

Weinberg and few-nucleon forces

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Abstract. Weinberg's contributions to the power counting and derivation of few-nucleon forces in Chiral EFT are briefly recalled. Subsequent improvements are reviewed, concluding with the recent suggestion of a combinatorial enhancement.

1. Introduction

About thirty years ago, Steven Weinberg [1, 2] set in motion a new nuclear physics [3] based on the framework of effective field theory (EFTs), which he had formulated earlier [4]. An EFT includes all interactions allowed by the symmetries that are supported by the degrees of freedom relevant to the energies of interest. These interactions involve arbitrary numbers of derivatives and fields. One of the advantages over more phenomenological approaches, which attracted immediate attention in the nuclear community, is that few-body forces can be constructed consistently with the two-body force. By the time EFT came into the scene, various excellent phenomenological parametrizations of the two-nucleon force existed which failed to describe the three- and four-nucleon systems better than within about 20%. Guessing the form of few-nucleon forces have proven to be nearly impossible without EFT.

Yet, the relative importance of few-body forces is not well established in the Chiral EFT [3] employed by Weinberg, which includes pions and nucleons. A crucial ingredient of any EFT is the power counting that orders interactions according to the magnitudes of their contributions to observables, but it requires assumptions which are rarely tested. Power counting is the rationale to neglect all but a few interactions at each order, thus enabling an *a priori* estimate of errors.

I arrived in Austin from São Paulo with an excellent background in physics, at a time which allowed me to participate in the formulation of nuclear EFTs. I have recently related the events surrounding the early developments [5]. In this brief report, I focus on the evolution of the ideas for power counting few-body forces, starting with Weinberg's work, continuing with various subsequent improvements, and ending with the recent suggestion of an environmental dependence on the number of nucleons.

2. The problem

QCD is the underlying theory of nuclear physics. It is characterized by the nonperturbative scale $M_{\text{QCD}} \sim 1$ GeV, which is reflected in the masses of most hadrons, including the nucleon's $m_N = \mathcal{O}(M_{\text{QCD}}) \simeq 940$ MeV. QCD has an approximate chiral symmetry, whose spontaneous breaking generates pions of mass $m_\pi \simeq 140$ MeV and interactions proportional to the inverse of the pion decay constant $f_\pi = \mathcal{O}(M_{\text{QCD}}/4\pi) \simeq 92$ MeV.



We are interested here in systems of A nucleons with typical momentum $Q \sim m_\pi \ll M_{\text{QCD}}$. This is the domain of Chiral EFT [3], where nucleons couple to pions according to the constraints of chiral symmetry. If we integrate out all heavy mesons and baryon excitations, observables can be calculated in an expansion in powers of Q/M_{QCD} . For $A = 0, 1$, this theory reduces to Chiral Perturbation Theory (ChPT). For perturbative amplitudes, the assumption of naturalness [6, 7]—according to which the magnitude of short-range interactions is set by their bare parameters with the regulator cutoff replaced by M_{QCD} —gives rise to the so-called naive dimensional analysis (NDA) [8], if one estimates that each loop contributes a factor of $(4\pi)^{-2}$. This factor combines with factors of f_π from the pion interactions to ensure a suppression of $(Q/M_{\text{QCD}})^2$ for each loop [4]. Thus amplitudes are indeed perturbative, consistently with the use of NDA in the first place.

For $A \geq 2$, Weinberg [1, 2] identified in A -nucleon reducible diagrams an infrared enhancement by a relative factor of m_N/Q . This was the first sign that nuclear amplitudes have a very different power counting than those for $A = 0, 1$, as the enhancement comes from nucleon recoil, a subleading effect in ChPT. Weinberg then defined the potential as the sum of irreducible subdiagrams, which are not infrared enhanced. The potential contains a -body components with $a = 2, \dots, A$. An a -body force cannot be reduced, within the resolution of the EFT, to an iteration of fewer-body forces; it is a force that disappears when any nucleon is removed. Even if the underlying theory were fundamental and its interactions of two-body character, the finite resolution of the EFT would require the existence of few-body forces. Few-nucleon forces are not forbidden by any symmetry, and therefore *must* appear at some order in the EFT expansion of amplitudes. The question is, which order?

In his first paper [1], Weinberg did not realize there was a price to pay for building higher-body forces and ended with the suggestion that few-body forces could be important. He was quick to rectify this oversight in his second paper [2], which announces the correction already in the abstract. The exchange of a single pion between two nucleons brings to a diagram a factor of at most $4\pi/m_N f_\pi$. Weinberg assumed that, since the potential is free of infrared enhancements, multi-nucleon contact interactions are also given by NDA, starting with the two-nucleon contact (containing four nucleon fields in the Lagrangian) of size $4\pi/m_N f_\pi$, too. These two-nucleon interactions are leading order (LO). When one adds a nucleon to the force, one changes the number of loops in the A -nucleon amplitude. Assuming the same factor of $(4\pi)^{-2}$ as for ChPT loops, Weinberg arrived at a cost of $(Q/M_{\text{QCD}})^2$ for each additional nucleon in the force.

