sample option takes the sampling on the peak, for the 2-sample option the sampling before the peak is added, for the 3-sample option the next sampling after the peak is added, for 4-sample and higher options the remaining samplings after the peak are added in consecutive order. The figure also shows the expected noise reduction if the noise correlation seen in the test beam is assumed, and the weights are calculated using the measured covariance matrix (see Section 3.2.2).

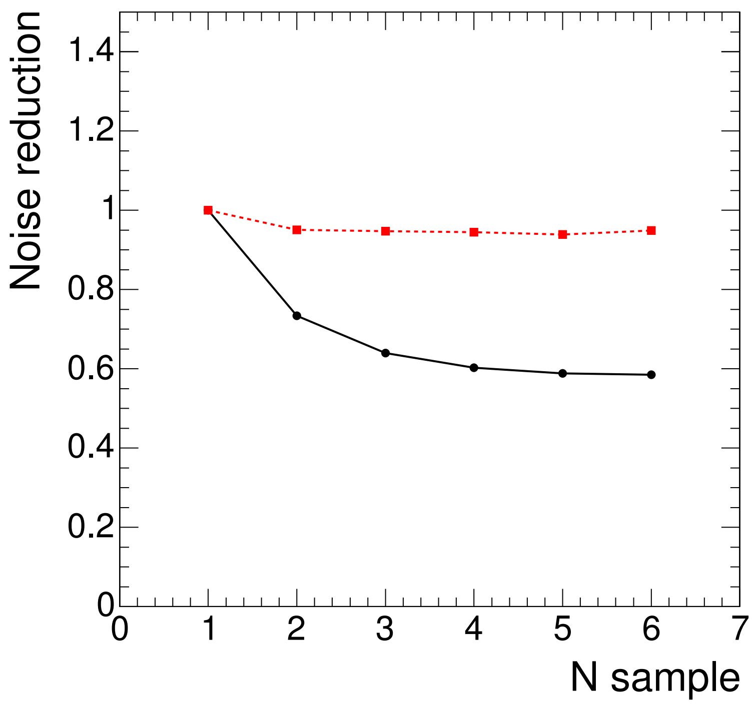


Figure 2: Noise reduction ($\sqrt{\sum w_i^2}$) as a function of the number of samples used for the reconstruction in the case of no noise correlations between time samples (solid line) and in the case of the correlations found in test beam data (dashed line).

3.2 Noise reduction

The noise reduction expected from using many samples in a situation where there is no noise correlation between samplings is not fully realized when the amplitude reconstruction algorithm is applied to real data. This is because there is noise which is correlated between samples. This is partly due to pickup noise which is also, to some degree, correlated between channels; it is at a much lower frequency than the 40 MHz sampling frequency and appears as a small event-to-event fluctuation in the level of the signal baseline or pedestal. The digital filter can be configured to subtract the pedestal using the information present in the presamples. Such a digital filter effectively removes the baseline fluctuation [3]. The noise performance of different implementations were compared using test beam data.

3.2.1 Measurement of the noise

The method was implemented in two different forms:

- The "5-weights" implementation:

  The 5-weights method uses 5 samples as defined in the previous section to reconstruct the signal amplitude. The pedestal is determined independently and its value is subtracted from the samples before reconstruction.

- The "pedestal-subtracting weights" implementation:

  The pedestal-subtracting weights method uses 3+5 weights applied to three samples before the signal pulse (the three presamples) and five samples during the pulse. The weights are calculated by minimizing Equation 2.3 for the amplitude A and the pedestal P where $G = G(A, P) = Af(t) + P$. It may be noted that in this situation the conservation of $\hat{A} = A$ implies that $\sum w_i = 0$. Such a set of weights performs a subtraction of the pedestal on an event by event basis.

  For completeness, a 3+1 weights pedestal-subtracting implementation has been studied. The single weight is applied to the sample on the peak. This allows the demonstration of the impact of dynamic baseline subtraction in its simplest form.

Table 1: Noise in ADC counts measured in a single channel and arrays of 3×3 and 5×5 channels for the 5-weights and the pedestals subtracting weights implementations of the amplitude reconstruction method, and for a single sampling.

Method	Noise (ADC counts)		
	1×1	3×3	5×5
Single sampling	1.20±0.01	3.7±0.04	6.5±0.1
5-weights	1.11±0.02	3.8±0.1	6.7±0.1
3+1 Pedestal subtracting weights	1.13±0.03	3.4±0.1	5.7±0.1
3+5 Pedestal subtracting weights	1.07±0.02	3.2±0.1	5.4±0.1

The noise is the root mean square deviation of the reconstructed amplitude when no signal is input. Thus the ECAL noise is measured by examining the variation of the reconstructed amplitude when random triggers, data taken with no signal (often called "pedestal runs"), are reconstructed. Table 1 shows the noise measured when using these implementations of the method for a single channel, and for sums of channels corresponding to matrices of 3×3 and 5×5 crystals (typical of ECAL shower reconstruction). Also shown, for comparison, is the single sampling noise (as defined in Section 3.1). Results for sums of channels are given because energy is reconstructed in the ECAL from such sums: the noise in the sums represents the noise seen when shower energy is measured. Noise coherent between channels will increase the noise measured in such sums above the quadratic sum of the noise measured in component channels. A total of 1200 events from 6 different runs were used and the mean values of noise in 8 different locations across the supermodule are given. When using the 5-weights method, the pedestal values for all the crystals were directly measured from the data by taking the average value, over a run, of the first presample.

The results given in the table show that the lowest noise is achieved by the pedestal-subtracting weights. This implies the presence of the low frequency noise, mentioned above, which is removed by the dynamic pedestal subtraction.

Comparing the noise seen in a single channel with that seen in sums of 3×3 and 5×5 channels the magnitude of the noise correlation between channels in a matrix of crystals can be seen. With the pedestal-subtracting weights filter the total noise seen in a sum of 9 (25) channels is almost exactly 3 (5) times the noise seen in a single channel, showing that the coherent noise between channels has been effectively removed. This noise is not suppressed when the average pedestal values for each channel are subtracted for each event. The total noise in a sum of 25 channels is reduced by 20% as compared to what is measured by reconstruction followed by pedestal subtraction using an average pedestal. Dynamic subtraction of the pedestal also avoids effects from variation of the pedestal over time.

The table also allows comparison between the use of 3+5 and 3+1 pedestal- subtracting weights. Using 5 samples in the signal, rather than 1, results in a slightly lower value of noise.

Using pedestal-subtracting weights the average value of the noise measured in 1000 channels of the supermodule is roughly 40 MeV/channel (1 ADC count ≅ 37 MeV). Nevertheless it can be seen from the table that this implementation of the weights method is not able to reduce the noise contribution by the factor of 0.6, expected when there is no correlation of noise. Clearly there is noise correlation, so the covariance matrix might be used to derive a more optimized set of weights. An investigation of the use of the covariance matrix is described in the next subsection.

3.2.2 Use of the covariance matrix

The covariance matrix represents the correlation of the noise between time samples. It is defined as:

$$\mathrm{C}_{i,j} = < n_i \times n_j > \tag{3.2}$$

where $n_i = S_i - P_i$ is the difference between the sample value and its mean (the pedestal) for sample i in the absence of a signal. The notation <> indicates an average over many events. Thus the diagonal elements $C_{i,i} = \sigma^2$ are the squared single sampling noise. This matrix can be built using a pedestal run and then used in the determination of an optimized set of weights (Equation 2.3).

The same 6 pedestal runs previously used for the results in Table 1 were processed to extract the matrix coefficients. An adequate number of events was used to limit the statistical error on these coefficients. An example of the normalized covariance matrix, $(\mathrm{C}_{i,j}/\sigma^2)$, is shown in Fig. 3. With this normalization it is referred to as the correlation matrix, and shows the noise correlations between the 10 readout samples.

