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Magnetic Properties of Quark Matter by Using of NJL Model with Pseudovector Interaction

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By using the two-flavor Nambu-Jona-Lasinio model with the pseudovector-type four-point interaction between quarks, spontaneous magnetization, originated from the pseudovector interaction, is calculated in quark matter with zero temperature and finite quark chemical potential. We showed that the chiral condensate and spin polarized condensate coexist in a narrow region of the quark chemical potential. In this narrow region, the spontaneous magnetization appears. Also, the magnetic susceptibility due to quarks with the positive energy is evaluated in the spin polarized phase.

KEYWORDS: quark matter, spontaneous magnetization.

1. Introduction

Behaviors of nucleons and quarks are described by Quantum Chromodynamics (QCD). To understand the QCD, clarifying the phase structure in the plane spanned by the temperature and baryon chemical potential is needed. We are interested in determining the phase structure and physical properties of the low-temperature and large baryon chemical potential region of QCD.

Also, it may be interesting to investigate magnetic properties in quark matter in the region of low temperature and large baryon chemical potential. In astrophysical systems, compact stars such as neutron stars, in particular magnetars [1, 2], show a very strong magnetic field. Thus, studying the magnetic properties of quark matter may lead to elucidate the origin of the magnetic field in magnetars. As for the magnetic properties of quark matter, it has been shown that the tensor-type four-point interaction between quarks in the Nambu-Jona-Lasinio(NJL) model leads to the anomalous magnetic moment [3], which may causes the spontaneous magnetization [4].

We employ the two-flavor NJL model with the pseudovector-type [5, 6] four-point interaction which causes spontaneous magnetization. We restrict ourselves to the system in quark matter with zero temperature and finite quark chemical potential. We investigate the magnetic properties due to the pseudovector interaction between quarks.

2. Formalization

We employed a two-flavor Nambu-Jona-Lasinio model with vector-pseudovector-type [5,6] four-point interactions between quarks under an external magnetic field. The Lagrangian density can be expressed as

$$\begin{aligned}\mathcal{L} = \bar{\psi}(i\gamma^\mu D_\mu - m_0)\psi + G_s[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2] \\ - G_p[(\bar{\psi}\gamma^\mu\vec{\tau}\psi)^2 + (\bar{\psi}i\gamma_5\gamma^\mu\vec{\tau}\psi)^2]\,, \qquad (1)\end{aligned}$$

$$D_\mu = \partial_\mu + iQA_\mu \,, \qquad A_\mu = \left(0,\ \frac{By}{2},\ -\frac{Bx}{2},\ 0\right) = (0,\ -\boldsymbol{A}) \,. \tag{2}$$

Here, Q is the electric charge. For up quark, Q has a value of $Q_u = +2e/3$, and for down quark, $Q_d = -e/3$, where e is the elementary charge. There is an external magnetic field B along z-axis.

Hereafter, we treat the model within the mean field approximation. In order to consider the spin polarization under the mean field approximation, the pseudovector condensate $\langle\bar{\psi}\gamma_5\gamma^3\tau_3\psi\rangle$ is taken into account.

Then, the Lagrangian density reduces to

$$\begin{aligned}\mathcal{L}_{MF} &= \bar{\psi}(i\gamma^\mu D_\mu - M_q)\psi + U_A\bar{\psi}\gamma_5\gamma^3\tau_3\psi - \frac{M^2}{4G_s} - \frac{U_A^2}{4G_p} \\ &= \bar{\psi}(i\gamma^\mu D_\mu - M_q)\psi - U_A\psi^\dagger\Sigma_3\tau_3\psi - \frac{M^2}{4G_s} - \frac{U^2}{4G_p} \,,\end{aligned} \tag{3}$$

where

$$\begin{aligned}\Sigma_3 &= -\gamma^0\gamma_5\gamma^3 = \begin{pmatrix} \sigma_3 & 0 \\ 0 & \sigma_3 \end{pmatrix} \\ M_q &= m_0 + M \,, \qquad M = -2G_s\langle\bar{\psi}\psi\rangle \,, \\ U_A &= 2G_p\langle\bar{\psi}\gamma_5\gamma^3\tau_3\psi\rangle = -2G_p\langle\psi^\dagger\Sigma_3\tau_3\psi\rangle \equiv U\tau_f \,.\end{aligned} \tag{4}$$

Here, $\tau_f = 1$ for up quark and -1 for down quark denote the eigenvalues of τ_3. Also, σ_3 is the third component of the Pauli spin matrices.

From this Lagrangian, we calculated the thermodynamic potential. Then, based on gap equations, we estimate chiral condensate M and spin polarization U.

3. Numerical results

The adopted model parameters are written in Table I. In these parameters, the dynamical quark mass M is obtained as $M = 0.322$ GeV. If we introduce the current quark mass $m_0 = 0.005$ GeV, the constituent quark mass $M_q = 0.335$ GeV is obtained under the same model parameters.

Table I. Model parameters.

G_s	G_p	m_0	Λ
5.5GeV^{-2}	$2G_s$	0	0.631GeV

By using these parameters, it has shown that both chiral condensate and pseudvector condensate, namely spin polarized condensate, coexist in the narrow region [7].

In the region with $\mu \leq \mu_{\rm cr,1}$ ($\approx$ 0.322GeV), the chiral symmetry is broken and chiral condensate or dynamical quark mass exists($M \neq 0$). But spin polarized condensate doesn't appear ($U = 0$).

