Symplectic Particle Tracking in a Thick Nonlinear McMillan Lens for the Fermilab Integrable Optics Test Accelerator (IOTA)

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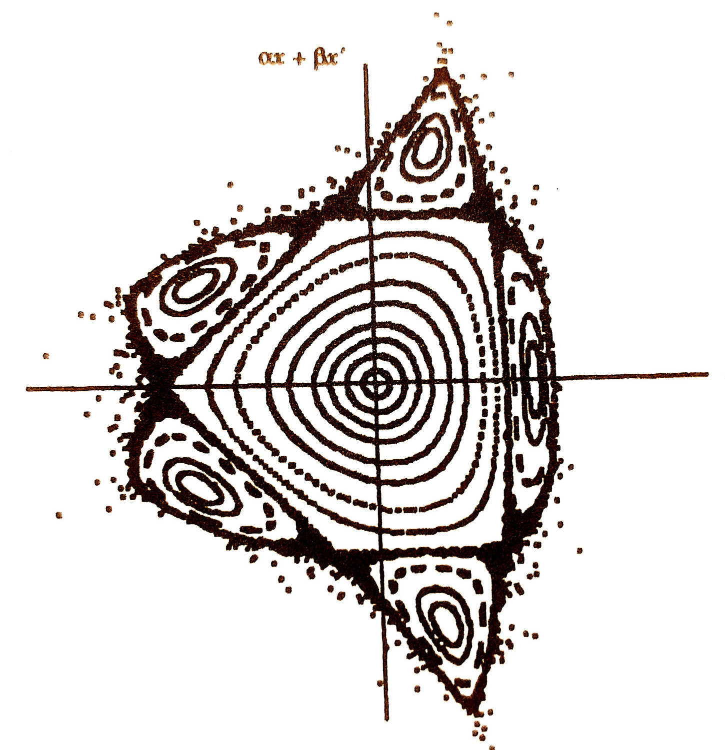
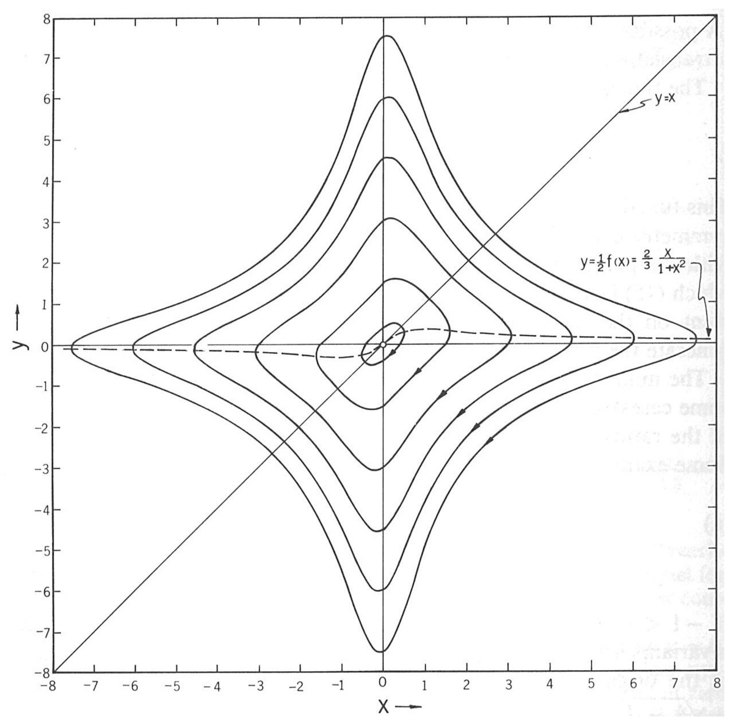
Nonlinear Optics

Landau damping is a way to mitigate beam instabilities. This is the use of tune spread to lower sensitivity to instabilities. To generate a tune spread nonlinear forces are required, such as octupole magnets. However, octupoles and other nonlinear elements can have a significant drawback in that they reduce the beam's dynamic aperture. Integrable nonlinear optics create tune spread without reducing dynamic aperture. This includes the McMillan system.