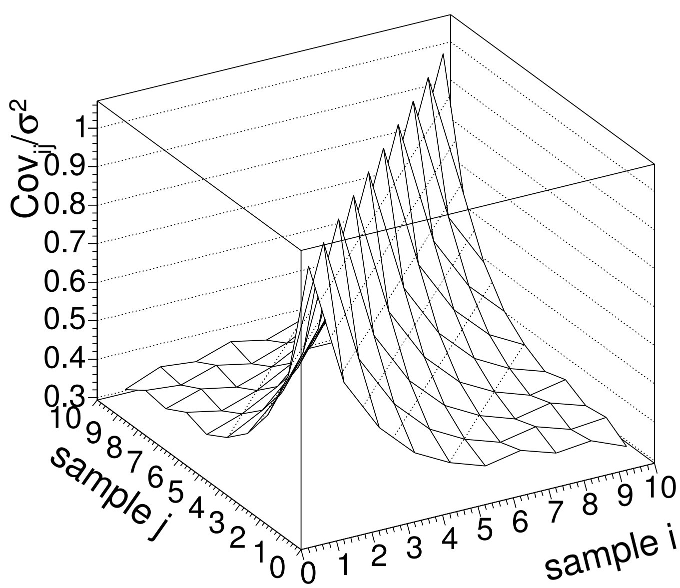


Figure 3: Normalized covariance matrix $(\mathrm{C}_{i,j}/\sigma^2)$, showing the correlation of the noise between the 10 samples of a given channel.

Strong correlations are present between samples which are close in time, as should be expected since the sampling period is shorter than the electronics shaping time. The MGPA preamplifier noise in the highest gain range is dominated by parallel noise from a feedback resistor and so the correlation between pairs of samples monotonically reduces as the time interval between them increases, until it reaches a residual value which corresponds to the low frequency pickup noise. The decrease of the correlation follows an exponential law whose time constant is related to the shaping time of the electronics.

The optimization of the weights using the noise covariance matrix results in giving more weight to the sample on the peak. The samples near the peak have less weight, and the weights applied to samples further away become almost insignificant. The expected noise reduction from the single sampling noise is 0.94 (see Fig. 2) and not 0.6 as calculated for the case of no noise correlation. Pedestal subtracting weights can also be derived using the covariance matrix.

Table 2: Noise in ADC counts measured in a single channel and arrays of 3×3 and 5×5 channels for the 5-weights and 3+5 pedestals subtracting weights implementations of the amplitude reconstruction method using the covariance matrix to derive the weights.

Method	Noise (ADC counts)		
	1×1	3×3	5×5
5-weights	1.05±0.02	3.3±0.1	5.9±0.1
3+5 Pedestal subtracting weights	1.05±0.02	3.0±0.1	5.2±0.1

Table 2 gives the noise measured when the covariance matrix is used to derive the weights. The results shown are the mean values of the noise measured in 8 different locations across the supermodule. Comparing the results in the table with those shown in Table 1 it can be seen that the use of the covariance matrix allows a small decrease in the contribution of noise, but the improvement over pedestal-subtracting weights without its use is marginal and all further results in this paper have been obtain without its use.

3.3 Effect of pileup events

Shaped signals cover several bunch crossings. When using the multi-weights method described above, up to 8 time samples are used. Pileup noise will occur if additional particles reaching the calorimeter cause signals which overlap these samples.

The magnitude of pileup noise expected at low luminosity ($\mathcal{L} = 2\times 10^{33}$ cm^{-2}s^{-1}) was simulated and studied using CMS reconstruction software. The single sampling electronics noise was set to 40 MeV per channel. Correlations between time samples were simulated to reproduce the correlations observed in the supermodule tested in the beam. Pileup noise was simulated using minimum bias events generated between -5 and +3 bunch crossings before and after signal. The average number of minimum bias events used per bunch crossing was 3.5. Figure 4 shows the reconstructed amplitude observed with and without pileup in the absence of any signal (the pileup is being considered as noise). The figure shows that at low luminosity the pileup noise is small with respect to electronics noise.

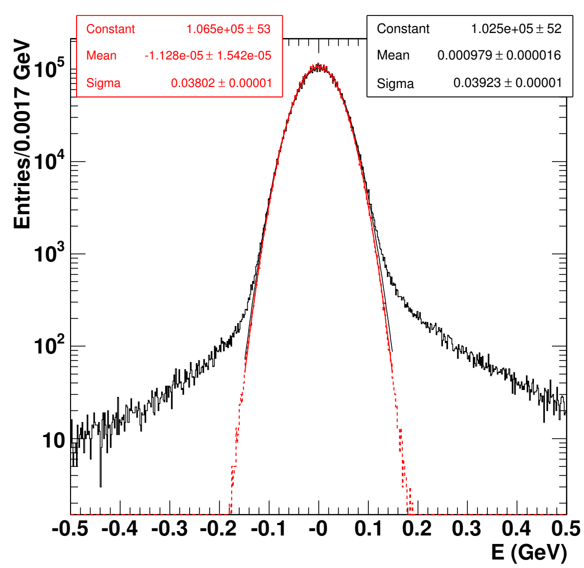


Figure 4: Reconstructed amplitude in ECAL barrel channels in the absence of a signal, without pileup (dashed histogram) and with pileup (solid histogram).

High energy pileup particles from different bunch crossings to the signal can be identified using cuts on χ^2_{Min} (defined in Appendix A) as well as the variable $\Delta\hat{P}$ which corresponds to the difference between the reconstructed baseline and the expected one (evaluated from pedestal runs). Distributions of reconstructed signal pulses with and without high energy (>200 MeV) pileup particles are shown in Fig. 5 as a function of χ^2_{Min} and $\Delta\hat{P}$. The bands shown in the scatter plot for events containing high energy pileup correspond to particles from different bunch crossings. By applying cuts on the variables χ^2_{Min} and $\Delta\hat{P}$ it is possible to remove out of time pileup signals that deposit a significant amount of energy.

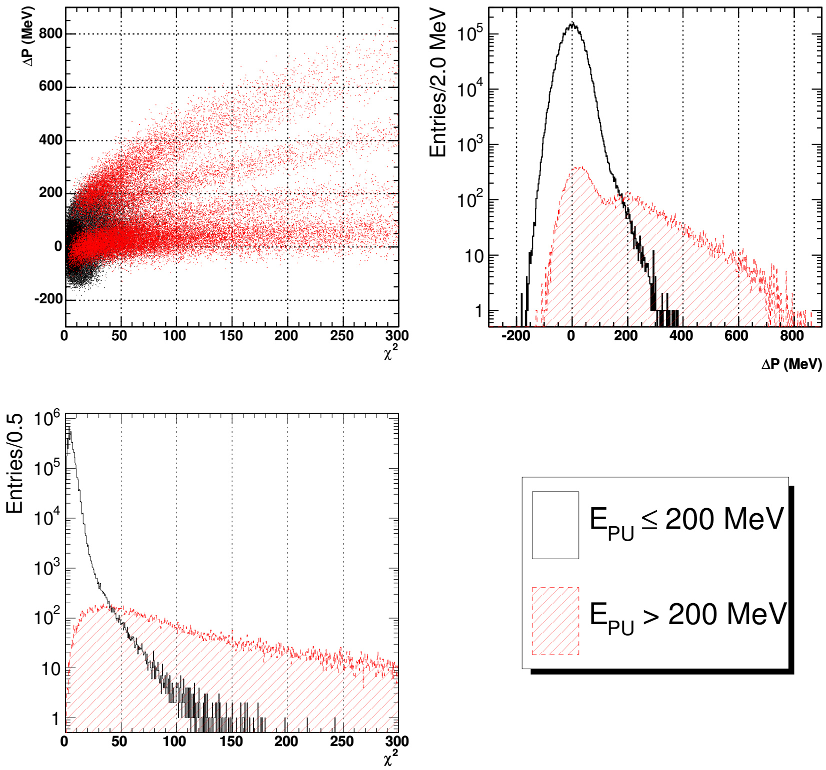


Figure 5: Distribution of $\Delta\hat{P}$ versus χ^2 in ECAL barrel, together the projections of these variable plotted as as histograms. Distributions are shown separately for pulses which include out of time pileup particles with an energy greater than 200 MeV and those which do not. The bands in the scatter plot correspond to particles from different bunch crossings.

