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ABSTRACT

High fidelity target control over quantum systems has been realized in noisy environments. However, with the development of quantum science and technology, much higher requirements have been placed on the control precision. Meanwhile, the open system dynamics can also be modulated via engineering the reservoirs. In this work, on a basis of high fidelity control over systems, we investigate the further enhancement of the fidelity via squeezing the reservoirs. Taking the adiabatic evolution of an open spin system as an example, we find that the squeezing direction determines whether the improvement of adiabatic fidelity occurs, while the squeezing strength determines how much the fidelity is improved. Therefore, a significant enhancement in fidelity can be obtained by choosing suitable squeezing parameters. In addition, squeezing is able to slow down the decline trend that the fidelity degrades with a longer evolution time, a larger system–bath interaction strength, or a more pronounced bath Markovianity. Our work shows that squeezing enables the further improvement of high fidelity and uses a combined strategy that simultaneously controls the system and its environment, which will have potential applications in various quantum information processing tasks.

I. INTRODUCTION

In reality, a quantum system is inevitably exposed to its environment, and consequently, the interaction between them not only complicates the system dynamics but also results in decoherence,[1] dissipation,[2] and disentanglement.[3] Precise control of system dynamics[4] against noise is a fundamental topic in quantum information processing. It normally refers to finding strategies to steer the quantum state evolution from an initial state toward a desired one. Control can roughly be divided into two methods: modulating the system directly or engineering its environment. For the first method, numerous schemes have been proposed, such as the leakage elimination operator (LEO),[1,5,6] quantum error correction,[7] and counter-diabatic driving.[8,9] LEO has been applied in closed or open systems and has been proven to be effective in adiabatic speedup,[10] quantum state transfer,[11] and environmental noise elimination.[1] Reservoir engineering and related techniques, adopting the second method, have been developed and earned ever-growing attention in recent years.[12–15] For instance, engineering certain Markov processes is capable of creating entangled states as stationary asymptotic states of the dynamics.[16–18] In addition, squeezing the reservoirs also provides an effective means for the quantum control.[19–24]

The environments in open quantum systems can be modeled as ensembles of harmonic oscillators in some cases, which in most situations are considered initially stationary. In other words, these oscillators are initialized in stationary states, for instance, coherent states.[25–27] If instead the oscillators are prepared in squeezed states,[28] the reservoirs will become non-stationary. The differences between coherent and squeezed states can be identified by the Heisenberg uncertainty principle: $\langle(\Delta x)^2\rangle\langle(\Delta p)^2\rangle \geq \hbar^2/4$. Coherent states saturate this principle with equal variances $\langle(\Delta x)^2\rangle = \langle(\Delta p)^2\rangle = \hbar/2$ in both quadratures of baths, which makes them closest to points in phase space and, therefore, the most classical quantum states. While for squeezed states, one of the variances can be reduced below the symmetry limit $\hbar/2$, along with the other one growing to saturate the uncertainty principle. Therefore, squeezing can influence quantum fluctuations, providing more adjustable parameters to modulate the system dynamics. Theoretical research has already demonstrated that squeezing the baths can inhibit the phase decay of a two-level atom.[29] From an application perspective, LIGO represents a remarkable achievement, where optical squeezed states are utilized to reduce quantum noise and improve the accuracy of gravitational wave detection.[30–32]

In general, solving the dynamics of open quantum systems is an intractable task. In the Markov regime, the Lindblad master equation[33] has been proposed and extensively adopted, which assumes the Born–Markov approximation and ignores the memory effects of the environments. Then some memory-related properties might be lost, involving quantum entanglement, inhibited spontaneous emission, quantum transport, and quantum tunneling.[34–39] The memory effects of the baths will introduce new challenges for the solution to system dynamics. Fortunately, there have been several methods that provide insights into solving the non-Markovian dynamics, such as hierarchical equations of motion (HEOM),[40] the quantum state diffusion (QSD) equation,[41,42] etc. Moreover, non-Markovian dynamics with squeezed initial states have also been investigated,[22,23,43] providing better understandings of both non-Markovianity and squeezing.

Recently, an accelerated adiabatic quantum search algorithm has been demonstrated, employing pulse control on the quantum system in a non-Markovian environment.[44] The results disclose that while a high success probability is achievable, it degrades with the increasing system–bath coupling strength or bath Markovianity. Is it possible for the adiabaticity to be further improved by squeezing the reservoirs? In this work, we investigate the effects of squeezing when the system is under pulse control. By combining the two methods, pulse control on the system and squeezing on the reservoirs, a better control quality has been obtained.

