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Sum squeezing, entanglement and quantum teleportation of the superposition of photon-added pair coherent state

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Abstract. In this paper, we study some non-classical properties of the superposition of photon-added pair coherent state (SPAPCS) such as two-mode sum squeezing and entanglement. The obtained results show that this state has two-mode sum squeezing behavior but the pair coherent state does not. The degree of squeezing of the SPAPCS is enhanced as more and more photons are added to the two modes of the original pair coherent state. Besides, this state is an entangled state and the entanglement degree can be enhanced by adding photons to both modes of the state. When using the SPAPCS as an entangled resource for a coherent state quantum teleportation, it is shown that the quantum teleportation process is successful and the average fidelity of the process can be higher than that of the case where the original pair coherent state is used if the parameters are selected appropriately.

1. Introduction

The non-classical properties of the coherent electromagnetic field such as squeezing and entanglement properties have many important applications in quantum optics and quantum information. For example, the application of squeezed light to improve the sensitivity of detectors in large laser interferometers at gravitational wave observatories [1]. Besides, the entanglement property is applied to areas such as quantum teleportation [2, 3, 4, 5, 6, 7], quantum computing [8], quantum cryptography [9, 10].

To determine the squeezing property as well as to measure the degree of entanglement of a two-mode state we can apply the corresponding criteria for that state. The squeezing criteria for two-mode states have been studied by Hillery since 1989 [11]. Recent studies on the non-classical properties of multimode states have used the two-mode sum or difference squeezing criteria [12], simultaneously they are developed to higher-order squeezing criteria [13] or squeezing criteria for three-mode states [14, 15]. Regarding quantum entanglement, to determine the degree of entanglement of the two-mode states we can use some criteria such as von Newman entropy [16, 17], linear entropy [4, 5, 14, 15, 18], and concurrence [17, 19, 20].

The pair coherent state (PCS) was proposed by Agarwal in 1986 [21] and later shown to be a non-classical state [18, 22]. The interesting thing is that many later studies have indicated that the adding or subtracting photons to the PCS can create new non-classical states with non-classical properties that can be enhanced [5, 23, 24] and hence the applying of the new states into quantum teleportation gives a higher degree of success when using the PCS [5, 23, 24, 25].

The PCS $|\xi, q\rangle$ is a state of the two modes a and b of the radiation field [21, 22], which is expanded in terms of the two-mode Fock state $|n, n+q\rangle \equiv |n\rangle_a |n+q\rangle_b$ as

$$|\xi, q\rangle = A_q \sum_{n=0}^{\infty} \frac{\xi^n}{[n!(n+q)!]^{1/2}} |n, n+q\rangle, \tag{1}$$

where $\xi = |\xi| \mathrm{e}^{i\phi}$ is a complex number, q is an integer that indicates the difference in the number of photons between two modes of the field, and A_q is the normalization coefficient determined by

$$A_q^{-2} = \sum_{n=0}^{\infty} \left[|\xi|^{2n} (n!(n+q)!)^{-1} \right]. \tag{2}$$

The PCS is the simultaneous eigenstate of the pair annihilation operator ab with a and b being boson operators, and the operator $Q = b^+b - a^+a$ with eigenvalues are ξ and q respectively, in which $ab\,|\xi, q\rangle = \xi\,|\xi, q\rangle$ and $Q\,|\xi, q\rangle = q\,|\xi, q\rangle$.