4 Amplitude reconstruction for CMS running

An investigation of the amplitude reconstruction method required for use with CMS data taken in LHC, where the sampling is synchronous with the signals, is described in this section.

4.1 Signal shape and timing

It has been observed in test beam data that the signal shapes and timing differ from crystal to crystal. The main difference being the parameter T_{Max} which shows an rms dispersion of roughly 3 ns across the supermodule. Using a a representation of the signal with a T_{Max} different from that of the actual signal to determine the weights changes the reconstructed amplitude. A timing difference of δt = 1 ns causes approximately 0.1% bias on the reconstructed amplitude when using the 3+5 weights method (Fig. 6). The 3+1 weights method is less sensitive to the timing because the single sample is centred on the peak, whereas the signal samplings of the 3+5 weights are not; they are mostly after the peak. The small bias, if constant with time, will be absorbed into the intercalibration of the ECAL channels. However systematic drift or variation of the time of the signal pulse maximum would result in a variation of the channel response with time. To avoid such variation the parameter T_{Max} needs to be carefully monitored and controlled. T_{Max} can be precisely measured during data taking (see Section 4.2) and adjusted for each front end card (25 channels) with a 1 ns precision, by changing a parameter downloaded to the card.

Figure 7 (left) shows the energy resolution as a function of the difference in timing between the signal represen-

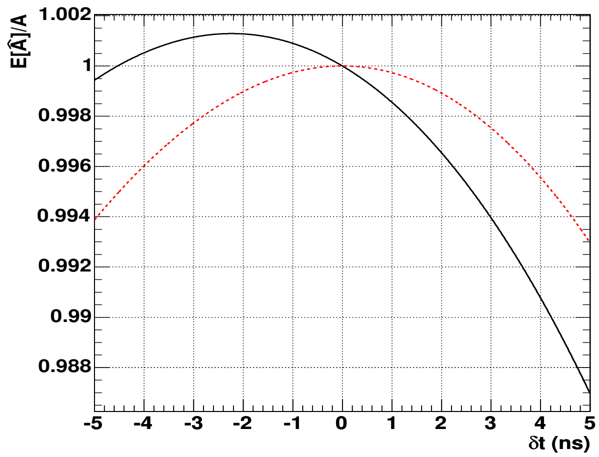


Figure 6: Reconstructed amplitude over true amplitude, $E[\hat{A}]/A$, as a function of timing difference δt. The solid line shows the value obtained with 3+5 pedestal-subtracting weights, and the dashed line shows the value obtained using 3+1 pedestal-subtracting weights.

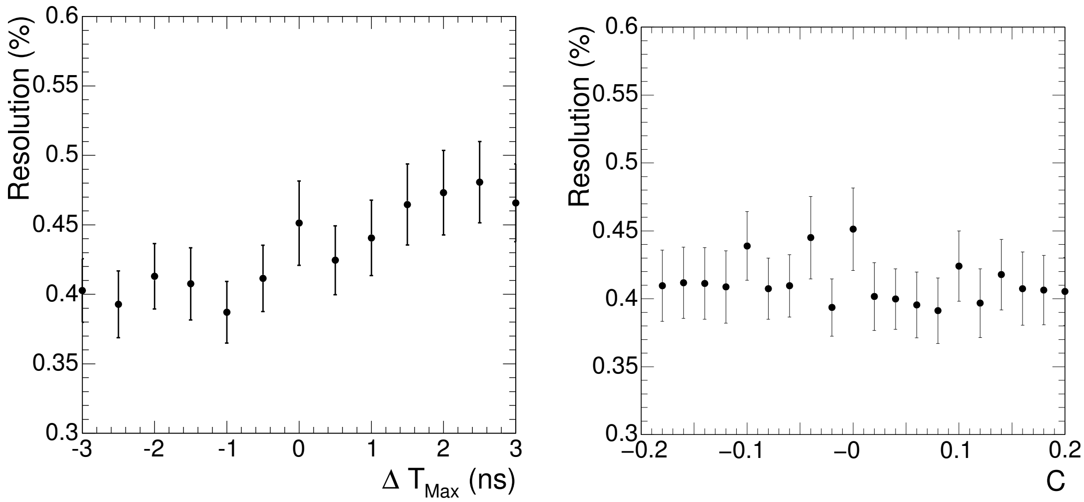


Figure 7: (Left) Energy resolution in a 3×3 matrix of crystals measured with a 120 GeV electron beam, as a function of $\Delta T_{\rm Max}$. (Right) Energy resolution as a function of the contraction factor, C.

tation used to determine the weights and the signal. To simulate synchronous running, test beam data have been analyzed taking only events in a single 1 ns bin of phase, where the sampling phase is such that the signal maximum coincides with the 2^{nd} signal sampling. The rms dispersion of T_{Max} observed in the supermodule is 3 ns, and a range of ± 3 ns was used for the scan. Within errors the resolution is not affected by variation of T_{Max} within this range. However, the resolution is on average better for $\Delta T_{\text{Max}} < 0$. This is a result of the large 0.65 ns jitter on the phase measurement. When $\Delta T_{\text{Max}} > 0$ there is more sensitivity to timing as has been seen in Fig. 6.

To study the sensitivity to differences between the pulse shape of the representation used to derive the weights and that of the signal, a homothetic transformation characterized by a contraction factor, C, has been applied to the time scale ($t \to t + C \times (t - T_{\text{Max}})$) of the representation used to derive the weights. Fits to the data indicate that the channel to channel pulse shape variation within the supermodule tested corresponds to an rms dispersion of 0.05 for the parameter C. Figure 7 (right) shows that the resolution is unaffected by variations of C within a range corresponding to a much larger dispersion in pulse shapes than observed in the supermodule. Other variations of the pulse shape have been tried (varying independently the rise and fall times) and result in the same conclusion.

These results suggest that, in the case of synchronous running, the same reference signal representation can be used to derive a single set of weights to be used for the signal amplitude reconstruction of all channels.

4.2 Time measurement

As already mentioned, the phase between pulse maximum and digitization clock can be adjusted for each front end card (25 channels) to 1 ns precision. In the supermodule tested in 2004 it was observed that the additional dispersion of the channels within each front end card was less than 1 ns. The time of maximum can be calculated precisely with weights if the signal timing variations are small. The set of weights which is derived to do this is different from the set of weights used to determine the amplitude (see Appendix A). The task can be described as the determination of the difference between a nominal time of maximum, T^0_{Max}, where the peak of the pulse corresponds to a sampling, and the actual time of maximum, T_{Max}:

$$\delta t = T_{\text{Max}} - T^0_{\text{Max}} \tag{4.1}$$

In this case, a linear expansion of the function $G(A, P, T_{\text{Max}})$ can be performed:

$$G_i(A, P, T^0_{\text{Max}}, \delta t) = Af(t^0_i + \delta t) + P \simeq Af(t^0_i) + (A\delta t)\frac{df}{dt}(t^0_i) + P. \tag{4.2}$$

As in Section 2, weights are obtained by minimizing χ^2 with respect to A, P and $A\delta t$. The estimated difference of the time of maximum from the nominal time of maximum, $\delta\hat{t}$, is biased by less than 20 ps if $|\delta t| < 1$ ns (see Fig. 8).