II. MODEL AND METHOD

A. Non-Markovian master equation

When a quantum system suffers from its environment, the total Hamiltonian $H_{\rm tot}$ then includes three parts

$$H_{\rm tot} = H_{\rm s} + H_{\rm b} + H_{\rm int}, \tag{1}$$

which symbolize Hamiltonians of the system, bath, and their interaction, in order. Without loss of generality, the bath is modeled as independent bosonic modes,

$$H_{\rm b} = \sum_{j=1}^{N} H_{\rm b}^{j} = \sum_{j,k} \omega_k^j b_k^{j\dagger} b_k^j, \tag{2}$$

where ω_k^j denotes the frequency, and $b_k^{j\dagger}$ (b_k^j) represents the creation (annihilation) operator of the kth mode in the jth individual bath, respectively,

$$H_{\rm int} = \sum_{j,k} (g_k^{j*} L_j^\dagger b_k^j + g_k^j L_j b_k^{j\dagger}), \tag{3}$$

suggests that the system is linearly coupled to its bosonic baths through the Lindblad operator L_j with associated coupling constant g_k^j. In this work, we consider $L = L^\dagger$ following the instructions from Refs. 22, 23, and 43.

In the context of two mode squeezed states,[22,23] a unitary squeezing operator S is used to squeeze the vacuum state: $|\phi\rangle = S|0\rangle$, which describes the environmental initial state. The squeezed state $|\phi^j\rangle$ of the jth bath can be characterized by the following correlations:

$$\langle\phi^j|b_k^j|\phi^j\rangle = 0, \tag{4}$$

$$\langle\phi^j|b_k^j b_{k'}^{j\dagger}|\phi^j\rangle = u_j^2\delta_{kk'}, \tag{5}$$

$$\langle\phi^j|b_k^{j\dagger} b_{k'}^{j}|\phi^j\rangle = |v_j|^2\delta_{kk'}, \tag{6}$$

$$\langle\phi^j|b_k^j b_{k'}^{j}|\phi^j\rangle = -v_j u_j \delta_{k',2k_0-k}, \tag{7}$$

where k_0 denotes the mode associated with the center frequency ω_0. The squeezing parameters u_j, v_j must adhere to $u_j^2 - |v_j|^2 = 1$, ensuring the unitarity of squeezing operator S. We also assume the broadband squeezing as in Refs. 22 and 23, i.e., the squeezing parameters are identical for all the environmental modes, and then $u_j = \cosh(r_j)$, $v_j = \sinh(r_j)\exp(i\theta_j)$. The squeezing strength $r_j \in [0,1]$, and θ_j represents the squeezing direction. When $r_j = 0$, the jth bath remains unsqueezed, resulting in an vacuum initial state $|0^j\rangle$. Moreover, $\theta_j = 0$ (π) corresponds to squeezing the p (x) quadrature of the jth bath.

The non-Markovian master equation is introduced to treat the system dynamics,[22,23,45] which reads

$$\begin{aligned}\frac{\partial}{\partial t}\rho_{\rm s} = -i[H_{\rm s},\rho_{\rm s}] + \sum_j \Big\{&[L_j,\rho_{\rm s}\overline{O}_1^{j\dagger}(t)] - [L_j,\overline{O}_1^{j}(t)\rho_{\rm s}] \\ &+ [L_j,\rho_{\rm s}\overline{O}_2^{j\dagger}(t)] - [L_j,\overline{O}_2^{j}(t)\rho_{\rm s}]\Big\}.\end{aligned} \tag{8}$$

It should be noted that within the non-Markovian master equation, the system Hamiltonian $H_{\rm s}$ is permitted to be time-dependent.[46] Here, the memory kernels $\overline{O}_{1,(2)}^j(t) = \int_0^t ds\alpha_{1,(2)}^j(t-s)O_j(t,s,z^*)$, with the bath correlation functions

$$\alpha_1^j(t,s) = \frac{\gamma_j\Gamma_j}{2}(u_j^2 - v_j u_j e^{-2iw_0^j s})e^{-iw_0^j(t-s)-\gamma_j|t-s|}, \tag{9}$$

$$\alpha_2^j(t,s) = \frac{\gamma_j\Gamma_j}{2}(|v_j|^2 - v_j^* u_j e^{2iw_0^j s})e^{iw_0^j(t-s)-\gamma_j|t-s|}, \tag{10}$$

and the operators O_j (for details see Refs. 47 and 48). Note that the correlation functions depend on not only the time difference $t - s$ but also the historical time s, indicating the initial non-stationarity of squeezed vacuum states. In the theoretical derivation, the weak system–bath coupling approximation $(\Gamma_j \ll 1)$[49,50] and the spectrum with a Lorentzian form $J(\omega^j) = \frac{1}{2}\frac{\Gamma_j\gamma_j^2}{(\omega_0^j-\omega^j)^2+\gamma_j^2}$ are assumed.[33,51] Among these, Γ_j symbolizes the system–bath coupling strength, and γ_j denotes the bandwidth of the spectral density. The associated environmental memory time γ_j^{-1} characterizes the memory capacity. $\gamma_j \to \infty$ $(\gamma_j \to 0)$ corresponds to a white (colored) noise situation, and the environment reaches the Markovian (non-Markovian) limit.