On the other hand, in larger region of the quark chemical potential $\mu_{\rm cr,1} < \mu < \mu_{\rm cr,2}$ ($\approx$ 0.325GeV), the chiral symmetry is still not restored and the solution of the gap equation for the dynamical quark mass with $M \neq 0$ appears with $U \neq 0$. The window of the quark chemical potential, in which the chiral condensate $M \neq 0$ and spin polarized condensate $U \neq 0$ coexist, is very narrow.

In the region with $\mu > \mu_{\rm cr,2}$, both the chiral condensate and the spin polarized condensate are vanished ($M = U = 0$). This region corresponds to the free quark gas.

The chiral condensate in these region is plotted in Fig1.

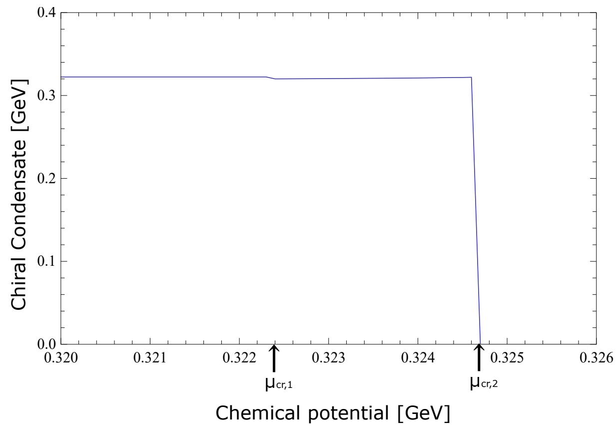

Fig. 1. The chiral condensate, M, is depicted as a function of the quark chemical potential μ.

3.1 Spontaneous magnetization

The spontaneous magnetization $\mathcal{M}$ is defined as

$$\mathcal{M} = -\left.\frac{\partial \Phi}{\partial B}\right|_{B=0}. \tag{5}$$

The spontaneous magnetization occurs only $U \neq 0$ region. The order of magnitude of the spontaneous magnetization is about 10^{17} and/or 10^{18} C/ms. These magnitude leads to the magnetic flux density with 10^{13} or 10^{14} Gauss in the surface of compact stars [8]. Also, the result of the spontaneous magnetization per unit volume is shown in Fig.2.

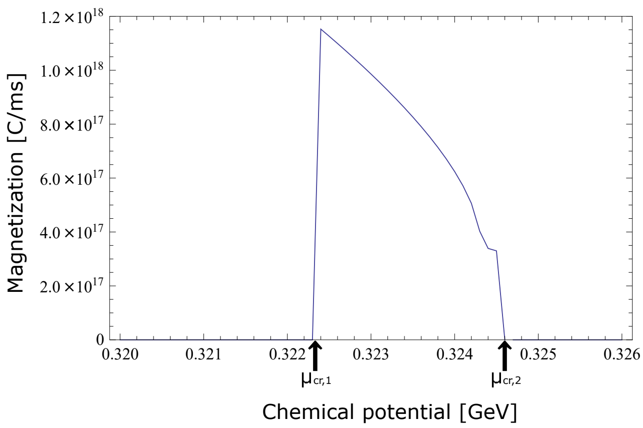

Fig. 2. The spontaneous magnetization per unit volume, $\mathcal{M}$, is depicted as a function of the quark chemical potential μ.

3.2 Magnetic susceptibility

The magnetic susceptibility χ is defined as

$$\chi = -\mu_0 \left.\frac{\partial^2 \Phi}{\partial B^2}\right|_{B=0}, \tag{6}$$

where μ_0 represents the vacuum permeability.

The result of the magnetic susceptibility per unit volume is shown in Fig.3. In this figure, we plot the value of $\Delta\chi \equiv \chi - \chi_{(\mu=0)}$. Here we subtract the $\chi_{(\mu=0)}$ in order to investigate the contribution only of the positive-energy particles due to the pesudvector-type interaction between quarks.

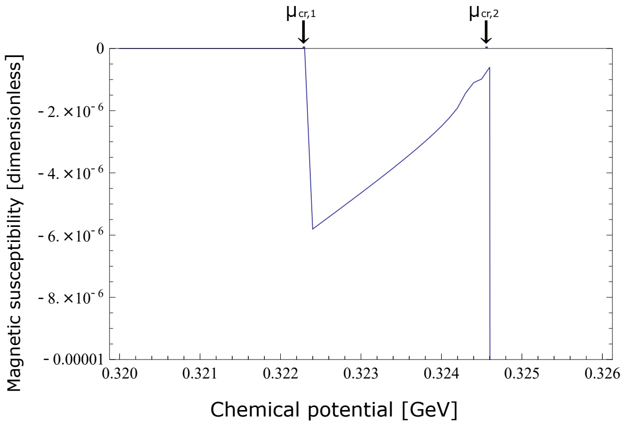

Fig. 3. The magnetic susceptibility compared with vacuum value $\chi_{(\mu=0)}$, $\Delta\chi$, is depicted as a function of the quark chemical potential μ.

4. Summary and concluding remarks

It has been shown that the spontaneous magnetization occurs due to the pseudovector-type four-point interaction between quarks in quark matter at zero temperature within the Nambu-Jona-Lainio model. We find that both the chiral condensate and pseudovector condensate, namely spin polarized condensate, coexist in a narrow region of the quark chemical potential, where the spontaneous magnetization appears. Also, we evaluated the magnetic susceptibility due to quarks with the positive energy in the spin polarized phase.

Further, the effects of the strange quark is not considered in this work. These are interesting future problems which are left in order to clarify the magnetic properties of high density quark matter.

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