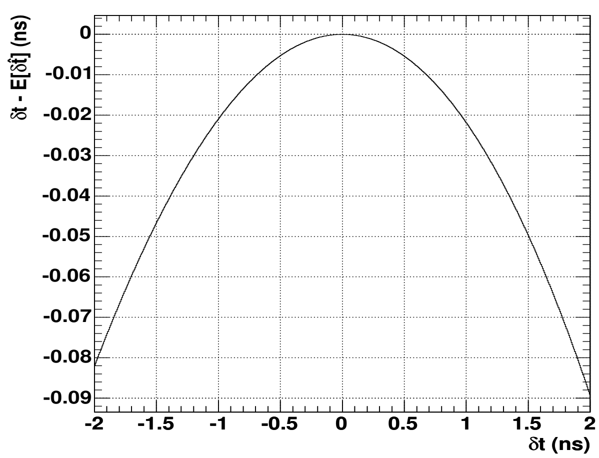


Figure 8: Time bias $\delta t - E[\delta\hat{t}]$ introduced by linear approximation as a function of δt.

Equation 4.3 relates the variance of $\delta\hat{t}$ to the amplitude of the signal:

$$V(\delta\hat{t}) \simeq \frac{a^2}{c^2 + (A/\sigma)^2} + b^2, \tag{4.3}$$

where a, b, c are three parameters and σ is the single sampling noise. Parameter $c = \sqrt{V(\hat{A})/\sigma^2} \simeq 0.94$ becomes negligible when $A \gg \sigma$. Parameter a is related to $\sqrt{V(\hat{A}\delta\hat{t})/\sigma^2}$ and strongly depends on the phase choice as $V(\hat{A}\delta\hat{t})$ is proportional to the sum of the derivatives of the signal as a function of time. Parameter b appears when the functional representation does not exactly reproduce the true signal.

During the 2004 test beam data taking it was not possible to directly measure all the parameters of $V(\delta\hat{t})$ as a function of $\hat{A}$ because of an additional jitter introduced by measurement of the trigger time. The resolution on $\hat{t} - T$, where $\hat{t}$ is the time estimated, and T is the trigger time measured by the TDC, is sensitive to the uncertainty on the measured trigger time ($V(\hat{t} - T) = V(\hat{t}) + V(T)$), as is shown in Fig. 9. The trigger time measurement is independent of the electron energy, so $V(T)$ increases the measured constant term of the time resolution. But the amplitude dependent part of the curve is related to parameter a in Equation 4.3.

In order to be sensitive to the parameter b (the constant term) in Equation 4.3, it is necessary to cancel the trigger time mismeasurement. This can be done by looking at the resolution of time difference between two channels. Figure 10 shows the resolution on the time difference between two channels as a function of the quadratic sum of the individual channel time resolutions with respect to the trigger time measured in the channels. The constant term b can be extracted from a fit to these data points assuming that parameters a, b and c are the same for both channels. With this assumption $V(\hat{t}_1 - \hat{t}_2)$ is:

$$V(\hat{t}_1 - \hat{t}_2) \simeq V(\hat{t}_1) + V(\hat{t}_2) \simeq a \times \left(\frac{\sigma_1^2}{E[\hat{A}_1]^2} + \frac{\sigma_2^2}{E[\hat{A}_2]^2} \right) + 2 \times b. \tag{4.4}$$

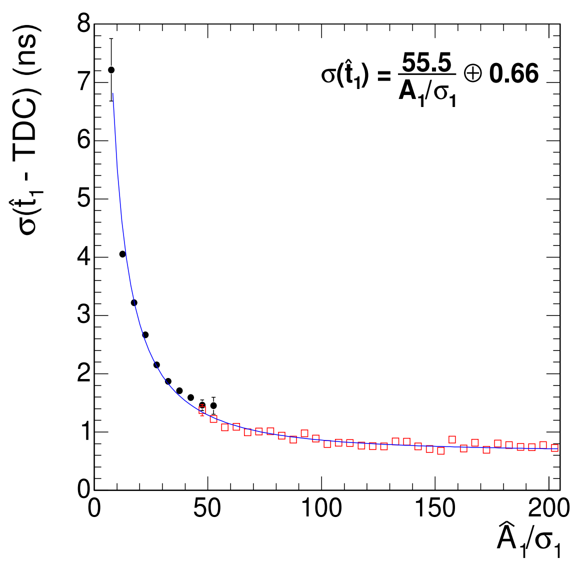


Figure 9: Time resolution versus amplitude for the crystal 703 studied during 2004 Test Beam. The circles and squares correspond to two different runs. The solid line is obtained when fitting all points.

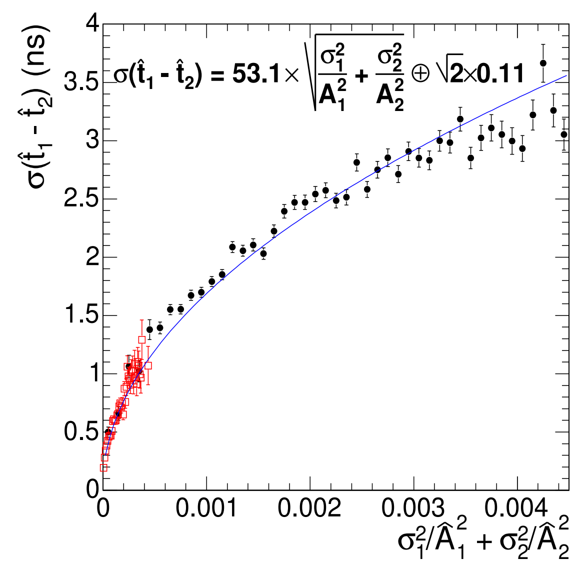


Figure 10: Resolution on time difference between two crystals 703 and 704 as a function of the sum of inverse square amplitudes. The fit is made assuming that the parameters a and b of Equation 4.3 are the same for both crystals.

It can be concluded that:

- taking reconstructed pulses with $\hat{A} > 50 \times \sigma$, i.e. energy > 2 GeV, T_{Max} can be measured to a precision better than 1 ns, allowing monitoring of peak position to the required precision (see Section 4.1);

- the time resolution is dominated by the constant term for pulses with $\hat{A} > \frac{a}{b}\sigma \simeq 500 \times \sigma$. The magnitude of the constant term depends on the precision to which the functional representation used in the computation of the derivatives matches the true signal. In the 2004 beam test the constant term was measured to be 0.11 ns.

4.3 Amplitude reconstruction at high energy

At high energy, the data read out contains samples recorded with different gains. The pedestal value of the ADC digitizing the lower gain signal is not the same as the pedestal value of the ADC digitizing the higher gain signal. The 3 presamplings thus do not give a measure of the pedestal of the ADC used for the most significant samplings of the signal (i.e. close to the peak). Thus a pedestal-subtracting weights method cannot be used. There is also little to be gained from using many weights since noise is negligible at these energies (the gain range change takes place at about 150 GeV for barrel channels). It is sufficient to measure the signal amplitude with a single sampling. In synchronous running the sample recorded on the peak is pedestal subtracted and then multiplied by the gain ratio (the relative calibration with respect to the highest gain range). The gain ratios must be determined precisely to avoid a degradation of the resolution and the introduction of non-linearity.

4.4 Summary of amplitude reconstruction for synchronous running

A summary of an implementation proposed for CMS running is outlined below:

- *Low energy*: For energies lower than 150 GeV, all the samples are digitizations of the highest gain signal. The reconstruction of the amplitude uses 3+5 pedestal-subtracting weights. These weights can all be determined from the same reference signal representation and are thus common to all channels of the ECAL.
- *High energy*: The saturation of the highest gain ADC occurs for a signal of about 150 GeV. For higher energy signals the reconstruction of signal amplitude uses a single sampling made on the signal pulse peak after pedestal subtraction. The value of the pedestal is the average value for that channel determined from special dedicated pedestal runs where the gain range is forced to be in a gain range other than the highest.