Recently, the superposition of photon-added pair coherent state (SPAPCS) $|\xi, q; k, l\rangle$ was proposed [25] by adding the superposition of two operators a^{+k} and b^{+l} to the original PCS as follows

$$\begin{aligned} |\xi, q; k, l\rangle &= A_{qkl} \left(a^{+k} + \varepsilon b^{+l} \right) \sum_{n=0}^{\infty} \frac{\xi^n}{[n!(n+q)!]^{1/2}} |n, n+q\rangle \\ &= \sum_{n=0}^{\infty} \left(C_{n,qkl} |n+k, n+q\rangle + D_{n,qkl} |n, n+q+l\rangle \right), \end{aligned} \tag{3}$$

where a^{+k} and b^{+l} are the creation operators of order k and l for modes a and b with k and l are non-negative integers, ϵ is a non-negative real number, and the coefficient A_{qkl} is determined from normalization condition as follows

$$A_{qkl}^{-2} = \sum_{n=0}^{\infty} |\xi|^{2n} \frac{1}{n!(n+q)!} \left(\frac{(n+k)!}{n!} + \frac{\varepsilon^2 (n+q+l)!}{(n+q)!} + 2\varepsilon \delta_{k+l,0} \right). \tag{4}$$

The coefficients $C_{n,qkl}$ and $D_{n,qkl}$ in Eq. (3) are determined by

$$C_{n,qkl} = A_{qkl} \frac{\xi^n \sqrt{(n+k)!}}{n! \sqrt{(n+q)!}}, \quad \text{and} \quad D_{n,qkl} = A_{qkl} \frac{\varepsilon \xi^n \sqrt{(n+q+l)!}}{(n+q)! \sqrt{n!}}. \tag{5}$$

Note that when $k = l = 0$, the SPAPCS in Eq. (3) is reverted to the PCS in Eq. (1).

In this paper, we investigate the two-mode sum squeezing of the SPAPCS in Section 2, and the degree of entanglement of the SPAPCS in Section 3. We use the SPAPCS as an entangled resource for the quantum teleportation process in Section 4. We conclude the results and discussions of the properties of the two-mode sum squeezing as well as the degree of entanglement of the SPAPCS in Section 5. Finally, the conclusions are given in Section 6.

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2. Two-mode sum squeezing

The criterion to detect two-mode sum squeezing of two-mode states of electromagnetic fields was introduced by Hillery in 1989 [11]. Later, to facilitate studying, a two-mode sum squeezing parameter $S(\phi)$ [12] is defined as

$$S(\phi) = \frac{4\left\langle (\Delta V(\phi))^2 \right\rangle - \langle N_a + N_b + 1\rangle}{\langle N_a + N_b + 1\rangle}, \tag{6}$$

where the variance $\left\langle (\Delta V(\phi))^2 \right\rangle = \left\langle V(\phi)^2 \right\rangle - \langle V(\phi)\rangle^2$ with $V(\phi) = \left(a^+b^+e^{i\phi} + abe^{-i\phi}\right)/2$ is the two-mode sum squeezing operator, ϕ is real, $\langle N_a\rangle$ and $\langle N_b\rangle$ are the average photon numbers of modes a and b. According to Ref. [12], a state has the two-mode sum squeezing behavior if squeezing parameter $S(\phi)$ of the state satisfies $-1 \le S(\phi) < 0$.

Note that the SPAPCS is reduced to the PCS when $k = l = 0$. From the PCS in Eq. (1) and the two-mode sum squeezing parameter in Eq. (6), it is easy to get $S(\phi) = 0$. We can conclude that the PCS has no two-mode sum squeezing behavior. For the SPAPCS in Eq. (3) and the condition in Eq. (6), we obtained $S(\phi)$ for the SPAPCS with any positive integer values of k and l as follows

