

Discrete symmetry tests using hyperon-antihyperon pairs

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Abstract. The BESIII experiment has accumulated millions of spin entangled hyperon–antihyperon pairs from the decays of the J/ψ and $\psi(2S)$ mesons produced in electron–positron collisions. Using multi-dimensional methods to analyze the angular distributions in the sequential decays the hyperon and antihyperon decay amplitudes are studied and tests of the combined charge-conjugation–parity (CP) symmetry are performed. Implications of the recent results using J/ψ decays into $\Lambda\bar{\Lambda}$ and $\Xi^-\bar{\Xi}^+$ are presented. For the cascade decay chain, the exclusive measurement allows for three independent CP tests and the first direct determination of the weak phases.

1 Introduction

The standard model of elementary particles explains most of the subatomic phenomena but it leaves out questions like the unification with the gravitational forces or the origin of the observed dominance of the matter over the antimatter in the universe. Any candidate for the underlying fundamental theory has to address these questions. A clue how to construct such theory might be found by scrutinizing the cornerstone symmetries of the standard model. In addition to the *natural* symmetries like space-time translations and space rotations, which lead to the four-momentum and angular momentum conservation, there are discrete symmetries related to parity (P), charge conjugation (C) and time reversal (T) transformations. Parity is the spatial inversion which changes directions of all particles to the opposite. Charge conjugation is the swap between all particles and antiparticles in a system. Separately the C and P symmetries are maximally violated in the weak interactions but their combination — the CP symmetry — is nearly preserved and a small observed violation is due to quantum interference effects only. The quantum interference can also enhance a signal of new interactions. For example, having the amplitude of a known process $M = |M| \exp(i\delta)$ and a new process amplitude $m = |m| \exp(i\xi)$ the probability of the combined process is $|M + m|^2 = |M|^2 + 2|M||m| \cos(\delta + \xi) + |m|^2$. If $|m|^2 \ll |M|^2$ the contribution of the new process is dominated by the interference term that is linear in $|m|$.

Here I will discuss direct CP violation that requires amplitudes for a process and the CP-transformed process to have different complex phases. For example if the amplitude for a particle decay is $\mathcal{A} = |\mathcal{A}| \exp(i\xi + i\delta)$ then the amplitude for the CP-transformed process is $\bar{\mathcal{A}} = |\mathcal{A}| \exp(-i\xi + i\delta)$, where ξ is the CP-odd phase that changes sign after the CP transformation. In order to observe an effect due to the CP-odd phases an interference with other amplitudes contributing to the process is needed. Since the CP-odd phases are small for the

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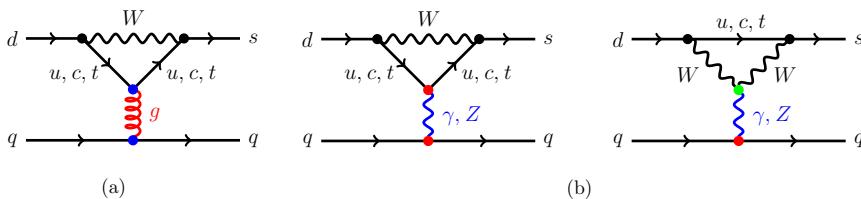


Figure 1. Quark level diagrams for the weak processes involving kaons or hyperons that contribute to the CP-odd phases in the standard model: (a) QCD-penguin operators and (b) electroweak penguin operators. This figure was created using a modified script from Ref. [7].