Figure 11 shows the distributions of the reconstructed energy obtained using the amplitude reconstruction implementation proposed above when electrons with energies of 120 and 250 GeV are incident on the ECAL. The shower energy is reconstructed in 3×3 matrices of crystals after applying intercalibration constants to equalize the responses, and scaled to the beam momentum.

To simulate synchronous running, test beam data have been analyzed taking only events in a single 1 ns bin of phase where the sampling phase is such that the signal maximum coincides with the 2^{nd} signal sampling. In order to limit the variation of shower containment, the impacts of the incident electrons are restricted to a $4 \times 4\mathrm{mm}^2$ region using cuts on the incident electron position measured by the beam hodoscopes. Long dedicated runs of 200 000 events were used to obtain adequate numbers of events despite the large losses when selecting events passing both the incident position cuts, and the strict phase requirement. The energy resolutions shown in the figure were obtained from the truncated Gaussian fits shown.

The test beam phase measurement has a jitter which is estimated to be about 0.65 ns. Compensating this effect using a method described in Section 5.2 gives a resolution of 0.40±0.03 % at 120 GeV. It should be noted that the measured energy resolution at 250 GeV contains a significant contribution from beam momentum spread; subtracting this contribution gives a calorimeter resolution of 0.39 %.

5 Application to asynchronous test beam data

In the test beam the ADC clock and the signal are asynchronous, and the implementation of the amplitude reconstruction needs to be elaborated to deal with this. The phase between the trigger (and thus the signal pulse) and the 40 MHz ADC clock was measured with a TDC. The data are sorted into 25 bins of 1 ns according to the measured phase. A different set of weights is determined and used for each of the bins. This imposes an additional severe constraint: the same amplitude must be reconstructed in all phase bins.

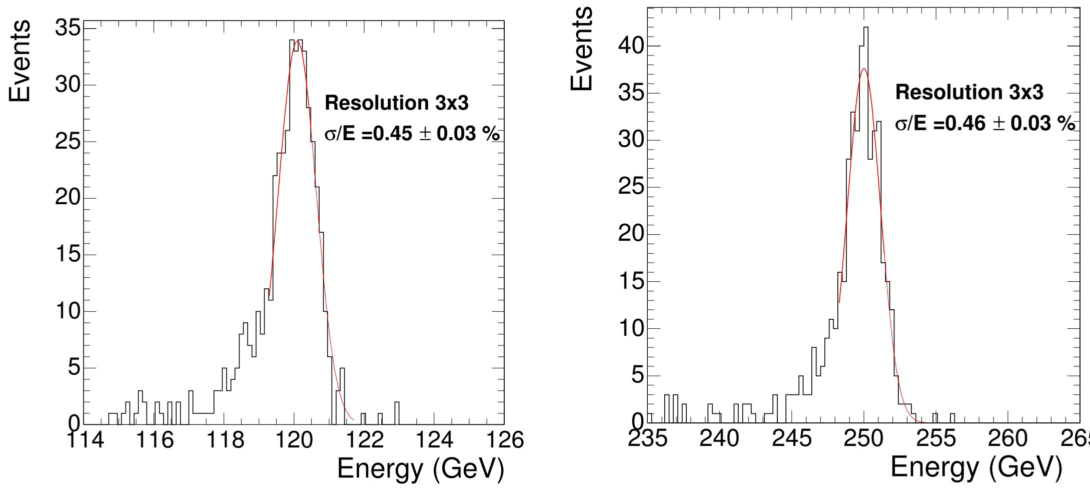


Figure 11: Distribution of the reconstructed amplitude with the 3+5 weights method summed in a 3×3 matrix of crystals for an electron energy of 120 GeV (left) and 250 GeV (right). The resolution extracted from the width of the superimposed fitted Gaussian (fit range is $[-1.5\sigma, 3\sigma]$) is shown for both distributions. The data taken is restricted to a single 1ns bin in phase, to simulate synchronous running.

5.1 Effect of channel to channel differences in signal timing and shape

The effect of differences in signal timing and shape between the signal pulse and the signal representation is dependent on the phase bin, and can be studied by comparing the results obtained using weights derived from an average representation of the signal pulse with those obtained using weights derived from a profile built using the data collected from the channel being investigated. The average representation is obtained by fitting the data from a large number of channels with an analytic formula and then taking the average of the fitted parameter values. A comparison between the average representation and a particular profile is shown in Fig. 12. The shapes are different in the rising edge, and the time of maximum also differs. Two collections of 25 sets of weights (one set for each phase bin) are extracted, one for each representation, and used to reconstruct the amplitude.

Figure 13 shows the average reconstructed amplitude summed in a 3×3 matrix of crystals in each of the 25 1 ns phase bins. The incident electron impact area is restricted to a 4×4 mm^2 window around the maximum containment point of the crystal. The average reconstructed amplitude, obtained from a Gaussian fit to the peak of the reconstructed amplitude distribution, is plotted as a function of the phase for both sets of weights. Using the weights derived from the average representation results in a bias in the reconstructed amplitude which varies with phase and ultimately degrades the energy resolution as can be seen in Fig. 14. When the weights are derived using the signal description specific to each channel, the resolution is as good as the one obtained in a single 1 ns bin. The bias visible in Fig. 13 when the weights used are those derived from the average representation, could also be corrected using the fitted line which would lead to a comparable resolution.

5.1.1 Relative importance of shape and timing

The relative importance of differences in signal timing and shape between the signal pulse and the signal representation used to derive the weights has been studied. As in Section 4.1 the signal in each channel is characterized by its peaking time $T_{\rm Max}$ and its width. Figure 15 shows the energy resolution measured when the weights used to reconstruct the amplitude are derived from signal representations with peaking times and widths which differ from those of the actual signal. The situation here differs from that shown in Fig. 7 in that here data in all 25 phase bins are being reconstructed, using 25 sets of weights. This is reconstruction of the full asynchronous test beam data set, wheres Fig. 7 was the reconstruction of data in a single phase bin, simulating synchronous data.

When the weights are derived using a signal representation that has a width 0.05 (the rms dispersion in a supermodule) different from the signal width the resolution is degraded from 0.45% to 0.52%. When the weights are derived using a signal representation that has a $T_{\rm Max}$ 3 ns (the rms dispersion in a supermodule) different from that of the signal, the resolution is degraded from 0.45% to 0.70%. Thus the most significant cause of the resolution degradation resulting from the use of an average signal pulse representation to derive the weights for reconstruc-

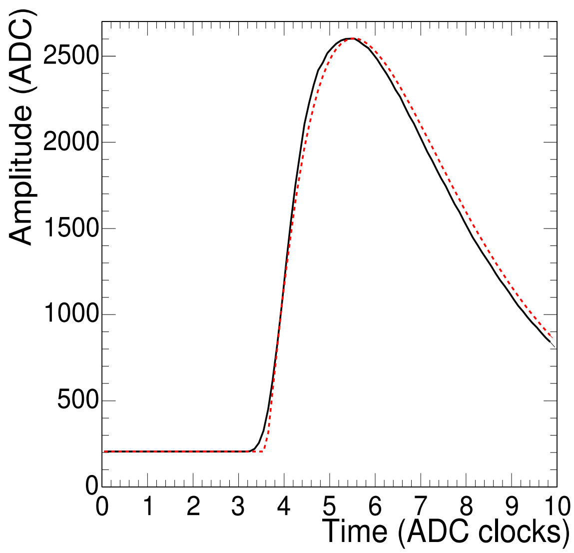


Figure 12: Signal pulses for a 120 GeV electron beam: the solid line shows the profile of the signal in the channel under study, and the dotted line shows the average representation of the signal.