Sextupole dynamic aperture (Edwards and Syphers, *An Introduction to the Physics of High Energy Accelerators*, p. 109)



Phase space of 2D McMillan system (E. McMillan, *Topics in Modern Physics*, p. 230)

The McMillan System

The McMillan system is a nonlinear, integrable system. It constitutes a linear transport with 0.25 phase advance followed by a radial nonlinear kick.

Linear Transport:

$$\begin{bmatrix} 0 & \beta & 0 & 0 \\ -\frac{1}{\beta} & \beta k & 0 & 0 \\ 0 & 0 & 0 & \beta \\ 0 & 0 & -\frac{1}{\beta} & \beta k \end{bmatrix}$$

Nonlinear Kick:

$$f(r) = \frac{kr}{\frac{r^2}{a^2} + 1}$$

McMillan Electron Lens

Current:

$$J(r) = \frac{j_0}{\left(\frac{r^2}{a^2} + 1\right)^2}$$

Force:

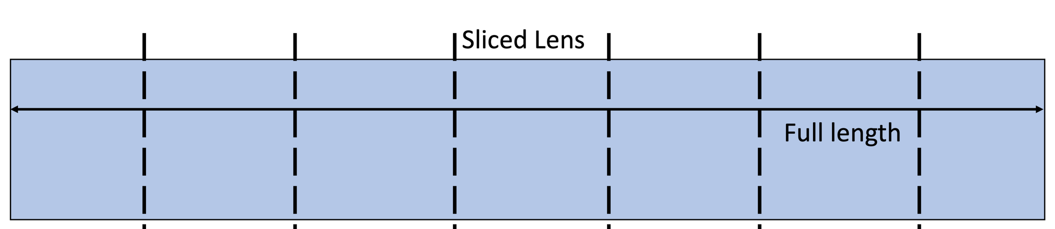
$$\vec{F}(r) = \kappa \frac{r}{\frac{r^2}{a^2} + 1} \hat{r}$$

Potential:

$$V(r) = \frac{-\kappa a^2}{2} \ln\left(\frac{r^2}{a^2} + 1\right)$$

Constants of Motion:

$$L_M = p_z r \theta' - \frac{eBr^2}{2} \qquad E_M = \frac{p_z^2}{2m}\left(r'^2 + \theta'^2\right) + V(r)$$



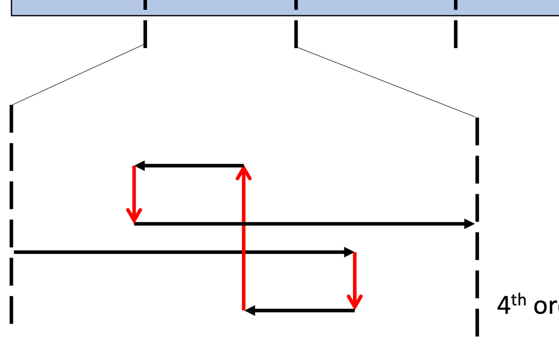
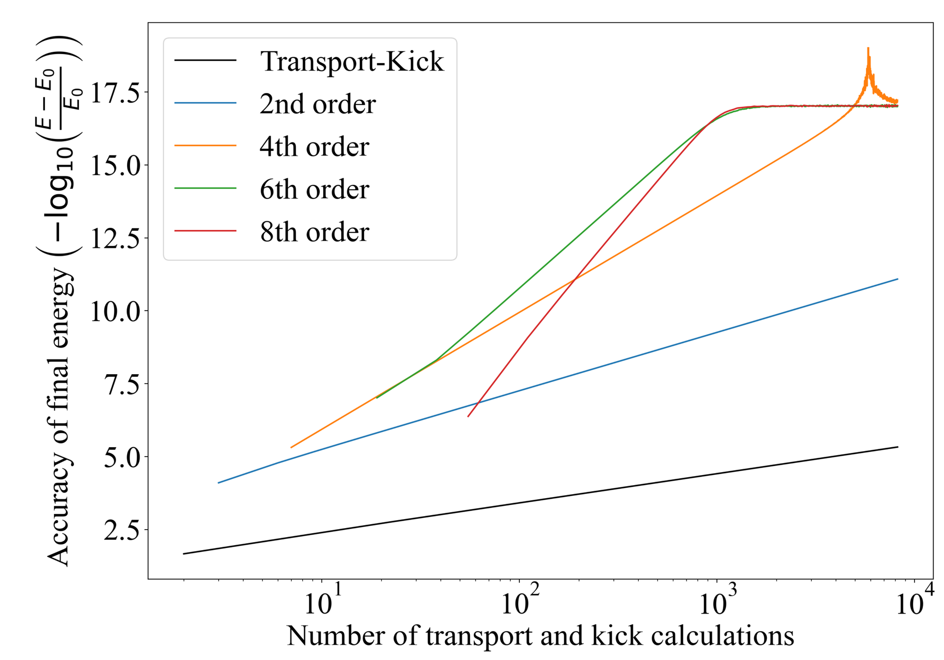