At that point we thought the first three-nucleon force appeared at relative $\mathcal{O}(Q^2/M_{\text{QCD}}^2)$ from diagrams where two nucleons interacted while there was already a pion “in the air” emitted by a third nucleon. Prompted by a remark by James Friar, we realized these diagrams cancel against the energy dependence in the one-pion-exchange two-nucleon force. However, this cancellation would only go through if there was an error in expressions for pion-in-the-air diagrams in Refs. [1, 2]. When I pointed this out to Weinberg he quickly agreed, an example of his utmost intellectual honesty that did wonders for my self-esteem. The correct expression was published shortly afterwards [9] and details of the cancellation were given in Refs. [10, 11]. As a consequence of this cancellation, the leading three-body force would come from interactions which are themselves suppressed by one power of Q/M_{QCD} . That is, the first three-body force would appear at relative $\mathcal{O}(Q^3/M_{\text{QCD}}^3)$, with four-body forces at $\mathcal{O}(Q^4/M_{\text{QCD}}^4)$ and so on.

The leading components of the three-nucleon potential according to this power counting were derived in Refs. [9, 11, 12]. Sometimes referred to as the Texas potential, it has two-pion, pion/short-range, and purely short-range components. The two-pion component is intimately related to pion-nucleon scattering and carries the imprints of chiral symmetry. It slightly corrects [13] the Tucson-Melbourne force [14] to a form closer to the Brazil force [15, 16]. The pion/short-range component, in turn, is related to p -wave pion production in nucleon-nucleon collisions [17], while the purely short-range component is intrinsically a three-body feature. The shorter-range

components have non-negligible effects on the three-body system [18, 12] and beyond. They have become very popular thanks to several successes, such as an improved description of light nuclei [19].

Unfortunately, there have not been extensive checks that these order assignments are supported by data. It is remarkable that many nuclear properties, such as those of nuclear matter [20, 21], are only described well with chiral potentials based on Weinberg's power counting when three-body forces are included. Most papers do not even report LO results. In fact, nuclei beyond $A = 4$ are not stable at LO [22]. Some of these problems are discussed in Ref. [23].

3. "... and then we learn something"

One of Weinberg's favorite remarks was that a theorist should insist on consistency with assumptions made, until evidence prompts their reevaluation "and then we learn something". Sadly, Weinberg's papers have been accepted like a gospel by most nuclear physicists, despite consistency issues that surfaced over time which I address in the following.

3.1. Role of the Delta

The first issue is the role of the Delta isobar, whose mass is only $\Delta \equiv m_\Delta - m_N \sim 300$ MeV above the nucleon's. If one does not include an explicit degree of freedom for the Delta in Chiral EFT, its effects are subsumed into contact interactions suppressed by powers of Δ^{-1} [24, 11, 25] instead of M_{QCD}^{-1} . Convergence is restricted.

There is really no reason not to include an explicit Delta field. When this is done, the leading three-nucleon force comes at relative $\mathcal{O}(Q^2/M_{\text{QCD}}^2)$ [11] in Weinberg's power counting. In the form of the Fujita-Miyazawa force [26], it is the dominant component of the force. One way [27] to see the importance of the Delta is to consider the relation between the two-pion component of the three-nucleon force and pion-nucleon scattering: one needs to extrapolate in energy by at least m_π which leads to errors no smaller than $\mathcal{O}(m_\pi^2/\Delta^2)$ when the Delta is integrated out. In contrast, with an explicit Delta one can extend the ChPT power counting to describe pion-nucleon scattering through the Delta peak [28] and firmly determine pion-nucleon couplings. Of course, the same argument holds for the two-pion components of two- and higher-body forces, which should be constructed consistently [24, 25].

3.2. Loop factors

The second shortcoming of Weinberg's power counting is the estimate of the powers of $(4\pi)^{-1}$. In the simpler Pionless EFT containing only nucleons [3], one can see explicitly that reducible loops have an additional enhancement of 4π relative to loops in ChPT. It is the combination of this enhancement with the infrared enhancement of Weinberg's that justifies [29] iterating the LO potential: a two-nucleon reducible loop contributes an $m_N Q/4\pi$ that compensates the additional $4\pi/m_N f_\pi$ from the potential, leading at LO to a series that needs resummation for $Q \sim f_\pi$ —incidentally, this generates naturally binding energies per nucleon $B_A/A \sim 10$ MeV, as typically observed. Counting 4π s *à la* Weinberg will simply not do.