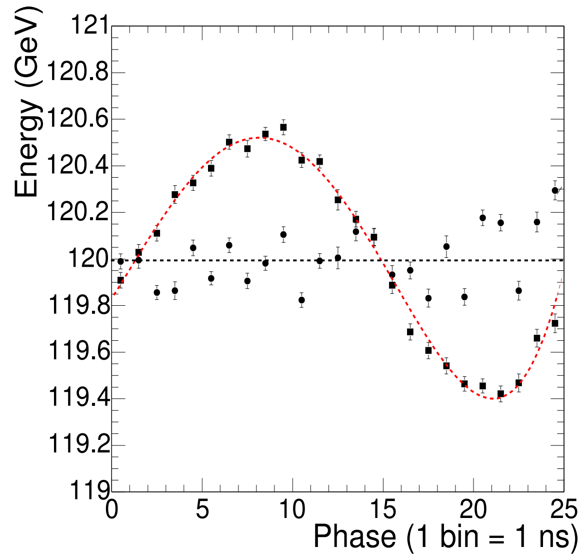


Figure 13: Average reconstructed amplitude summed in a 3×3 matrix of crystals as a function of the phase using weights obtained with the average representation of the signal (squares) and the signal profile from the channel under study (circles). The polynomial fit and straight line are shown to guide the eye.

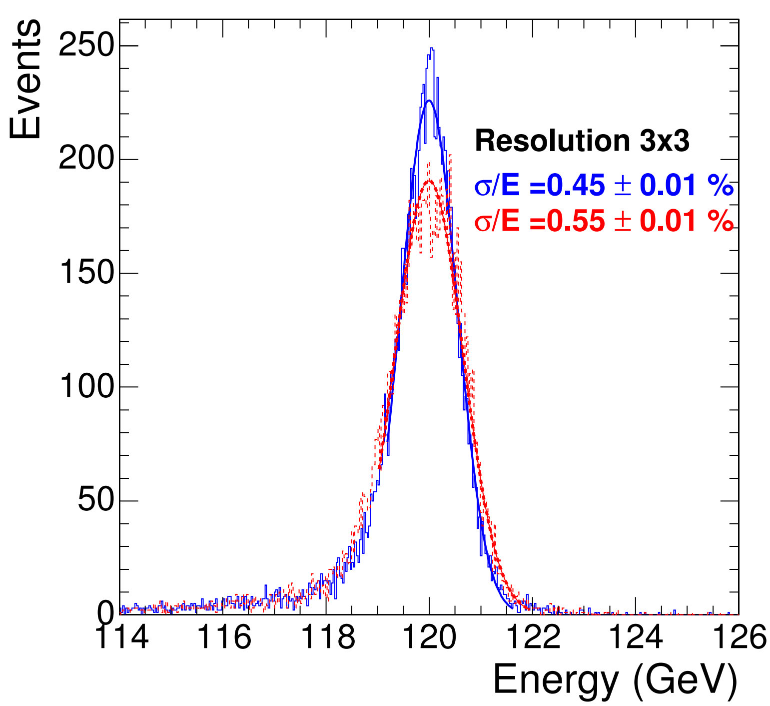


Figure 14: Distribution of the reconstructed amplitude in a 3×3 matrix for a 120 GeV beam using the weights derived from an average representation of the signal pulse (dashed histogram) and the true profile of the signal in the crystal studied (solid histogram). Truncated Gaussian fits are superimposed on both distributions and are used to derive the energy resolution: dashed histogram 0.55%, solid histogram 0.45%. (Asynchronous data using all phase bins).

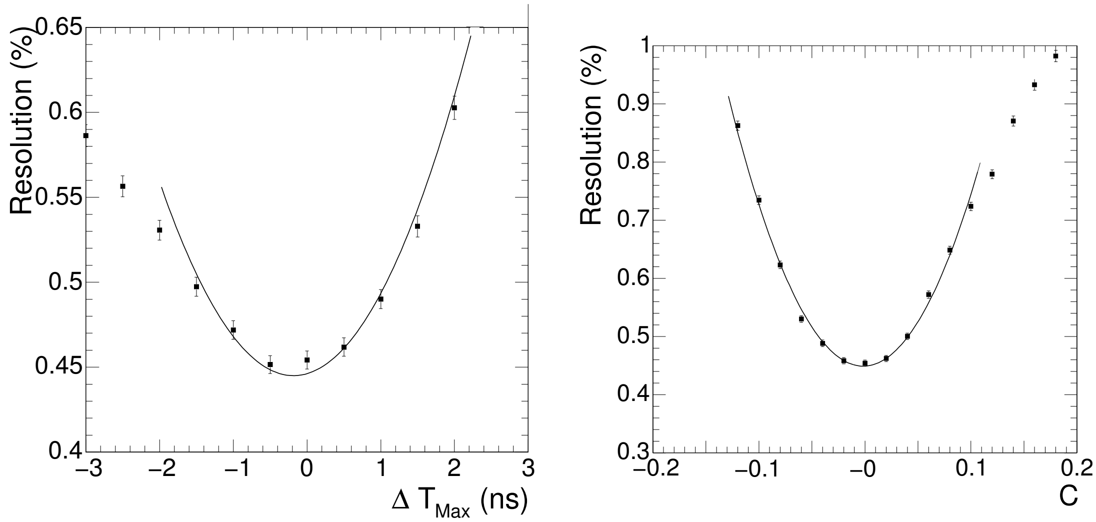


Figure 15: (Left) Energy resolution in a 3×3 matrix of crystals for a 120 GeV electron beam, as a function of ΔT_{Max}. (Right) Energy resolution as a function of the contraction factor, C.

tion of asynchronous data is mismatch of signal timing. It should be possible, in the future test beam data taking, to adjust the signal timing to within 1 ns. Alternatively, the mistimings could be categorized, and a number of different sets of weights used.

5.2 Timing jitter

As described in Section 5, different sets of weights are used for events in different bins of phase. The TDC measurement is used to decide which set of weights should be used. However an rms uncertainty of roughly 0.65 ns on the TDC phase measurement was observed. This results in an additional smearing of amplitudes which degrades the energy resolution.

The derivation of the weights can be modified to account for small variations of T_{Max}, or jitter, and produce what we call "jitter-compensating" weights (see Appendix A). Figure 16 shows the results of a study using simulated signal pulses to determine the effect of jitter on the constant term as a function of pulse maximum time for both standard amplitude reconstruction method and jitter-compensating weights. The effect of jitter in the case of CMS running ($\simeq$ 0.2 ns due to the bunch length) is also shown. The results demonstrate that the affect of jitter will be negligible in CMS running, without the use of jitter-compensating weights, especially if the sampling is performed on the peak.

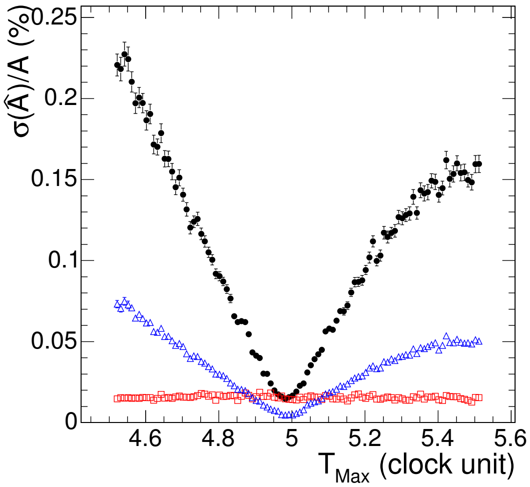
Figure 17 shows the distribution of reconstructed amplitude summed in a 3 × 3 matrix of crystals using pedestal subtracting weights and pedestal-subtracting weights with jitter compensation. The improvement observed when using jitter-compensating weights, has motivated the choice of using these special sets of weights to extract performance results from test beam data taken in 2004.