$$\begin{aligned} S(\phi) = 2\Bigg\{ & A_{qkl}^2 \sum_{n=0}^{\infty} \frac{|\xi|^{2n}}{n!(n+q)!} \left[\frac{(n+k+2)!}{n!} \left(\frac{|\xi|^2\cos^2 2\phi}{(n+1)(n+2)} + \frac{(n+k)(n+q)}{(n+k+1)(n+k+2)} \right) \right. \\ & \left. + \frac{\varepsilon^2 (n+q+l+2)!}{(n+q)!} \left(\frac{|\xi|^2\cos^2 2\phi}{(n+q+1)(n+q+2)} + \frac{n(n+q+l)}{(n+q+l+1)(n+q+l+2)} \right) \right] \\ & - \left(1+\cos^2 2\phi\right) \left(A_{qkl}^2 \sum_{n=0}^{\infty} \frac{|\xi|^{2n+1}}{n!(n+q)!} \left[\frac{(n+k+1)!}{(n+1)!} + \frac{\varepsilon^2(n+q+l+1)!}{(n+q+1)!} \right] \right)^2 \Bigg\} \\ & \times \left\{ 1 + A_{qkl}^2 \sum_{n=0}^{\infty} \frac{|\xi|^{2n}}{n!(n+q)!} \left[\frac{(n+k)!(2n+q+k)}{n!} + \frac{\varepsilon^2 (n+q+l)!\,(2n+q+l)}{(n+q)!} \right] \right\}^{-1}, \end{aligned} \tag{7}$$

where coefficient A_{qkl} is given by Eq. (4).

3. Entanglement degree

There are many criteria to quantify the degree of entanglement of a two-mode state. In this section, we have chosen the linear entropy criterion [18] to investigate the degree of entanglement for the SPAPCS. Accordingly, the linear entropy parameter E_{lin} is defined as

$$E_{lin} = 1 - Tr\left(\rho_k^2\right), \tag{8}$$

where E_{lin} always satisfies $0 \le E_{lin} \le 1$. Any non-zero E_{lin} provides a signature of entanglement presenting in the state. The bigger E_{lin} is, the higher the entanglement degree of the state becomes. If $E_{lin} = 1$, the state has an ideal entanglement degree. To determine E_{lin} for the SPAPCS, we put $\rho_{ab} = |\xi, q; k, l\rangle_{ab}\langle \xi, q; k, l|$ in first, and then we examine $\rho_a = Tr_b(\rho_{ab})$. Finally, we investigate $E_{lin} = 1 - Tr\left(\rho_a^2\right) = 1 - Tr\left(\rho_b^2\right)$. After the calculations, the final result of the linear entropy parameter E_{lin} is as follows

$$\begin{aligned} E_{lin} = 1 - A_{qkl}^4 \sum_{n=0}^{\infty} \frac{|\xi|^{4n}}{(n!)^2((n+q)!)^2} & \left\{ \frac{((n+k)!)^2}{(n!)^2} + \frac{\varepsilon^4((n+q+l)!)^2}{((n+q)!)^2} \right. \\ & \left. + \frac{2\varepsilon^2|\xi|^{2l} n!(n+k+l)!}{((n+l)!)^2} + \frac{2\varepsilon^2|\xi|^{2k}(n+q)!\,(n+q+k+l)!}{((n+q+k)!)^2} \right\}, \end{aligned} \tag{9}$$

where coefficient A_{qkl} is given by Eq. (4).

4. Quantum teleportation uses the measuring of the orthogonal quadrature components

The orthogonal quadrature components measurements protocol in quantum teleportation was proposed by Furusawa *et. al.* in 1998 [26]. Later, it was improved by several authors [2, 27, 28, 29]. In this section, we use the SPAPCS as an entanglement resource for the teleportation process in the measuring of orthogonal quadrature components protocol [2], where the SPAPCS is shared by Alice and Bob. Alice possesses mode a, and mode b to Bob. Suppose the task of the process requires Alice to teleport to Bob a coherent state $|\alpha\rangle_c$ corresponding to mode c. The process is begun by combining the coherent state $|\alpha\rangle_c$ with the SPAPCS state as

$$\begin{aligned} |\Phi_{in}\rangle_{abc} &= |\xi, q; k, l\rangle_{ab}|\alpha\rangle_c \\ &= \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} d_m \left(C_{n,qkl}|n+k, n+q, m\rangle_{abc} + D_{n,qkl}|n, n+q+l, m\rangle_{abc}\right), \end{aligned} \tag{10}$$

where $|\alpha\rangle_c = \sum_{m=0}^{\infty} d_m |m\rangle_c$, $d_m = e^{-|\alpha|^2/2}\alpha^m \big/ \sqrt{m!}$.

