Generalized particle dynamics in anti de Sitter spaces: mimicking a phantom dark energy

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Abstract.
We consider a generalized particle dynamics in brane world formalism in an (4+1)-asymptotically anti de Sitter background. This generalized particle dynamics model induces an effective (4+1)-dimensional spacetime and (3+1)-dimensional physical universe is then embedded in this effective spacetime. The embedding framework induces an effective Dark Energy equation of state which is dynamical in nature and exhibits a phantom like behavior. We then corroborate our results with present day observed cosmological parameters.

1. Introduction

Observational evidence of the present-day acceleration of the universe [1], and the subsequent precise measurements of observable parameters [2] indicates that the entity Dark Energy (DE) is responsible for this recent acceleration. The DE density-fraction of total cosmic density ($\Omega_{\rm DE}$) is determined conclusively from several independent probes ($\Omega_{\rm DE} = 0.734 \pm 0.029$ at 95% C.L. from latest WMAP7 data [2]). On the other hand, the analysis of the most reliable SNIa Gold dataset show strong indication that the DE effective Equation of State (EOS) $w_{\rm DE} < -1$ (the lower bound being $-1.12 < w_{\rm DE}$ from WMAP7 data [2]), leading to the conclusion that models with phantom divider crossing are preferred over ΛCDM (or quintessential candidates) at 2σ level. This clearly weakens the claims of cosmological constant Λ or dynamical scalar field models as viable DE candidates. One can resurrect the scalar field models only at the cost of phantom fields with a negative kinetic term but they bring in severe instability problems. In this perspective, it seems reasonable to explore modified gravity theories [3] which do not suffer from any such major drawbacks. It is well-known [3, 4] that by embedding techniques one can relate cosmological surface dynamics (Friedmann equations) in lower (3+1)-dimensions with particle motion in a higher (4+1)-dimensional black-hole like space-time. Our introduction of generalized particle dynamics [5] in such space-times can give rise to an effective negative pressure resulting in cosmic acceleration and the subsequent crossover of the phantom divide. We further establish our model by an analysis of the equation of state and a determination of the relevant parameters describing the evolution of the observable universe.

2. Mathematical Formulation and Observable Parameters

We start with the reparametrization-invariant action [5]

$$S = \int L d\tau = m \int d\tau \left[\frac{1}{2e} g_{\mu\nu} \dot{x}^\mu \dot{x}^\nu - \frac{e}{2} - \lambda g_{\mu\nu} \xi^\mu \dot{x}^\nu + \frac{e\lambda^2}{2} g_{\mu\nu} \xi^\mu \xi^\nu + \frac{e\beta\lambda^2}{2} \right]. \quad (1)$$

where τ is the worldline parameter, $\lambda(\tau)$ is an auxiliary scalar variable, e is the worldline einbein, β is a numerical constant. In a (4+1)-dimensional Schwarzschild-anti de Sitter (Sch-AdS) spacetime $g_{\mu\nu}dx^\mu dx^\nu = -\left(k - \frac{2M}{r^2} + \Lambda_5 r^2\right) dt^2 + \frac{dr^2}{k - \frac{2M}{r^2} + \Lambda_5 r^2} + r^2 d\Omega_3^2$, this action (1) becomes

$$S = \int L d\tau = \frac{m}{2} \int d\tau \, e \left[\frac{1}{e^2} \left(-f(r)\dot{t}^2 + \frac{\dot{r}^2}{f(r)} + r^2 \dot{\varphi}^2 \right) - \frac{2\lambda}{e} r^2 \dot{\varphi} + \lambda^2 (r^2 + \beta) - 1 \right], \quad (2)$$

where $f(r) = k - \frac{2M}{r^2} + \Lambda_5 r^2$, k is the curvature scalar and Λ_5 is the constant curvature of the space-time. Here, the energy and angular momentum of a test particle of mass m are conserved, which implies $p_t = -mF(r)\dot{t} = m\varepsilon, p_\varphi = mr^2(\dot{\varphi} - \lambda) = m\beta\lambda = ml$, where the overdot represents a proper-time derivative. The radial geodesic equation for (2) turns out to be

$$\dot{r}^2 + V_{eff}(r) = \varepsilon^2 \quad , \quad V_{eff} = F(r)\left(1 + l^2/r^2\right) \quad , \quad F(r) = \left(k - \frac{2M}{r^2} + \Lambda_5 r^2\right)\left(\frac{\mu^2 r^2 + l^2}{r^2 + l^2}\right), \quad (3)$$

which induces the $4+1$-dimensional effective metric of the form

$$ds_{4+1}^2 = -F(r)dt^2 + \frac{dr^2}{F(r)} + r^2 d\Omega_3^2. \quad (4)$$

Here μ ($\mu^2 = 1 + \frac{l^2}{\beta}$) is a dimensionless parameter. For $\mu^2 = 1$ (i.e. $\beta \longrightarrow \infty$), this reduces to standard Schwarzschild-AdS metric.