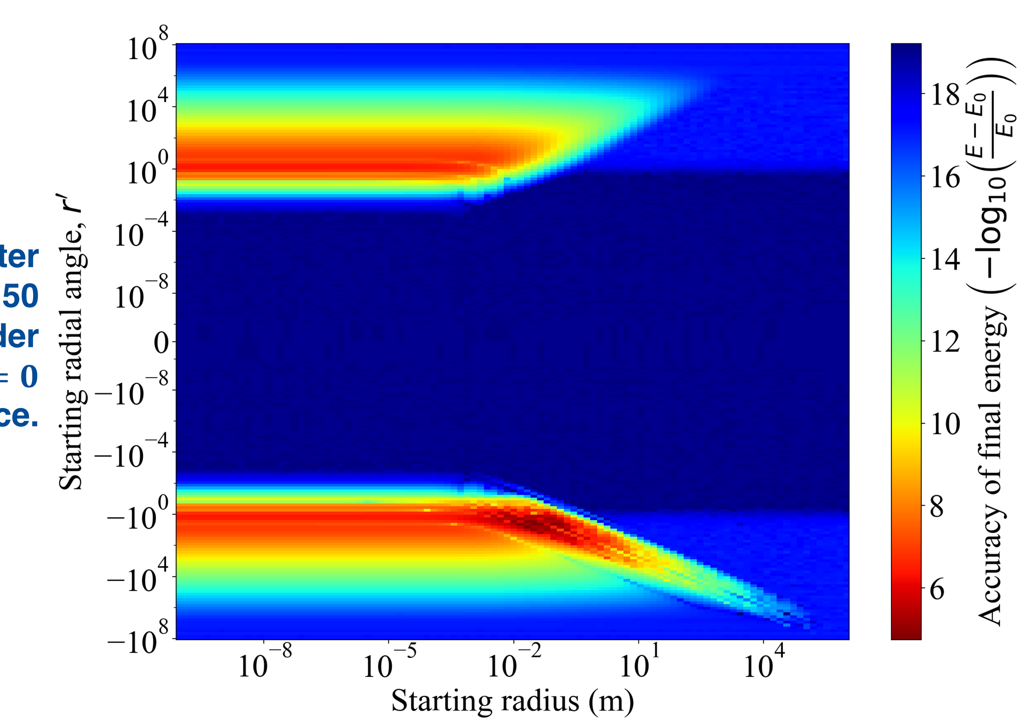
Diagram showing how Yoshida's method was implanted for the thick McMillan Lens.

The Integrable Optics Test Accelerator (IOTA) Electron Lens

The McMillan system is being achieved using an electron lens. Unlike the ideal system, the real one is complicated by a thick nonlinear force instead of the thick kick and a solenoid. For IOTA, the lens length is 0.7m. The force used to approximate the kick has no known analytic solution. As such, Yoshida's method for constructing symplectic integrators was used to simulate motion through the lens. This was done by splitting the Hamiltonian into the solenoid transport and the McMillan kick. Accuracy was determined by comparing the final and initial constants of motion.



Convergence for different integrator orders with increasing slicing of the lens.