Next, Alice measures the orthogonal quadrature components on two modes a and c. After this measurement, the state of Bob becomes

$$|\Phi\rangle_b = {}_{ac}\langle \beta \mid \xi, q; k, l\rangle_{ab}|\alpha\rangle_c, \tag{11}$$

where $|\beta\rangle_{ac}$ are the eigenstates of two commuting operators $x_- = x_c - x_a$ and $y_+ = y_a + y_c$ [2]. Such states are expressed in the Fock space as $|\beta\rangle_{ac} = \pi^{-1/2}\sum_{k=0}^{\infty} D_c(\beta)\,|k,k\rangle_{ac}$ with $x_-|\beta\rangle_{ac} = \mathrm{Re}(\beta)\,|\beta\rangle_{ac}$, $y_+|\beta\rangle_{ac} = \mathrm{Im}(\beta)\,|\beta\rangle_{ac}$, and $D_c(\beta)$ is the displacement operator acting on the state of mode c. The state in Eq. (11) is expanded as

$$|\Phi\rangle_b = \frac{e^{-|\alpha-\beta|^2/2} e^{(\alpha\beta^* - \alpha^*\beta)/2}}{\sqrt{\pi}} \sum_{n=0}^{\infty} \left(\frac{(\alpha-\beta)^{n+k}}{\sqrt{(n+k)!}} C_{n,qkl}|n+q\rangle_b + \frac{(\alpha-\beta)^n}{\sqrt{n!}} D_{n,qkl}|n+q+l\rangle_b \right). \tag{12}$$

Then, by using a classical channel, Alice sends Bob the information about the measurement results. After receiving the information, first, Bob performs a photon number shifting by using the operator $U_p = \sum_{j=0}^{\infty} |j\rangle\langle j+p|$ [30]. In this process, we add a photon displacement at the receiver party [6]. Thus, the Bob state becomes

$$\begin{aligned} |\Phi\rangle = U_p|\Phi\rangle_b = & \frac{e^{-|\alpha-\beta|^2/2} e^{(\alpha\beta^* - \alpha^*\beta)/2}}{\sqrt{\pi}} \sum_{j=0}^{\infty}\sum_{n=0}^{\infty} \left(\frac{(\alpha-\beta)^{n+k}}{\sqrt{(n+k)!}} C_{n,qkl}\,|j\rangle\,\delta_{j+p,n+q} \right. \\ & \left. + \frac{(\alpha-\beta)^n}{\sqrt{n!}} D_{n,qkl}\,|j\rangle\,\delta_{j+p,n+q+l} \right). \end{aligned} \tag{13}$$

The quantum teleportation process is finished after Bob restores the teleported state by using an operator $D(\beta)$ acting on his state. In the case $p \geq q+l$, the output state reads

$$\begin{aligned} |\Phi_{out}\rangle = & \frac{e^{-|\alpha-\beta|^2/2} e^{(\alpha\beta^* - \alpha^*\beta)/2}}{\sqrt{\pi}} \sum_{n=0}^{\infty} \left[\frac{(\alpha-\beta)^{n+k+p-q}}{\sqrt{(n+k+p-q)!}} C_{n+p-q,qkl} \right. \\ & \left. + \frac{(\alpha-\beta)^{n+p-q-l}}{\sqrt{(n+p-q-l)!}} D_{n+p-q-l,qkl} \right] D(\beta)\,|n\rangle\,. \end{aligned} \tag{14}$$