The non-Markovian master equation in (8) can be numerically solved with the assistance of two closed equations[23]

$$\frac{\partial}{\partial t}\overline{O}_1^j = \alpha_1^j(0,0)L_j - (iw_0 + \gamma_j)\overline{O}_1^j + \sum_j\left[-iH_s - L_j\overline{O}_1^j - L_j\overline{O}_2^j, \overline{O}_1^j\right], \tag{11}$$

$$\frac{\partial}{\partial t}\overline{O}_2^j = \alpha_2^j(0,0)L_j - (-iw_0 + \gamma_j)\overline{O}_2^j + \sum_j\left[-iH_s - L_j\overline{O}_1^j - L_j\overline{O}_2^j, \overline{O}_2^j\right]. \tag{12}$$

In the Markov limit (i.e., $\gamma_j \to \infty$), from the consistency condition $\partial_t \frac{\delta|\psi_t\rangle}{\delta z_s^*} = \frac{\delta}{\delta z_s^*}\partial_t|\psi_t\rangle$, the operator $O_j = L_j$. Here, z_s^* (or z_t^*) represents a complex Gaussian stochastic process, and the functional derivative $\frac{\delta}{\delta z_s^*}$ describes the influence of environmental noise z_s^* on the system state $|\psi_t\rangle$ or its instantaneous variation $\partial_t|\psi_t\rangle$ (for more details see Refs. 41 and 47). Therefore, the memory kernels are $\overline{O}_1^j = \frac{\Gamma_j}{2}(u_j^2 - v_j u_j)L_j$ and $\overline{O}_2^j = \frac{\Gamma_j}{2}(|v_j|^2 - v_j^* u_j)L_j$. Accordingly, the master equation in (8) reduces to the Lindblad form[23]

$$\begin{aligned}\frac{\partial}{\partial t}\rho_s = -i[H_s,\rho_s] &- \sum_j \Gamma_j(u_j^2 - v_j u_j)\left(\frac{1}{2}\{L_j^\dagger L_j,\rho_s\} - L_j\rho_s L_j^\dagger\right)\\ &- \sum_j \Gamma_j(|v_j|^2 - v_j^* u_j)\left(\frac{1}{2}\{L_j^\dagger L_j,\rho_s\} - L_j\rho_s L_j^\dagger\right).\end{aligned} \tag{13}$$

B. System Hamiltonian

In this work, we take the adiabatic evolution of a time-dependent Hamiltonian as an example to analyze the roles of squeezing and its synergy with pulse control. Adiabaticity is always required in adiabatic quantum computation,[52–55] which is polynomially equivalent to circuit quantum computation, aiming to solve NP-hard and NP-complete problems from its inception. In virtue of the quantum adiabatic theorem, a computational problem can be transformed into the task of finding the ground state of a quantum system. The considered Hamiltonian reads

$$H_s(t) = \left(1 - \frac{t}{T}\right)H_0 + \frac{t}{T}H_T. \tag{14}$$

Here, $H_0 = \mathbb{1} - |\psi_0\rangle\langle\psi_0|$ is the initial Hamiltonian with the uniform superposition state $|\psi_0\rangle = \frac{1}{\sqrt{N}}\sum_{x=0}^{N-1}|x\rangle$ as its ground state. In addition, the final one

$$H_T = \sigma_1^z - \sigma_2^z + \sigma_3^z + \sigma_1^z\sigma_2^z - \sigma_1^z\sigma_3^z + \sigma_2^z\sigma_3^z - \sigma_1^z\sigma_2^z\sigma_3^z. \tag{15}$$

The ground state $|\psi_T\rangle$ of the problem Hamiltonian H_T actually encapsulates the solution to a 3-SAT problem,[52–54,56] the details of which are as follows.