weak transitions of strange quarks a significant contribution might be due to a new physics (NP) effect. One process where the direct CP-violation was observed is the combined weak and strong decay of K^0 meson into a pion pair $K^0 \rightarrow \pi^+ \pi^-$. There are two final states that have distinct properties with respect to the strong interactions¹. The initial weak transitions can proceed to one of the two states and are represented by the amplitudes \mathcal{A}_0 and \mathcal{A}_2 where $|\mathcal{A}_2/\mathcal{A}_0| \approx 0.05$. The strong interaction of the pions is described by two strong phases δ_0 and δ_2 . The complete amplitude for the $K^0 \rightarrow \pi^+ \pi^-$ process can then be written as $\mathcal{A} = |\mathcal{A}_0| \exp(i\xi_0 + i\delta_0) + |\mathcal{A}_2| \exp(i\xi_2 + i\delta_2)$. The amplitude of the CP-transformed process $\bar{K}^0 \rightarrow \pi^- \pi^+$, $\bar{\mathcal{A}}$, differs only by the signs of the CP-odd phases ξ_0 and ξ_2 . The decay rates $|\mathcal{A}|^2$ and $|\bar{\mathcal{A}}|^2$ can be different if CP is violated. The direct CP violation observable is expressed as [1, 2]:

$$\text{Re}(\epsilon') := \frac{1}{2} \frac{|\mathcal{A}|^2 - |\bar{\mathcal{A}}|^2}{|\mathcal{A}|^2 + |\bar{\mathcal{A}}|^2} \approx (\xi_0 - \xi_2) \sin(\delta_0 - \delta_2) \left| \frac{\mathcal{A}_2}{\mathcal{A}_0} \right|, \quad (1)$$

where the measured value of $\text{Re}(\epsilon')$ is $(3.7 \pm 0.5) \times 10^{-6}$ [3] corresponding to the weak-phase difference $\xi_0 - \xi_2 \approx 10^{-4}$ rad. In the standard model the contributions to the CP-odd phases $\xi_{0,2}$ are given by the loops that involve all three quark generations as shown in the diagrams in Fig. 1. An order of magnitude estimate for these contributions is determined from the Cabibbo–Kobayashi–Maskawa matrix [4, 5] and can be expressed by the product of the Wolfenstein parameters [6] as $\lambda^4 A^2 \eta \approx -6 \times 10^{-4}$ [3]. Recent standard model predictions for the CP-violation in kaon decays that include hadronic effects are given in the framework of the low energy effective field theory [7] and the lattice quantum chromodynamics [8, 9]. The agreement with the $\text{Re}(\epsilon')$ parameter measurement is satisfactory but due to large uncertainties there is a room for a new physics contribution.

2 Baryon two-body weak decays

Similar mechanism of the direct CP violation is possible in the main decay modes of the baryons with one or more strange quarks — hyperons. The spin of a baryon provides an additional degree of freedom that can be used for more sensitive CP tests. A quantum state of a spin-1/2 baryon B can be described by Pauli matrices:

$$|B\rangle = \sigma_0 + \mathbf{P}_B \cdot \boldsymbol{\sigma}, \quad (2)$$

where σ_0 is the 2×2 unit matrix and $\boldsymbol{\sigma} := (\sigma_1, \sigma_2, \sigma_3)$ represents spin x, y, z projections in the baryon rest frame. The polarization vector \mathbf{P}_B describes the preferred spin direction

¹They have different values of the isospin quantum number $I = 0$ and $I = 2$ for the two-pion system.

for an ensemble of the B -baryons. Consider the $B \rightarrow b\pi$ transition between two spin-1/2 baryons with positive internal parities where a negative parity pion is emitted. Examples of such processes are the decays $\Xi^- \rightarrow \Lambda\pi^-$ or $\Lambda \rightarrow p\pi^-$. The baryon b can have the same or opposite direction of spin as the baryon B and the angular momentum conservation requires s - or p -wave for the $b\pi$ system. The parity of the final state is negative for the s -wave and positive for the p -wave. Both are allowed since parity is not conserved in weak decays. The decay amplitude is given by the transition operator

$$\mathcal{A}(B \rightarrow b\pi) \propto \mathcal{S}\sigma_0 + \mathcal{P}\sigma \cdot \hat{\mathbf{n}}, \quad (3)$$