The quality of the quantum teleportation process is shown by the average fidelity as

$$F_{av} = \int |_c\langle \alpha \mid \Phi_{out}\rangle|^2 d^2\beta. \tag{15}$$

The quantum teleportation process is successful when $F_{av} > 0.5$ and perfect at $F_{av} = 1$. In case $\phi = 0$, the our calculating result for the average fidelity of the quantum teleportation process is

$$\begin{aligned} F_{av} = A_{qkl}^2 \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} \Bigg\{ \frac{2^{-(n+m+p-q+1)}|\xi|^{n+m+2(p-q)}}{\sqrt{n!m!(n+p)!(m+p)!}} \Bigg(\frac{2^{-k}\,(n+m+k+p-q)!}{(n+p-q)!(m+p-q)!} \\ + \frac{\varepsilon^2|\xi|^{-2l}2^l(n+p)!(m+p)!\,(n+m+p-q-l)!}{(n+p-l)!(m+p-l)!\,(n+p-q-l)!\,(m+p-q-l)!} \Bigg) \Bigg\}, \end{aligned} \tag{16}$$

where coefficient A_{qkl} is given by Eq. (4).

5. Results and discussions

We use the analytical expression in Eq. (7) to clarify the two-mode sum squeezing property in the SPAPCS. In Fig. 1, we plot the dependence of $S(\phi)$ as a function of $|\xi|$ for several values of ϕ, k, q, and l. Fig. 1 (a) shows that when $\phi = m\pi/4$ with m is an integer, the SPAPCS reveals the best two-mode sum squeezing property. Besides, Figs. 1 (b) and (c) show that if the parameter k is fixed and l is increased, or the parameter l is fixed and k is increased, the degree of two-mode sum squeezing in the SPAPCS will be increased. In addition, in these cases, the two-mode sum squeezing behaviors depend on the parameter k more strongly than parameter l. The above results exhibit that the SPAPCS has the two-mode sum squeezing property while the PCS does not. More importantly, the two-mode sum squeezing characteristics in the SPAPCS is improved if the number of photons added to the modes increases.

In the entanglement property, Eq. (9) allows us to investigate the entanglement degree of the SPAPCS. In Fig.2, we plot about dependence of E_{lin} on $|\xi|$ with $\phi = \pi$ and $\epsilon = 1$ for several values, in which the blue solid lines ($k = l = 0$) correspond to the PCS and the others are the SPAPCS. In Fig. 2 (a), the graphs show that when l is fixed and k is increased, E_{lin} increases. The entanglement degree is enhanced if k is fixed and l is increased (see Fig. 2 (b)). It is easy to see that E_{lin} of the SPAPCS is always higher than that of the PCS. It is interesting that in the SPAPCS when values of $|\xi|$ are very large, the values of E_{lin} tend to the unit.

With respect to quantum teleportation, we evaluate the success of the quantum teleportation process of a coherent state by using Eq. (16). In Fig. 3, we plot the average fidelity F_{av} as a function $|\xi|$ for some values of ϵ, p, q, k, l, in which we have chosen $p \geq q + l, \epsilon = 0.25$, and $k = 1$. The graphs in Fig. 3 (a) show that when the parameters p, k, q are fixed, the average fidelity F_{av} is increased by increasing l. In addition, Fig. 3 (b) shows that when the values of p, q, l satisfied $p = q + l$ and $q = l$, if l increases then F_{av} increases. We can easily see that the F_{av} of the SPAPCS is greater than that of the PCS in some regions of $|\xi|$. More importantly, the average fidelity F_{av} of the SPAPCS reaches 0.81 at $|\xi| = 3.5$ while this factor of the PCS is maximum at 0.76 when $|\xi| = 1.25$ (see Fig. 3 (b)).

6. Conclusions

In this paper, we have studied the two-mode sum squeezing, entanglement, and quantum teleportation in the orthogonal quadrature components measurements protocol of the SPAPCS. The obtained results show that the SPAPCS exhibits the two-mode sum squeezing property while the PCS does not. The degree of the two-mode sum squeezing is enhanced by increasing the number of photons added to two modes of the original PCS. Besides, it is shown that the