A Boolean formula Φ depends on n literals, $x_i \in \{0,1\}$ or their negations $\neg x_i$

$$\Phi = C_1 \wedge C_2 \wedge \cdots \wedge C_m, \tag{16}$$

with the clauses $C_i = (\neg)^{s_1}x_1 \vee (\neg)^{s_2}x_2 \vee \cdots \vee (\neg)^{s_k}x_k$. Here, $s_i = 1$ if x_i is negated in its clause; otherwise, it is 0. The Boolean satisfiability problem[52] is to decide whether there exists an assignment of values to the literals that satisfies the Boolean formula, i.e., $\Phi = 1$. If such an assignment exists, then the problem is satisfiable; otherwise, it is unsatisfiable. It has significance in not only in industrial optimization but also the theoretical foundation of computer science.[57] Specifically, the problem with $k = 3$, also called 3-SAT, is NP-complete. Since any NP-hard problem can be transformed to a 3-SAT, it is often employed in complexity theoretical proofs.[56]

In general, a literal in a 3-SAT can be mapped to a Pauli operator through the relation[58]

$$(\neg)^{s_i}x_i \leftrightarrow \frac{\mathbb{1} + (-1)^{s_i}\sigma_i^z}{2}. \tag{17}$$

For instance, we consider the following 3-SAT problem:

$$\begin{aligned}\Phi = &(x_1 \vee x_2 \vee x_3) \wedge (\neg x_1 \vee x_2 \vee x_3) \wedge (x_1 \vee \neg x_2 \vee x_3)\\ &\wedge (x_1 \vee x_2 \vee \neg x_3) \wedge (\neg x_1 \vee \neg x_2 \vee x_3) \wedge (x_1 \vee \neg x_2 \vee \neg x_3)\\ &\wedge (\neg x_1 \vee \neg x_2 \vee \neg x_3).\end{aligned} \tag{18}$$

Correspondingly, the problem Hamiltonian reads $H_T = \sigma_1^z - \sigma_2^z + \sigma_3^z + \sigma_1^z\sigma_2^z - \sigma_1^z\sigma_3^z + \sigma_2^z\sigma_3^z - \sigma_1^z\sigma_2^z\sigma_3^z$, i.e., the Hamiltonian in Eq. (15).

To summarize, from an easily prepared $|\psi_0\rangle$ as our system's initial state, the dynamical evolution process is expected to conclude with a state sufficiently close to the target state $|\psi_T\rangle$. Here, the adiabatic fidelity evolution is monitored as $F(t) = \sqrt{|\langle\psi_s(t)|\rho_s(t)|\psi_s(t)\rangle|}$, where $\rho_s(t)$ is the reduced density matrix and $|\psi_s(t)\rangle$ denotes the instantaneous ground state, derived from $H_s(t)|\psi_s(t)\rangle = E_0(t)|\psi_s(t)\rangle$.

III. RESULTS AND DISCUSSIONS

In this section, we will first analyze the roles of squeezing, i.e., how it affects the free evolution of the system. Then we simultaneously control the system and squeeze its reservoirs to investigate whether squeezing enables the further enhancement of adiabatic fidelity on a basis of LEO control. Throughout the work, EVIS (ESIS) stands for environments with vacuum (squeezed) initial states. The Lindblad operator $L_j = \sigma_j^x$ describes the linear coupling type between the system and the jth bath. To simplify, we also assume all the individual baths share the identical environmental parameters: $\Gamma_j = \Gamma$, $\gamma_j = \gamma$, $r_j = r$, and $\theta_j = \theta$.

A. Free evolution

We first analyze the free evolution case, and in Fig. 1, we plot the fidelity F vs rescaled time t/T without and with environments (EVIS) for given total evolution times $T = 5, 100$, respectively. Other parameters, system–bath interaction strength Γ and non-Markovianity parameter γ, are annotated in Fig. 1. First, for the closed system, the final fidelity $F(T) \approx 1$ with $T = 100$ implies that the system is in an adiabatic regime. $F(T) \approx 0.566$ with $T = 5$ implies a non-adiabatic regime. Second, in the presence of baths, adiabaticity is destroyed, and the fidelities drop dramatically. Meanwhile, the fidelities also decrease with increasing Γ and γ as expected.[10] Furthermore, the fidelities $F(t/T)$ with $T = 100$ are less than those with $T = 5$, suggesting the accumulative deleterious effects of baths on the adiabatic evolution. This is in line with the intuition that the baths will destroy more adiabaticity when the system is exposed to the baths for a longer time.