where $\hat{\mathbf{n}}$ is the direction of the b -baryon in the B -baryon rest frame. The complex parameters \mathcal{S} and \mathcal{P} can be represented as $\mathcal{S} = |\mathcal{S}| \exp(i\xi_S + i\delta_S)$ and $\mathcal{P} = |\mathcal{P}| \exp(i\xi_P + i\delta_P)$. The strong interaction of the $b\pi$ system is given by the δ_P and δ_S phases and the CP-odd phases are ξ_P and ξ_S . The amplitude ratios $|\mathcal{P}/\mathcal{S}|$ for $\Lambda \rightarrow p\pi^-$ and $\Xi^- \rightarrow \Lambda\pi^-$ are 0.442(4) and 0.188(2), respectively [10]. Since only the angular distributions will be discussed the probability of the decay $\Gamma \propto |\mathcal{S}|^2 + |\mathcal{P}|^2$ is set to $|\mathcal{S}|^2 + |\mathcal{P}|^2 = 1$. The real and imaginary parts of the amplitude interference term are

$$\alpha = 2\text{Re}(\mathcal{S}^*\mathcal{P}) \propto 2|\mathcal{S}||\mathcal{P}|\cos(\xi_P + \delta_P - \xi_S - \delta_S) \quad (4)$$

$$\beta = 2\text{Im}(\mathcal{S}^*\mathcal{P}) \propto 2|\mathcal{S}||\mathcal{P}|\sin(\xi_P + \delta_P - \xi_S - \delta_S). \quad (5)$$

The real part given by the α parameter can be determined from the angular distribution of the baryon b when the baryon B has known non-zero polarization or by measuring the polarization of the daughter baryon. For example, the proton angular distribution in the $\Lambda(\Lambda \rightarrow p\pi^-)$ decay is given as

$$\frac{1}{\Gamma} \frac{d\Gamma}{d\Omega} = \frac{1}{4\pi} (1 + \alpha_\Lambda \mathbf{P}_\Lambda \cdot \hat{\mathbf{n}}), \quad (6)$$

where \mathbf{P}_Λ is the Λ polarization vector. Measurement of the imaginary part of the interference term given by the parameter β requires that the polarization of both baryon B and the daughter baryon is determined. For the decay $\Xi(\Xi^- \rightarrow \Lambda\pi^-)$, where the cascade is polarized, the β_Ξ parameter can be determined using the subsequent $\Lambda \rightarrow p\pi^-$ decay that acts as Λ polarimeter. The relation between the initial Ξ^- polarization \mathbf{P}_Ξ and the daughter Λ polarization \mathbf{P}_Λ is given by the Lee–Yang formula [11]. If the $\hat{\mathbf{z}}$ axis in the Ξ^- rest frame is defined along the $\hat{\mathbf{n}}$ direction than the relation between the polarization vectors is

$$\begin{bmatrix} P_\Lambda^x \\ P_\Lambda^y \\ P_\Lambda^z \end{bmatrix} = \frac{1}{1 + \alpha_\Xi P_\Xi^z} \begin{pmatrix} \gamma_\Xi P_\Xi^x - \beta_\Xi P_\Xi^y \\ \beta_\Xi P_\Xi^x + \gamma_\Xi P_\Xi^y \\ \alpha_\Xi + P_\Xi^z \end{pmatrix}, \quad (7)$$

where the parameter $\gamma = |\mathcal{S}|^2 - |\mathcal{P}|^2$. The equation implies that Λ in decay of unpolarized Ξ^- has the longitudinal polarization vector component $P_\Lambda^z = \alpha_\Xi$. Using a polar angle parameterization such that $\beta = \sqrt{1 - \alpha^2} \sin \phi$ and $\gamma = \sqrt{1 - \alpha^2} \cos \phi$, the ϕ parameter has interpretation of the rotation angle between the Ξ and Λ polarization vectors in the transversal x - y plane.