With this effective metric (4) in hand we now turn to its cosmological implications. We embed a $(3+1)$-dimensional FLRW space-time $(T, \sigma, \theta, \varphi)$:

$$ds_{3+1}^2 = -dT^2 + a(T)^2 \left[\frac{d\sigma^2}{1 - k\sigma^2} + \sigma^2 (d\theta^2 + \sin^2\theta \, d\varphi^2) \right], \quad (5)$$

into the $(4+1)$-dimensional effective metric (4) [5]. Employing the Gauss-Codazzi junction conditions and Z_2 symmetry [5, 4] and identifying $r(T)$ with $a(T)$, the Friedmann equations for spatially flat universe (k=0) takes the form:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G_4}{3}\rho + \frac{2M}{a^4} + \left[\alpha - \Lambda_5 + \frac{l^2(2M/a^2 - \Lambda_5 a^2)}{\beta(a^2 + l^2)} \right] \quad (6)$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G_4}{3}(\rho + 3p) - \frac{2M}{a^4} + \left[\alpha - \Lambda_5 - \frac{l^2(\Lambda_5 a^4 + 2M + 2\Lambda_5 l^2 a^2)}{\beta(a^2 + l^2)^2} \right] \quad (7)$$

where $\dot{a} = da/dT$, $8\pi G_4/3 = 2\rho_0 \left(8\pi G_5/3\right)^2$ and $\alpha = (8\pi G_5 \rho_0/3)^2$. The terms containing M ($\propto a^{-4}$) contributes to radiation energy density of the universe which is negligibly small for present day universe. Thus we express the Hubble parameter $H = \dot{a}/a$ in terms of redshift z ($1 + z = a_0/a = 1/a$) and neglect the radiation contributions (the terms of the order of $(1+z)^4$ or higher) to express the Friedmann equation (6) in a convenient form:

$$H^2 = H_0^2 \left[\Omega_X \left(1 + b(1+z)^2\right) + \Omega_M (1+z)^3 \right] \quad (8)$$

where $\Omega_M = 8\pi G_4\rho/3H_0^2$ is the density parameter for the matter sector and $\Omega_X = (\alpha - \Lambda_5\mu^2)/H_0^2$ is the density parameter for the dark energy as induced by our generalized particle model. As we will see explicitly, the dimensionless parameter $b = \Lambda_5\beta(\mu^2-1)^2/(\alpha - \Lambda_5\mu^2)$, is responsible for the phantom behavior of the DE EOS. The most crucial part is to develop and estimate the observable parameters to show that we have a late accelerating universe where Ω_X accounts for the dark energy density. We also show that, consistent with observations, our candidate DE EOS (w_X) indeed crosses the phantom divider. As observations indicate a universe close to the ΛCDM model, this forces b to be small and β large (μ contains β in the denominator). These values can be restricted by χ^2 fitting.

Luminosity-redshift relation $d_L(z)$ determines dark energy density Ω_X from observations, and for our model it is given by

$$d_L(z) = (1+z)\int_0^z \frac{dz'}{H(z')} = \frac{(1+z)}{H_0}\int_0^z \frac{dz'}{\left[\Omega_X\left(1+b(1+z')^2\right)+\Omega_M(1+z')^3\right]^{1/2}}. \tag{9}$$

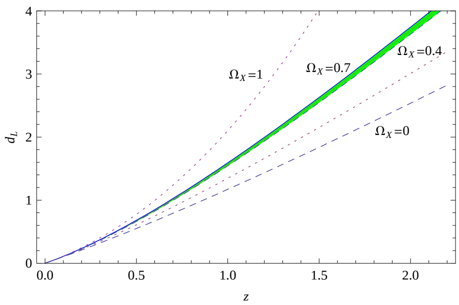

Figure 1. Variation of the luminosity distance with redshift for different Ω_X with $\Omega_M + \Omega_X = 1$

The plots in Figure 1 show that a universe with $\Omega_X \sim 0.7$, $\Omega_M \sim 0.3$ is favored, confirming that Ω_X accounts for the dark energy density. The allowed region (green shade) gives the bound for b as $-0.07 \le b < 0$. Throughout the rest of the paper, we take a representative small negative value for $b = -0.05$. Once the luminosity distance is estimated, the apparent magnitude of the Supernovae can be calculated from the Hubble constant-free distance modulus $\mu(z) = 5\log_{10}[d_L(z)/\mathrm{Mpc}] + 25$.