5.3 Reconstruction in full dynamic range

If a single sampling is used for amplitude reconstruction in asynchronous running, the resolution obtained is seriously degraded by the jitter on the phase measurement when the sampling occurs away from the maximum. In order to reconstruct the amplitude in asynchronous running when a range change has occured, a "jitter-compensating" set of weights must be used, and this necessitates the use of more than one sampling.

The MGPA system implements a "hysteresis" in the choice of gain range so that the range, once changed, does not revert to the higher gain range for at least 5 samplings. A 5-weights method is implemented for reconstruction of the signal amplitude. The first 5 samples in the lower gain are fed into the algorithm after pedestal subtraction using stored average values.

Figure 18 shows the distributions of the reconstructed amplitude summed in a 3×3 matrix of crystals, and compares the use of different implementations to reconstruct the amplitude of the crystal for which the range has changed.



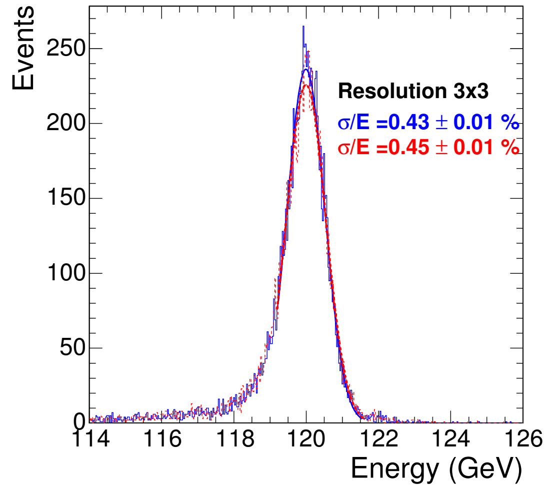


Figure 16: Amplitude measurement spread due to jitter as a function of pulse maximum time. Solid circles and open squares correspond to simulated events with a timing jitter of 0.65 ns. The solid circles are obtained when standard amplitude reconstruction is used, whereas the open squares result from use of jitter-compensating weights. The open triangles correspond to standard amplitude reconstruction when the jitter is equal to 0.2 ns as expected in CMS.

Figure 17: Distribution of the reconstructed amplitude summed in a 3 $\times$ 3 matrix of crystals using pedestal-subtracting weights with (solid histogram, σ/E= 0.43%) and without (dashed histogram, σ/E= 0.45%) jitter compensation. (Asynchronous data using all phase bins).

The best resolution is obtained using jitter-compensating weights.

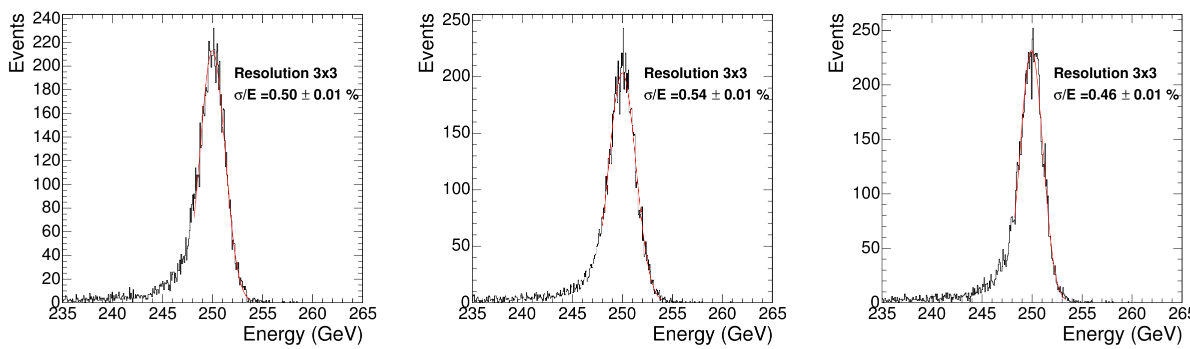


Figure 18: Distributions of the reconstructed amplitude summed in a 3 $\times$ 3 matrix of crystals for data taken with 250 GeV electrons. The amplitude reconstruction uses (left) 1-weight, (centre) 5-weights, and (right) 5-weights with jitter compensation. (Asynchronous data using all phase bins).

5.4 Summary

The implementation of the weights method for reconstruction of data taken in the test beam is as follows:

- *Low energy*: The reconstruction is performed by 3+5 pedestal-subtracting and jitter-compensating weights. A different set of weights is used for each crystal of the supermodule to avoid degrading the resolution due to a systematic bias of the reconstructed amplitude as a function of phase. Such a systematic bias arises when using a signal representation with an offset time of maximum, or a very different pulse width from the actual signal.
- *High energy*: Amplitude reconstruction uses a set of 5 jitter-compensating weights. The first 5 samples

measured in the lower gain are used with a 5-weights method. Compensation of the jitter is necessary when analyzing 2004 test beam data to prevent the degradation of the resolution due to the jitter of 0.65 ns on the trigger time measurement.

6 Conclusion

Reconstruction of the signal amplitude in the CMS ECAL can be performed using digital filtering techniques. Using an implementation with dynamic pedestal subtraction, the measured noise in a single channel is 40 MeV, and coherent noise between channels is effectively removed so that the noise measured in a 3×3 (5×5) matrix of crystal is about 120 MeV (200 MeV). Additional noise induced by pileup events was investigated with simulated data and shown to be negligible at low luminosity.

Studies of the test beam data collected in 2004 with a fully equipped supermodule allowed investigation of the importance of the shape and timing of the reference signal representation used to derive the weights, as well as the effect of timing jitter on the resolution. The impact on the resolution of the shape and timing of the reference signal representation used to derive the weights has been clearly demonstrated to be caused by the asynchronous data taking mode in the test beam. The necessity to reconstruct the same amplitude for all values of the phase imposes severe constraints on the tolerable variation of timing and pulse shape between channels. The poor time resolution of the phase measurement used in the 2004 test beam data taking also requires use of more complex and sophisticated techniques to avoid degradation of the resolution.

In synchronous running a single set of weights can be used to reconstruct the amplitude in all the ECAL channels and timing information can be provided using an additional set of weights. The time measurement can be used to monitor possible shifts of the signal timing.

7 Acknowledgments

We would like to express our gratitude to the people who worked long hours on the set-up of the super-module, its readout electronics and data acquisition system which made these studies possible. Thanks are also due to the CERN staff who operated the accelerator to deliver the beam in the H4 beam line. We thank the ECAL Test Beam community coordinated by Yves Sirois for numerous and useful interactions on the subjects and issues presented in this note. Thanks to Yves for his useful suggestions and encouragements during this work. We specially thank Chris Seez for his important input in the development of the method as well as his useful comments and advice throughout these studies. Thanks to Patrick Jarry and Pascal Paganini for sharing their knowledge from previous experience on the subject. We had fruitful discussions with Mark Raymond and Nicolo Cartiglia about the intrinsic functioning of the electronics, thanks for their help.