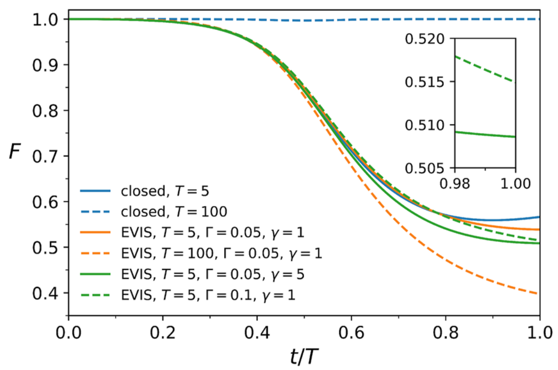


FIG. 1. Fidelity F of free evolutions vs rescaled time t/T in the closed and open (EVIS) systems for $T = 5, 100$.

From now on, we fix the total evolution time $T = 5$ and investigate adiabaticity improvement via squeezing in the non-adiabatic regime. To clearly illustrate to what extent the fidelity F can be boosted via squeezing, we define the fidelity enhancement as

$$\Delta F(t) = F_{\rm s}(t) - F_{\rm v}(t), \tag{19}$$

where $F_{\rm s}(t)$ $(F_{\rm v}(t))$ denotes the fidelities obtained in ESIS (EVIS).

Next, we explore the effects of squeezing on fidelity enhancement ΔF in the free evolution cases. Figure 2(a) shows the trend of ΔF changing with evolution time t and squeezing direction θ. Here the interaction strength $\Gamma = 0.05$, spectrum width $\gamma = 5$, and squeezing strength $r = 1$. Recall that $\theta = 0$ (π) corresponds to squeezing the p (x) quadrature of the baths. Reference 23 has elucidated that when $0 < \theta < \pi/2$ $(\pi/2 < \theta < \pi)$, squeezing the p (x) quadrature predominates. Evidently, in Fig. 2(a), a longer evolution time t or a smaller squeezing direction θ corresponds to a more distinct ΔF. That is to say, the fidelity enhancement provided by squeezing becomes more pronounced when the system is exposed to its baths for a longer evolution time t and when squeezing on their p quadrature prevails. On the one hand, a circumstance with a longer interaction time can provide more scope for squeezing to improve the adiabaticity, due to the terrible performances in EVIS. On the other hand, Refs. 22 and 23 have already revealed that the squeezing on p quadrature of baths is more favorable for maintaining the system's quantumness, as it weakens the system–bath interaction, resulting in a reduction in their effective interaction strength. In Fig. 2(b), we proceed to adjust the squeezing strength r and investigate the collaborative implications of squeezing strength and direction on the final fidelity enhancement $\Delta F(T)$ under free evolution. It turns out that whether the squeezing is overall supportive depends on the squeezing direction θ and the degree of support on the squeezing strength r.

A natural question comes that to what extent is squeezing the baths able to improve the adiabaticity of our system? In Fig. 3, we demonstrate the maximal final fidelity enhancement $\Delta F_{\rm max}(T)$ that can be implemented by squeezing the baths over all possible choices of squeezing parameters. On the bright side, as reported earlier, $\Delta F_{\rm max}(T)$ becomes more pronounced as the system–bath interaction strength Γ and the spectrum bandwidth γ grow. Nonetheless, even with the assistance of squeezing, the final fidelity $F(T)$ is still not sufficiently satisfying. It should be emphasized is that in this work, we consider a simple case, i.e., fixed squeezing parameters in the whole evolution, and explore whether bath squeezing can have positive effects on the overall evolution process. If the squeezing parameters are allowed to change in some way, a better improvement will be achieved, as we can speculate. For example, Ref. 32 has reported that the fluctuations in the amplitude quadrature and phase quadrature contribute to the quantum noise differently at different frequencies and, therefore, making the squeezing direction frequency dependent would optimize the detector sensitivity. In this