3 CP violation in baryon decays

For the C-transformed baryon decay process $\bar{B} \rightarrow \bar{b}\bar{\pi}$ the amplitude is:

$$\bar{\mathcal{A}}(\bar{B} \rightarrow \bar{b}\bar{\pi}) \propto \bar{\mathcal{S}}\sigma_0 - \bar{\mathcal{P}}\sigma \cdot \hat{\mathbf{n}}, \quad (8)$$

where the complex parameters \bar{S} and \bar{P} are obtained from S and P by reversing the sign for the weak CP-odd phases ξ_S and ξ_P . Since the product of the P-odd and P-even terms changes sign, the decay parameters are:

$$\bar{\alpha} \propto -2|S||P| \cos(-\xi_P + \delta_P + \xi_S - \delta_S) \quad (9)$$

$$\bar{\beta} \propto -2|S||P| \sin(-\xi_P + \delta_P + \xi_S - \delta_S). \quad (10)$$

The weak phase difference $\xi_P - \xi_S$ can be determined using two independent experimental observables:

$$A_{CP} = \frac{\alpha + \bar{\alpha}}{\alpha - \bar{\alpha}} = (\xi_P - \xi_S) \tan(\delta_P - \delta_S), \quad B_{CP} = \frac{\beta + \bar{\beta}}{\alpha - \bar{\alpha}} = (\xi_P - \xi_S). \quad (11)$$

4 Spin entangled baryon–antibaryon systems

A novel method to study hyperon decays is to use the entangled baryon–antibaryon pair from J/ψ resonance decays. In general the spin state of a baryon–antibaryon system can be represented as

$$|B\bar{B}\rangle = \sum_{\mu, \nu=0}^3 C_{\mu\nu}^{B\bar{B}} \sigma_{\mu}^{(B)} \otimes \sigma_{\nu}^{(\bar{B})}, \quad (12)$$

where the Pauli operators $\sigma_{\mu}^{(B)}$ and $\sigma_{\nu}^{(\bar{B})}$ act in the rest frames of the baryon and antibaryon, respectively. The coefficients of the spin correlation–polarization matrix $C_{\mu\nu}^{B\bar{B}}$ depend on the pair production process. Two reactions provide best sensitivity for the CP tests in hyperon decays: $e^+e^- \rightarrow J/\psi \rightarrow \Lambda\bar{\Lambda}$ [12, 13] and $e^+e^- \rightarrow J/\psi \rightarrow \Xi\bar{\Xi}$ [14]. At the BESIII experiment for every million of the produced J/ψ resonances, 320 and 56 of fully reconstructed $\Lambda\bar{\Lambda}$ and $\Xi\bar{\Xi}$ pairs are selected, respectively. With 10^{10} J/ψ mesons available at BESIII, precision measurements of the hyperon decay parameters are possible. The elements of the $C_{\mu\nu}^{B\bar{B}}$ matrix for the $e^+e^- \rightarrow B\bar{B}$ processes are known functions of the baryon B production angle θ and depend on two parameters that need to be determined from data [15, 16]. The baryons can be polarized in the direction perpendicular to the reaction plane given by the \hat{y} unit vector. The polarization vector component $P_y(\theta)$ and the spin correlation terms are $C_{ij}(\theta)$, $i, j = x, y, z$:

$$C_{\mu\nu}^{B\bar{B}} = \frac{1}{\sigma d\cos\theta} \begin{pmatrix} 1 & 0 & P_y & 0 \\ 0 & C_{xx} & 0 & C_{xz} \\ -P_y & 0 & C_{yy} & 0 \\ 0 & -C_{xz} & 0 & C_{zz} \end{pmatrix}. \quad (13)$$