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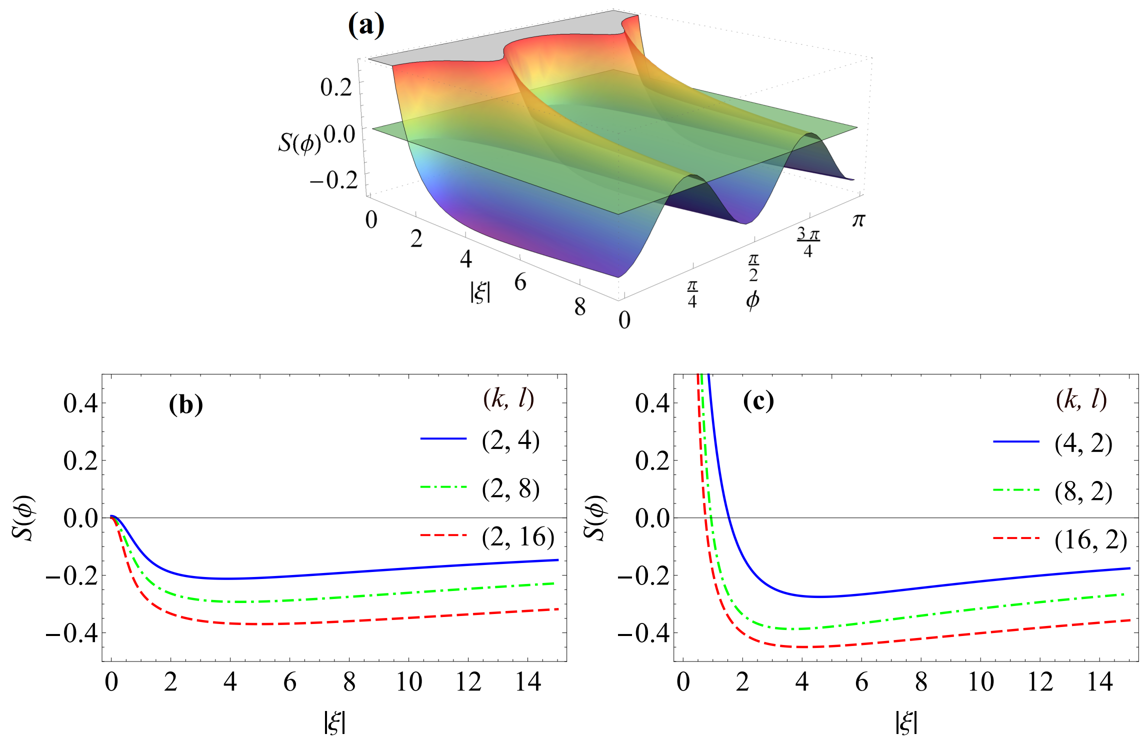

Figure 1. The squeezing parameter $S(\phi)$ as a function of $|\xi|$ and ϕ with $\epsilon = 1, q = 2$ for in (a) $(k,l) = (4,2)$, in (b) $\phi = \pi$ and $(k,l) = (2,4)$ (the blue solid line), (2, 8) (the green dot-dashed curve), and (2, 16) (the red dashed curve), in (c) $\phi = \pi$ and $(k,h) = (4,2)$ (the blue solid line), (8, 2) (the green dot-dashed curve), and (16, 2) (the red dashed curve).