The deceleration parameter $q(z)$ explicitly shows the late accelerating behavior and for our model (8) we have

$$q(z) = \frac{-\ddot{a}/a}{\dot{a}^2/a^2} = \frac{H'(z)}{H(z)}(1+z) - 1 = \frac{\Omega_M(1+z)^3 - 2\Omega_X}{2(\Omega_X(1+b(1+z)^2)+\Omega_M(1+z)^3)}. \tag{10}$$

The plot in Figure 2 confirms that our model indeed results in an early decelerating and late accelerating universe. Moreover, onset of the recent accelerating phase, when the universe was $\sim 60\%$ of its present size ($z = 0.6$), is also confirmed by our model.

The effective DE EOS in our model is given by

$$w_X(z) = \frac{2q(z)-1}{3[1-\Omega_M(z)]} = -1 + \frac{2b(1+z)^2}{3}. \tag{11}$$

since b is negative and nonzero, it can be clearly seen from (11) that the effective EOS of dark energy candidate satisfies $w_X < -1$, exhibiting phantom behavior. Latest WMAP7 data

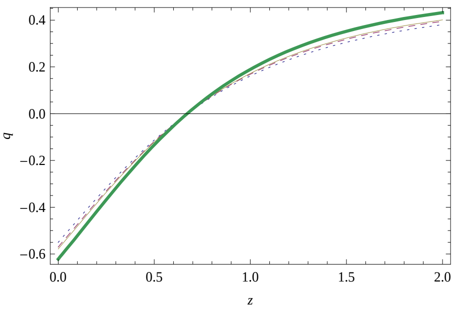

Figure 2. Variation of the deceleration parameter q with redshift

constrains the lower bound of the dark energy EOS to be $-1.12 < w_{DE}$ [2]. This sets the lower bound of b at $(-.16 \leq b)$, which is way below its lower bound $(-.07 \leq b)$ as predicted before from the luminosity-redshift relation. This model with $(-.07 \leq b < 0)$ will thus fit well with more precise bound for the EOS available in future.

The statefinder parameters $\{r, s\}$ [6] are defined as

$$r = \frac{\dddot{a}/a}{(\dot{a}/a)^3} = 1 + \left[\frac{H''}{H} + \left(\frac{H'}{H}\right)^2\right](1+z)^2 - 2\frac{H'}{H}(1+z)\ ;\ s = \frac{2}{3}\frac{r-1}{2q-1}$$

and can distinguish dynamical models from ΛCDM. In our model, $\{r, s\}$ pair is given by

$$r = 1 - \frac{b\Omega_X(1+z)^2}{\Omega_X(1+b(1+z)^2) + \Omega_M(1+z)^3}\ ;\ s = \frac{2}{3}\frac{b(1+z)^2}{[3+b(1+z)^2]}.$$

Hence, so far as b is non-trivial (as in the present case), the $\{r, s\}$ pair can distinguish our alternative gravity model from ΛCDM (for which $r = 1, s = 0$).

3. Summary and Future Prospects

To summarize, in our modified gravity scenario, induced by a higher dimensional non-minimal particle dynamics framework, one can have a phantom dark energy model *without the phantom*. Our model is qualitatively distinct, but not quantitatively far off, from ΛCDM model. Hence all the positive features of ΛCDM along with a phantom behavior (without the problems related to the negative kinetic term) can be accommodated in our model.

Several aspects of the proposed framework can be investigated further. The parameters used in the model can be constrained observationally by using maximum likelihood method involving the minimization of the function $\chi^2 = \sum_{i=1}^{N} [d_L(z)_{\text{obs}} - d_L(z)_{\text{th}}]^2 / \sigma_i^2$, where $d_L(z)_{\text{th}}$ contains the parameters used in a specific theory, N is the number of Supernovae (taken as 157 for the most reliable Gold dataset) and σ_i are the 1σ error from observational method used. Observationally, this is the most accurate probe of Ω_{DE} and will further constrain the parameters used in our model. Further, the variable EOS may be reflected in Integrated Sachs-Wolfe (ISW) effect, which will serve as another test for the model. Studying features related to perturbations in this cosmological framework is another open issue.

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