The average baryon polarizations $|P_y|$ in the $e^+e^- \rightarrow J/\psi \rightarrow \Lambda\bar{\Lambda}$ and $e^+e^- \rightarrow J/\psi \rightarrow \Xi\bar{\Xi}$ reactions are 18% and 23%, respectively. The two production reaction parameters, the decay parameters α_{Λ} , α_{Ξ} and β_{Ξ} , as well as the CP-violating variables A_{CP}^{Λ} , A_{CP}^{Ξ} and B_{CP}^{Ξ} , can be determined using unbinned maximum likelihood fit to the measured angular distributions. The statistical uncertainty of the observable A_{CP}^{Λ} in the process $e^+e^- \rightarrow J/\psi \rightarrow \Lambda\bar{\Lambda}$ is inversely proportional to the Λ polarization [10]. The weak phase difference for the Ξ baryon decay is directly given by the measurement of the B_{CP}^{Ξ} observable in the process $e^+e^- \rightarrow J/\psi \rightarrow \Xi\bar{\Xi}$. The sensitivity of such measurement depends on the average squared of the Ξ polarization and the $\Xi\bar{\Xi}$ spin correlation terms [10].

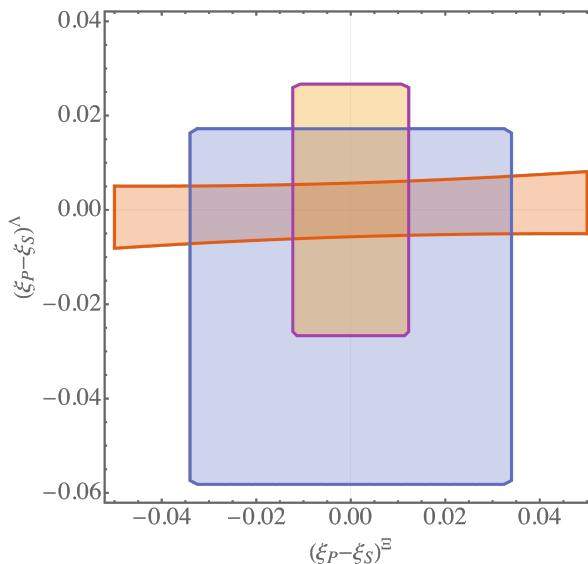


Figure 2. Summary of the results for the weak phases $(\xi_P - \xi_S)^\Xi$ and $(\xi_P - \xi_S)^\Lambda$. The present BESIII results are given by the blue rectangle [13, 14]. The red horizontal band is the HyperCP result [17]. The brown rectangle is the projection of the statistical uncertainties for the complete BESIII data set.

5 Weak phases

The goal of the CP tests is to determine the weak-phase differences for the two decays. The BESIII results based on 1.3×10^9 and 10^{10} J/ψ data for $(\xi_P - \xi_S)^\Xi$ [14] and $(\xi_P - \xi_S)^\Lambda$ [13], respectively, are presented in Fig. 2 by the blue rectangle. The red band represents the HyperCP experiment result on $A_{CP}^\Lambda + A_{CP}^\Xi$ [17]. Interpretation of this measurement requires the value of $\tan(\delta_P - \delta_S)_\Xi$ that is poorly known. Projection for the statistical uncertainties of the analysis using full BESIII data set for $e^+e^- \rightarrow J/\psi \rightarrow \Xi\bar{\Xi}$ is given by the brown rectangle. The results should be compared to the standard model predictions of $(\xi_P - \xi_S)^\Lambda = (-0.2 \pm 2.2) \times 10^{-4}$ and $(\xi_P - \xi_S)^\Xi = (-2.1 \pm 1.7) \times 10^{-4}$. In case of hyperon decays the standard model contribution is dominated by QCD-penguin contributions shown in Fig. 1(a). The results on the weak phases in kaon and in the hyperon decays can be combined in order to search for deviations from the standard model. The present kaon data imply the limits $|\xi_P - \xi_S|_{NP}^\Lambda \leq 5.3 \times 10^{-3}$ and $|\xi_P - \xi_S|_{NP}^\Xi \leq 3.7 \times 10^{-3}$ [18]. Clearly, the hyperon CP-violation measurements with much improved precision will provide an independent constraint on the NP contributions in the strange quark sector. However, a lot also remains to be done on the theory side, as the present predictions suffer from considerable uncertainties. It is hoped that the lattice analyses [19] could help solve this problem in future.

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