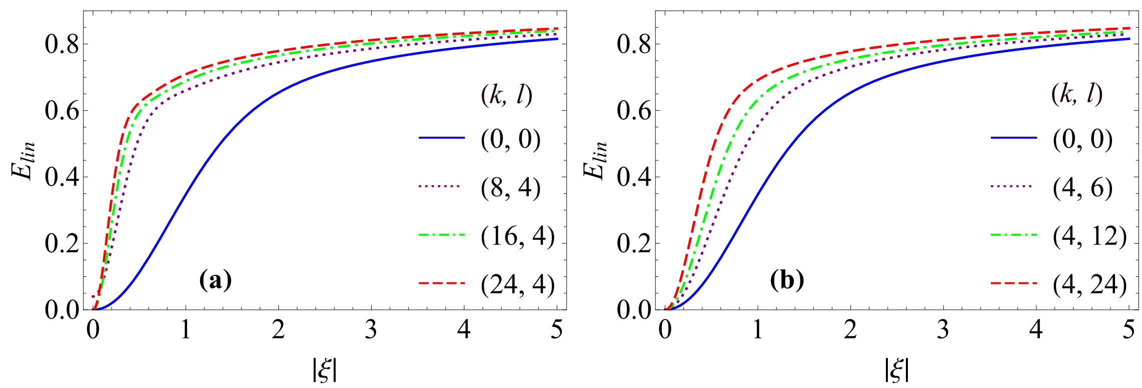

Figure 2. The linear entropy parameter E_{lin} as a function of $|\xi|$ with $\phi = \pi, \epsilon = 1, q = 3$ for in (a) $(k,l) = (0,0)$ (the blue solid line), $(k,l) = (8,4)$ (the purple dotted curve), $(k,l) = (16,4)$ (the green dot-dashed curve), and $(k,l) = (24,4)$ (the red dashed curve), in (b) $(k,l) = (0,0)$ (the blue solid line), $(k,l) = (4,6)$ (the purple dotted curve), $(k,l) = (4,12)$ (the green dot-dashed curve), and $(k,l) = (4,24)$ (the red dashed curve).

SPAPCS is an entangled state with strong entanglement property, and the its entanglement degree is always greater than the PCS's one. Thus, the non-classical properties of the SPAPCS such as squeezing and entanglement are enhanced with the addition of photons. When using the SPAPCS as an entangled resource to teleport a coherent state, the average fidelity of the

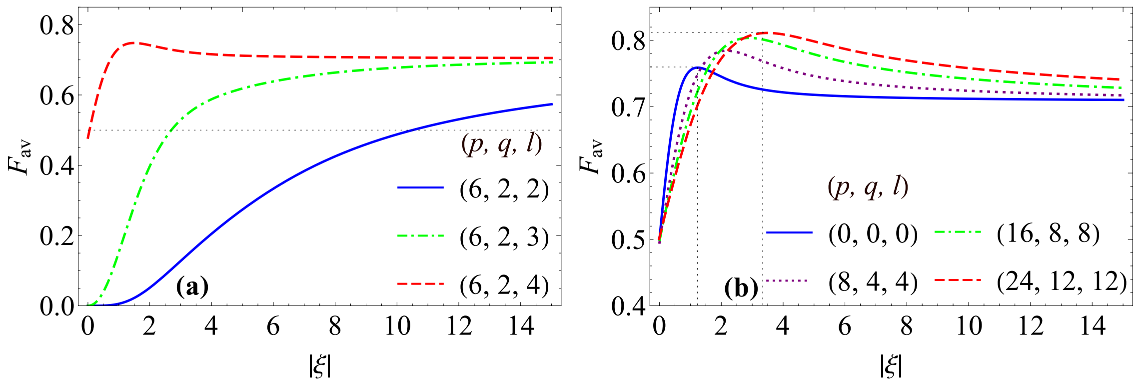

Figure 3. The average fidelity F_{av} as a function of $|\xi|$ with $p \geq q+l, \epsilon = 0.25, k = 1$ for in (a) $(p,q,l) = (6,2,2)$ (the blue solid line), $(6,2,3)$ (the green dot-dashed curve), and $(6,2,4)$ (the red dashed curve), in (b) $(p,q,l) = (0,0,0)$ (the blue solid line), $(8,4,4)$ (the purple dotted curve), $(16,8,8)$ (the green dot-dashed curve), and $(24,12,12)$ (the red dashed curve).

quantum teleportation process in the SPAPCS is improved compared to the PCS. If the number of added photons k and l, the degeneracy parameter q, and the photon number shift parameter p are appropriately chosen, the average fidelity of the quantum teleportation process can reach over 0.85 while that is only 0.76 maximum in the case using the PCS [29]. Therefore, we conclude that the role of photon addition is very important not only for the enhancement of non-classical properties of the SPAPCS but also for the quality of the teleportation processing.

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