A Appendix : Weights derivation

In order to derive optimal weights, a least squares method is used. The χ^2, defined in Section 2, can be written in matrix notation as:

$$\chi^2 = (\mathbf{S} - \mathbf{G}(A, \delta t, P))^T \mathbf{C^{-1}} (\mathbf{S} - \mathbf{G}(A, \delta t, P)) \tag{A.1}$$

where $\mathbf{S}$ is a vector of the time samples S_i with N elements, $\mathbf{C}$ is the covariance matrix of noise, and $\mathbf{G}(A, \delta t, P)$ is a vector describing the mean of the measurements and is modeled by:

$$G(t_i; A, \delta t, P) = Af(t_i + \delta t) + P \tag{A.2}$$

where A is the amplitude of the signal, $f(t)$ is the function which corresponds to the time development of the signal pulse, δt is a possible timing jitter and P is the pedestal. When δt is small enough, Equation A.2 can be linearized:

$$G(t_i; A, \delta t, P) \simeq Af(t_i) + (A\delta t)\frac{df}{dt}(t_i) + P \tag{A.3}$$

restricting the problem to the situation in which $\mathbf{G}(A, \delta t, P)$ is a linear function of the free parameters:

$$\chi^2 = (\mathbf{S} - A\mathbf{F} - (A\delta t)\mathbf{F'} - P\mathbf{1})^T \mathbf{C^{-1}} (\mathbf{S} - A\mathbf{F} - (A\delta t)\mathbf{F'} - P\mathbf{1}) \tag{A.4}$$

where $\mathbf{F}$ is a vector of $f(t_i)$, and $\mathbf{F}'$ is a vector of $\frac{df}{dt}(t_i)$ and $\mathbf{1}$ has all its vector elements equal to 1. Minimizing χ^2 with respect to A, $A\delta t$ and P, a linear system of three equations is obtained:

$$\begin{pmatrix} \mathbf{F}^T\mathbf{C}^{-1}\mathbf{F} & \mathbf{F}^T\mathbf{C}^{-1}\mathbf{F}' & \mathbf{F}^T\mathbf{C}^{-1}\mathbf{1} \\ \mathbf{F}'^T\mathbf{C}^{-1}\mathbf{F} & \mathbf{F}'^T\mathbf{C}^{-1}\mathbf{F}' & \mathbf{F}'^T\mathbf{C}^{-1}\mathbf{1} \\ \mathbf{1}^T\mathbf{C}^{-1}\mathbf{F} & \mathbf{1}^T\mathbf{C}^{-1}\mathbf{F}' & \mathbf{1}^T\mathbf{C}^{-1}\mathbf{1} \end{pmatrix} \begin{pmatrix} \hat{A} \\ \hat{A}\hat{\delta t} \\ \hat{P} \end{pmatrix} = \begin{pmatrix} \mathbf{F}^T \\ \mathbf{F}'^T \\ \mathbf{1}^T \end{pmatrix} \mathbf{C}^{-1}\mathbf{S} \tag{A.5}$$

Denoting the matrix on the left of Equation A.5 as $\mathbf{M}$, the solution of the system can be expressed as:

$$\begin{pmatrix} \hat{A} \\ \hat{A}\hat{\delta t} \\ \hat{P} \end{pmatrix} = \mathbf{M}^{-1} \begin{pmatrix} \mathbf{F}^T \\ \mathbf{F}'^T \\ \mathbf{1}^T \end{pmatrix} \mathbf{C}^{-1}\mathbf{S} = \mathbf{WS} \tag{A.6}$$

$\mathbf{W}$ is the matrix of weights.
The covariance matrix $\mathbf{V}$ between the estimators is equal to:

$$\mathbf{V} = E[(\mathbf{WS} - E[\mathbf{WS}])(\mathbf{WS} - E[\mathbf{WS}])^T] = \mathbf{WCW}^T = \mathbf{M}^{-1} \tag{A.7}$$

Replacing the parameters by the solutions of Equation A.6 in Equation A.4, the minimal χ^2 value can be computed:

$$\chi^2_{Min} = \mathbf{S}^T(\mathbb{1} - \begin{pmatrix} \mathbf{F} & \mathbf{F}' & \mathbf{1} \end{pmatrix}\mathbf{W})^T\mathbf{C}^{-1}(\mathbb{1} - \begin{pmatrix} \mathbf{F} & \mathbf{F}' & \mathbf{1} \end{pmatrix}\mathbf{W})\mathbf{S} = \mathbf{S}^T\mathbf{M}_{\chi^2}\mathbf{S} \tag{A.8}$$

$\mathbf{M}_{\chi^2}$ is a matrix which can be used to compute χ^2_{Min} event by event without minimizing χ^2.
All simpler cases can be derived from Equations A.5 and A.6. Two common cases are:

1. Parameter A is the only free parameter. Simplifying Equation A.5 gives:

$$\mathbf{W} = \frac{\mathbf{F}^T\mathbf{C}^{-1}}{\mathbf{F}^T\mathbf{C}^{-1}\mathbf{F}}. \tag{A.9}$$

   In addition, if $\mathbf{C} = \sigma^2\mathbb{1}$:

$$W_i = \frac{f_i}{\sum_j^N f_j^2}. \tag{A.10}$$

   The variance of $\hat{A}$ is obtained from Equation A.7 : $V(\hat{A}) = \frac{1}{\mathbf{F}^T\mathbf{C}^{-1}\mathbf{F}} = \frac{\sigma^2}{\sum_j^N f_j^2}$.

2. Parameter A and P are free parameters and $\mathbf{C} = \sigma^2\mathbb{1}$. Solving Equation A.5 gives two sets of weights used to compute the two estimators

$$\begin{pmatrix} \hat{A} \\ \hat{P} \end{pmatrix} = \frac{1}{N\mathbf{F}^T\mathbf{F} - (\mathbf{F}^T\mathbf{1})^2} \begin{pmatrix} N\mathbf{F}^T - (\mathbf{F}^T\mathbf{1})\mathbf{1}^T \\ (\mathbf{F}^T\mathbf{F})\mathbf{1}^T - (\mathbf{F}^T\mathbf{1})\mathbf{F}^T \end{pmatrix} \mathbf{S} = \mathbf{WS} \tag{A.11}$$

   The weights can also be written as:

$$W_{A,i} = \frac{f_i - \sum_j^N f_j}{\sum_j^N f_j^2 - (\sum_j^N f_j)^2/N}, \; W_{P,i} = \frac{\sum_j^N f_j^2 - (\sum_j^N f_j)f_i}{N\sum_j^N f_j^2 - (\sum_j^N f_j)^2} \tag{A.12}$$

   Equation A.7 gives: $V(\hat{A}) = \frac{\sigma^2}{\sum_j^N f_j^2 - (\sum_j^N f_j)^2/N}$.

3. Jitter compensating weights can be obtained by leaving δt as a free parameter. In the case A and δt are the only two free parameters and $\mathbf{C} = \sigma^2\mathbb{1}$, weights are computed as

$$\begin{pmatrix} \hat{A} \\ \hat{A}\hat{\delta t} \end{pmatrix} = \frac{1}{(\mathbf{F}'^T\mathbf{F}')(\mathbf{F}^T\mathbf{F}) - (\mathbf{F}^T\mathbf{F}')^2} \begin{pmatrix} (\mathbf{F}'^T\mathbf{F}')\mathbf{F}^T - (\mathbf{F}^T\mathbf{F}')\mathbf{F}'^T \\ (\mathbf{F}^T\mathbf{F})\mathbf{F}'^T - (\mathbf{F}^T\mathbf{F}')\mathbf{F}^T \end{pmatrix} \mathbf{S} = \mathbf{WS}, \tag{A.13}$$

   which gives

$$W_{A,i} = \frac{(\sum_j^N \frac{df_j}{dt}^2)f_i - (\sum_j^N f_j\frac{df_j}{dt})\frac{df_i}{dt}}{(\sum_j^N f_j^2)(\sum_j^N \frac{df_j}{dt}^2) - (\sum_j^N f_j\frac{df_j}{dt})^2}. \tag{A.14}$$

References

[1] CMS Collaboration, *"The Electromagnetic Calorimeter Technical Design Report"*, CERN/LHCC 1997-033, 1997

[2] Raymond, M. *et al.*, *"The MGPA Electromagnetic Calorimeter Readout Chip for CMS"*, Proceedings of the 9th Workshop on Electronics for the LHC Experiments, CERN-LHCC-2003-055, 2003

[3] P. Paganini and I. vanVulpen, *"Pulse amplitude reconstruction in the CMS ECAL using the weights method"*, **CMS Note 2004/025**