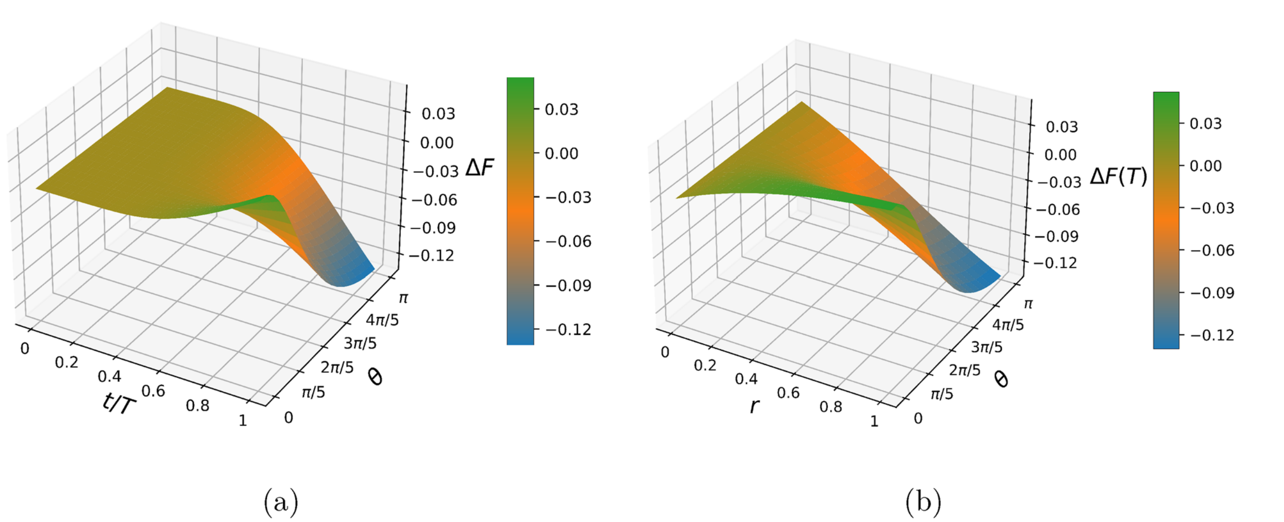


FIG. 2. (a) Fidelity enhancement ΔF vs rescaled time t/T and squeezing direction θ in free evolutions. $r = 1$. (b) Final fidelity enhancement $\Delta F(T)$ via squeezing vs strength r and direction θ in free evolutions. $\Gamma = 0.05$, and $\gamma = 5$.

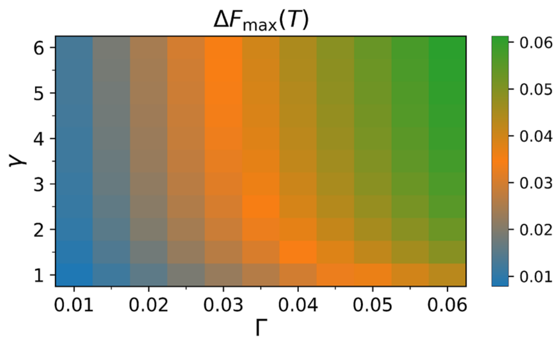


FIG. 3. Maximal final fidelity enhancement $\Delta F_{\max}(T)$ via squeezing vs the system–bath interaction strength Γ and spectrum bandwidth γ in free evolutions.

case, the squeezing direction is determined by the relative phase between the carrier of the main interferometer and the pump field, which produces the squeezed light.

B. Controlled evolution

As mentioned before, the pulse control technique[5,6,44] has been applied to speed up the adiabatic evolution process, and consequently, the detrimental effects of the baths can be partially eliminated. Its main idea is to introduce an LEO Hamiltonian to the system one,

$$H_{\rm c}(t) = H_{\rm s}(t) + H_{\rm LEO}(t), \tag{20}$$

where $H_{\rm LEO}(t) = c(t)H_{\rm s}(t)$, and it can be implemented by a sequence of fast pulses, denoted as $c(t)$. In this work, we assume finite pulse intensity and duration time, which is more realistic for the sake of experimental implementations. The pulse conditions for effective control have been theoretically derived by the P-Q partitioning technique in closed systems.[11] For instance, considering such pulses

$$c(t) = \begin{cases} I, & n\tau < t < (n+1)\tau, \quad n \text{ is even,} \\ -I, & \text{otherwise,} \end{cases} \tag{21}$$

the pulse condition is

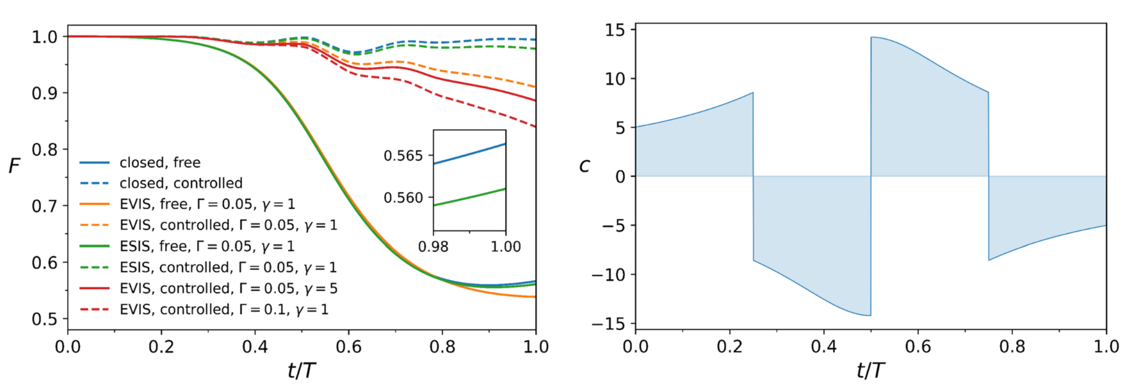
$$I\tau = 2\pi m, \quad m \in N, \tag{22}$$