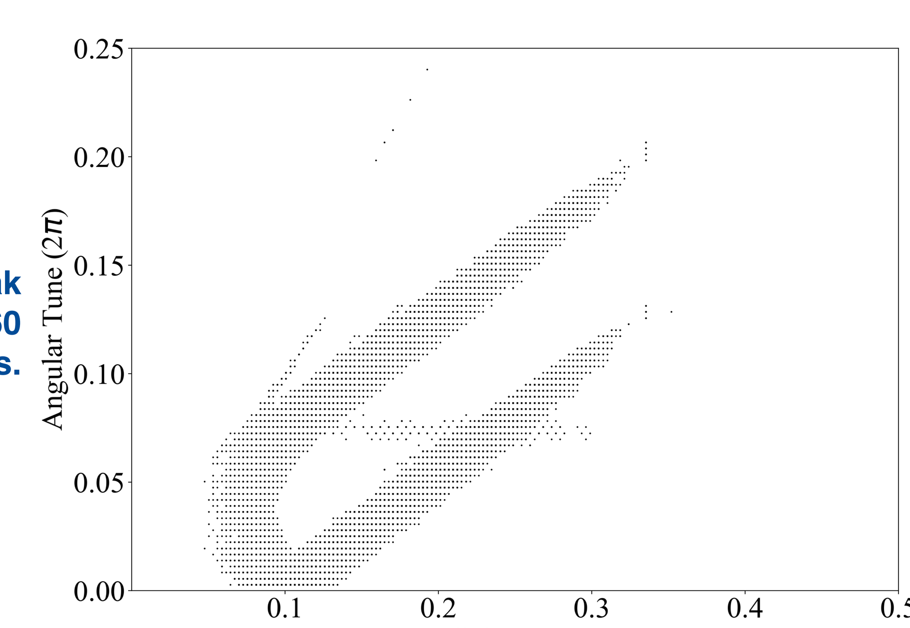


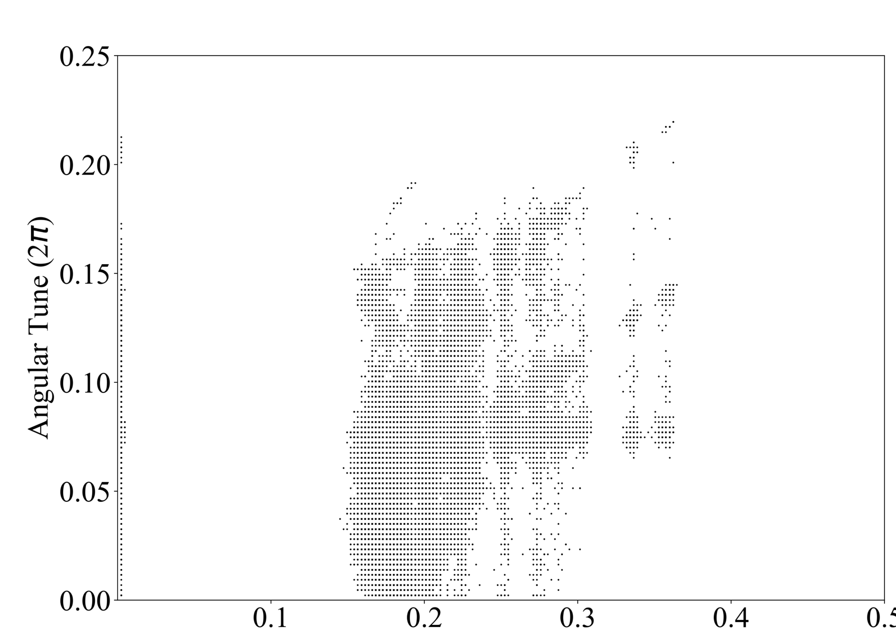
Error in final energy after transporting through 50 slices of the 6th order integrator for the $\theta' = 0$ phase space.

Tune Spread with the Thick McMillan Lens

Simulations were done with a weak and strong lens with the ideal linear transport. The orbiting beam started with an uncorrelated Gaussian distribution with horizontal and vertical emittances of 2.95 mm mrad. While normally the weak lens would not reach the integer resonance, the thick lens with a solenoid did.

Tune spread from a weak ($k = 0.538$m^{-1}) lens for 360 turns.





Tune spread from a strong ($k = 1.076$m^{-1}) lens for 430 turns.

Future Work

Using this method, long term simulations will be done to see how the thick lens affects beam stability. Studies using different IOTA configurations can be done to see if better performance can be achieved.

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