Taking into account the proper factor of $(4\pi)^{-1}$ for reducible loops, Friar [30] arrived at an improved power counting where few-nucleon forces are enhanced with respect to Weinberg's. For more details, see Ref. [31]. With an explicit Delta and Friar's counting the three-body force first appears at next-to-leading order (NLO), that is, a relative $\mathcal{O}(Q/M_{\text{QCD}})$ with respect to the LO two-body force. Unfortunately, Friar's work is usually ignored by the nuclear community.

3.3. NDA failure

The third concern is the assumption of NDA. It is now well known that Weinberg's power counting is not consistent with the renormalization group (RG) at the two-body level [32, 33, 34].

The LO two-body potential in Chiral EFT is singular and its renormalization requires more contact interactions than supplied by NDA [31]. In hindsight, this might not be entirely surprising, as NDA is based on perturbative renormalization. Continuing to assume naturalness, but now in the appropriate nonperturbative context, leads to departures from NDA [35].

One could then reasonably expect that NDA might breakdown also in the many-body sector. However, there is no RG evidence that either Weinberg's or Friar's countings fail for more-body forces once the two-nucleon amplitude is renormalized at LO and NLO [33, 36, 22]. This is in stark contrast with Pionless EFT where the RG demands a three-body force at LO [37, 38, 39]. Based on continuity with Pionless EFT, Kievsky and collaborators [40] suggested that three-nucleon forces should be included at LO also in Chiral EFT. While there is an improvement in the description of data, a power-counting rationale is missing.

4. A solution?

The description of $A = 3, 4$ nuclei in properly renormalized Deltaless EFT up to (and including) NLO—that is, before three-nucleon forces enter according to either Weinberg or Friar—is actually very good [22]. Thus in light nuclei few-body forces do not seem to be necessary at LO in Chiral EFT, consistently with a lack of RG enhancement. However, just as for potentials in Weinberg's power counting, larger nuclei are not stable at LO [22]. While one cannot exclude that stability will emerge at higher orders, which should be perturbative, instability could be a clue for the growing importance of three-nucleon forces as A increases.

This led Jerry Yang and collaborators [41] to propose that the ordering of few-nucleon forces depends on the number of nucleons present. It is not impossible that the power counting needs to be modified for $A \gg 1$, as we then have an additional, large dimensionless factor. The basic idea is very simple: for $2 < a < A/2$, there are ${}_A C_a / {}_A C_2$ more ways to construct an a -body than a two-body interaction, where ${}_A C_a = A! / a!(A - a)!$ is the binomial coefficient. Of course, one needs to account as well for a suppression by powers of Q/M_{QCD} , and the question arises of the dependence of the typical bound-state momentum Q on A . There is no obvious answer, except for $A = 2$ where the position of the pole in the imaginary axis of the complex-momentum plane is $(2m_N B_A / A)^{1/2}$. This is the same as one would naively guess by assuming each nucleon contributes Q^2/m_N to B_A . With this assumption, the fact that B_A/A is essentially constant for $A \geq 4$ would lead to a constant Q . With Friar's counting in Deltaless Chiral EFT, the suppression is $(Q/M_{\text{QCD}})^2$. If $Q \sim 3f_\pi$ for nuclear matter, this suppression can be alternatively written as $\rho_0/f_\pi^2 M_{\text{QCD}}$, where $\rho_0 \simeq 0.16 \text{ fm}^{-3}$ is the saturation density. With these very rough estimates, one expects three-nucleon forces to become comparable to two-nucleon forces for $A \sim 20$, quickly followed by four-body forces at $A \sim 25$.

While these critical values of A cannot be taken very seriously, they suggest there might be a range of nuclei for $A > 4$ where three-body forces should be included at LO, despite the fact that they are subleading (and thus perturbative) for $A \leq 4$. It is encouraging that then ^{16}O and even ^{40}Ca become stable [22]. However, for the latter the single-particle states indicate a disfavored deformation, which could be a consequence of the inappropriate neglect of four-body forces at such large A . Since on account of the exclusion principle five- and more-nucleon forces have additional Q/M_{QCD} suppression, it is possible that they never become important.

Even if the combinatorial factor is not the root cause of an enhancement of three-body forces, it could be that resumming these forces into LO for $A > 4$ is justified as an “improved action” in the sense of of lattice QCD: an interaction that is introduced to accelerate convergence—in this case, to obtain stability already at LO without breaking RG invariance and to enable a perturbative treatment of corrections.

5. Conclusion

Despite the importance of few-nucleon forces for a consistent description of nuclei and the many years of development in Chiral EFT, the order at which they should first be included remains to be conclusively established. We hope that a better understanding of the leading order—which should give the correct physics within the error of the EFT expansion but is usually avoided by potential modelers—will soon emerge thanks to improved “*ab initio*” methods [42] for the solution the many-body Schrödinger equation.

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