where I is the pulse intensity and τ is the half period. In addition, when the energy gap between the ground state and first excited state ΔE_{10} is time dependent, instead of constant, the control intensity I needs further modulations,[10]

$$I(t) = \frac{I}{\Delta E_{10}(t)}. \tag{23}$$

Notice that integrating such pulses in (22) and (23) over a period yields zero, i.e., the zero-area condition as in Refs. 10, 11, and 44.

In what follows, we will first analyze the adiabaticity improvement via pulse control in EVIS, then collaborate on it with squeezing to make further enhancements. In Fig. 4(a), we plot the free and controlled fidelity time evolution $F(t)$ in closed systems, EVIS, and ESIS, respectively. For squeezing the baths, the strength is $r = 1$, and direction $\theta = 0$. Other parameters, system–bath interaction strength Γ and non-Markovianity parameter γ, are annotated in Fig. 4(a). In the closed cases, LEO supports the system in achieving an adiabatic evolution even in its non-adiabatic regime. In all EVIS situations, a significant fidelity enhancement ΔF can be achieved via pulse control.[10] However, even in such controlled evolutions, the fidelity F still declines evidently with increasing t, Γ, or γ, which shows again the deleterious powerful influences of environmental noise as in free evolutions. Fortunately, the green dashed line in Fig. 4(a) showcases that squeezing the baths is able to slow down this declining



(a) (b)

FIG. 4. (a) Fidelity F of free and controlled evolutions vs rescaled time t/T in closed systems, EVIS, and ESIS. For the squeezing, $r = 1$, and $\theta = 0$. (b) Modulated pulses in (a). $\tau = T/4$, and $m = 1$.

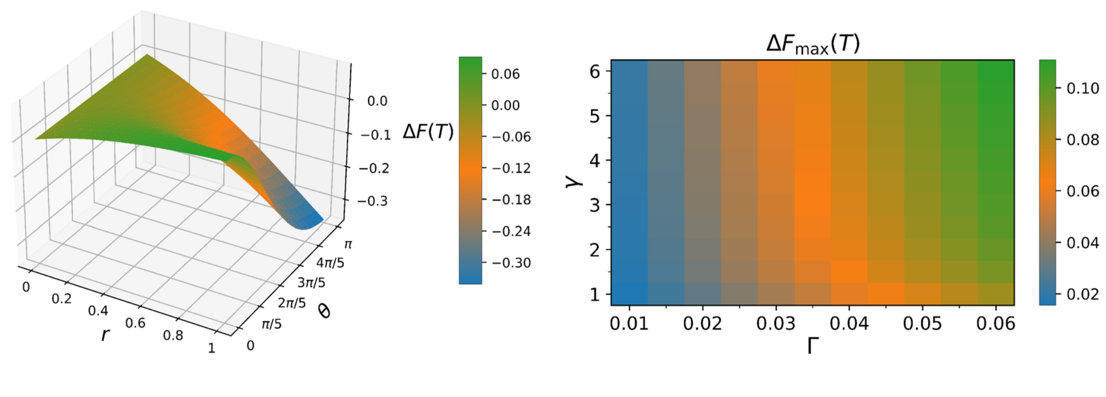


FIG. 5. (a) Final fidelity enhancement $\Delta F(T)$ via squeezing vs strength r and direction θ in controlled evolutions. $\Gamma = 0.05$, and $\gamma = 5$. (b) Maximal final fidelity enhancement $\Delta F_{\max}(T)$ via squeezing, vs system–bath interaction strength Γ and spectrum bandwidth γ in controlled evolutions.

trend. In other words, as the evolution time t, interaction strength Γ, and spectrum bandwidth γ increase, the decline degree of adiabatic fidelity $F(t)$ of controlled evolutions becomes smaller with the help of squeezing. Finally, LEO is also able to visibly reverse the environmental consequences in ESIS as we desire. In Fig. 4(b), the modulated pulses are illustrated. Here, the pulse half period $\tau = T/4$, and $m = 1$ in (22).

Subsequently, as in free evolution scenarios, we plot the final fidelity enhancement $\Delta F(T)$ vs squeezing strength r and direction θ under control in Fig. 5(a). Here, mutual interaction strength $\Gamma = 0.05$, and non-Markovianity parameter $\gamma = 5$. Identical trends with the free evolutions are observed: squeezing effectiveness relies on the squeezing direction θ, and the level of effectiveness is determined by the strength r. In Fig. 5(b), we explore the maximal enhancement available $\Delta F_{\max}(T)$ via squeezing in controlled evolutions. A precedent conclusion once again gets illustrated that an ESIS with a larger Γ or γ leads to a more significant enhancement in fidelity. In addition, note that the environmental parameter region in Fig. 5(b) is the same as that in Fig. 3, but the attainable maximal improvement becomes clearly amplified. This implies that the pulse control also plays a helpful role for fidelity enhancement via squeezing.

Finally, we consider the Markov limit (i.e., $\gamma \to \infty$) when the dynamics are governed by (13). In Fig. 6, we present the final fidelity $F(T)$ in EVIS and the attainable maximal final fidelity $F_{\max}(T)$ in ESIS vs the system–bath interaction strength Γ. In both scenarios, we study the free and controlled evolutions and find that the pulse control keeps its efficacy in the Markov limit, regardless of EVIS or ESIS. In addition, squeezing the Markov baths also allows for a distinct fidelity enhancement compared with free and controlled evolutions in EVIS. Meanwhile, this enhancement becomes more pronounced as Γ grows. It is also visible that $F(T)$ and $F_{\max}(T)$ decrease with the increasing Γ for all cases, but the decrease decline in ESIS is observably reduced. Furthermore, a same bath squeezing results in a more significant fidelity improvement in controlled evolution processes. This proves again the mutual support between bath squeezing and pulse control.

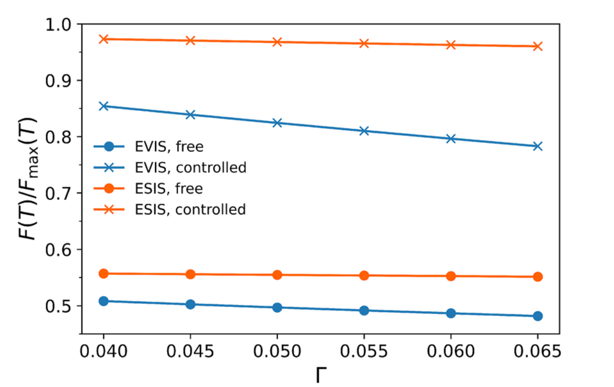


FIG. 6. Final fidelity $F(T)$ in Markov EVIS and the attainable maximal final fidelity $F_{\max}(T)$ in Markov ESIS vs system–bath interaction strength Γ. The blue (orange) lines indicate the EVIS (ESIS), and the dots (crosses) mark the free (controlled) evolutions.

IV. CONCLUSIONS

In this work, we analyze the roles of bath squeezing in the open quantum system dynamics and apply it to further improve the adiabatic fidelity on a basis of LEO control. Specifically, the adiabatic evolution process of an open spin system is taken for a demonstration, and the non-Markovian QSD equation is employed to treat the system dynamics. For both free and controlled evolutions, we find that the squeezing direction determines whether the improvement of the adiabatic fidelity occurs or not, while the squeezing strength determines how much it improves. Therefore, with suitable strength and direction, the adiabatic fidelity can be enhanced via squeezing. In addition, the fidelity enhancement available via squeezing is more significant for a longer system–bath interaction time, a larger interaction strength, and a stronger bath Markovianity. It is also worth mentioning that the integration of squeezing and LEO can offer improved performance compared to their individual applications. Squeezing can help LEO slow down the fidelity degradation trend as the evolution time, interaction strength, and Markovianity increase, and LEO can in turn amplify the helpful roles of bath squeezing in fidelity improvement. It should be noted that in this work we only consider a simple case, i.e., fixed squeezing in the whole evolution. If squeezing is allowed to vary with, for example, frequency, there are supposed to be better performances. Our study not only demonstrates the efficacy of squeezing in further fidelity enhancement but also provides ideas of combined control. We believe that this combined control strategy will have potential applications in performing quantum information processing tasks.

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AUTHOR DECLARATIONS

Conflict of Interest

The authors have no conflicts to disclose.

Author Contributions

Yang-Yang Xie: Data curation (lead); Formal analysis (lead); Writing – original draft (equal); Writing – review & editing (equal). **Zhao-Ming Wang**: Conceptualization (equal); Funding acquisition (equal); Supervision (equal); Writing – original draft (equal); Writing – review & editing (equal).

DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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