15th International Particle Accelerator Conference,Nashville, TN JACoW Publishing
ISBN: 978-3-95450-247-9 ISSN: 2673-5490 doi: 10.18429/JACoW-IPAC2024-WEPG44

USING CT ALGORITHM TO RECONSTRUCT ELECTRON BEAMS TRANSVERSE PHASE SPACE IN HUST-UED *

Y. Xu, H. Wang, J. Wang, C. Tsai, Z. Liu, K. Fan†, School of Electric and Electronic Engineering, Huazhong University of Science and Technology, Wuhan, China
J. Yang, Osaka University, Osaka, Japan

Abstract

Accurate beam emittance and transverse phase space measurement are crucial for obtaining high-quality sample information in Ultrafast Electron Diffraction (UED). Traditional methods rely on general initial assumptions about the electron beam's phase space and lack specific distributions. The transverse phase space reconstruction technique based on the Computed Tomography (CT) algorithm eliminates the need for prior assumptions, resulting in more precise measurements. In this paper, we utilize an Algebraic Reconstruction Technique (ART) algorithm for HUST-UED, enabling the reconstruction of the beam transverse phase space distribution at the sample location and further facilitating system optimization.

INTRODUCTION

Ultrafast Electron Diffraction (UED) technology has emerged as a pivotal research tool for investigating atomic-scale ultrafast dynamics, undergoing significant developments over recent decades [1-3]. Employing high-brightness electron beams as probes requires low emittance and short bunch lengths to preserve the temporal and spatial resolution of sample information. However, the propagation of electron beams incurs nonlinear changes, including space charge effects, leading to increased emittance. Numerous studies focus on reducing electron beam emittance, employing techniques such as laser longitudinal shaping [4-5] and optimizing electron gun acceleration [6]. A comprehensive understanding of the specific distribution of electron beams in phase space is imperative for advancing such research endeavors.

Traditional methods for measuring electron beam emittance typically rely on general assumptions about the electron beam's phase space, often assuming Gaussian or elliptical distributions. However, these assumptions often fail to describe the actual phase space of the electron beam accurately. To overcome this challenge, the transverse phase space reconstruction technique based on the Computed Tomography (CT) algorithm has garnered significant attention in recent years [7]. Originally developed as a classical image reconstruction method for medical imaging purposes to create three-dimensional representations of human anatomy, the CT algorithm operates by analyzing X-ray absorption within a medium to ascertain its internal density distribution. Subsequently, it utilizes inverse projection algorithms to back-project the projection data from various directions onto a two-dimensional plane, ultimately reconstructing a three-dimensional image. The CT algorithm does not rely on prior assumptions about the electron beam's phase space and has been found to be applicable in many acceleration areas.

This paper investigates the application of the Algebraic Reconstruction Technique (ART) algorithm for CT imaging on the HUST-UED system. The objective is to accurately reconstruct the transverse phase space distribution of the electron beam at the sample position and further promote system optimization. Two methods, multi-slit interpolation and CT were employed to reconstruct the electron beam's phase space at the sample position, emphasizing the distinct advantages of the CT technique. This research contributes to a deeper comprehension of electron beam behavior in UED experiments, which is essential for exploring emittance growth mechanisms further.

* Work supported by NSFC 12235005 and National Key Research and Development Program of China 2022YFA1602202
†kjfan@hust.edu.cn

BASIC PRINCIPLES

Computed Tomography

CT involves gathering projection data of a section at different angles and utilizing appropriate algorithms to reconstruct the image of the section. Its mathematical foundation is the Radon transform, which essentially states that a two-dimensional or three-dimensional object can be fully described by its infinite projections.

Reconstruction algorithms form the cornerstone of CT technology. These algorithms are mainly categorized as analytical reconstruction algorithms and iterative reconstruction algorithms. Analytical reconstruction algorithms are based on the Radon transform, with the filtered back projection (FBP) algorithm being a typical representative [8]. The analytical algorithms offer rapid reconstruction but are susceptible to noise and require high data completeness. In cases of limited projection data, serious artifacts may appear in the reconstructed images.

Iterative reconstruction algorithms formulate a set of algebraic equations for unknown vectors based on projection data and subsequently solve these unknown vectors. The ART and Maximum Entropy Technique (MENT) are examples of iterative algorithms [9-10]. Iterative methods excel in noise resistance and demand lower data completeness. However, they are time-consuming.

The greatest challenge in utilizing CT technology for reconstructing electron beam phase space is the complexity of obtaining projection data over a large angular range. Therefore, iterative algorithms like ART have advantages.

ISBN: 978-3-95450-247-9 ISSN: 2673-5490 doi: 10.18429/JACoW-IPAC2024-WEPG44

Phase Space Reconstruction

Based on CT technology principles, reconstructing a specific image requires knowledge of the image's projection values at various angles. In accelerator systems, we cannot directly observe the phase space of an electron beam at a certain location. However, beam transport theory provides methods to reconstruct the beam's phase space. When an electron beam passes through quadrupole magnets, drift sections, or other transport elements, its phase space undergoes rotations, stretching, and shearing. The beam profile image can be obtained by placing a detector downstream of the transport elements. Adjusting the parameters of the transport elements allows us to obtain phase space distribution at different rotation angles. Then, utilizing the transfer matrix facilitates obtaining projection values at the reconstruction point, enabling the reconstruction of the phase space distribution through an appropriate algorithm.

The specific principles are as follows: Suppose at position z where the transverse phase space distribution of the electron beam is represented by $f(x, x')$, and the spatial distribution in real space is denoted as $h(x, y)$. If the electron beam's transverse phase space and real space are projected onto the x-axis, the distribution in the x-direction remains consistent. This projection corresponds to the Radon projection value of the electron beam phase space at a certain rotation angle.

The rotation angle of the electron beam is determined by the beam transport line, which typically consists of quadrupole magnets and drift sections. Assuming the phase space distribution at position z_0 is denoted as $f(x_0, x_0, z_0)$, and the transfer matrix from z_0 to z is denoted as R, according to the principles of beam transport,

$$\begin{pmatrix} x \\ x' \end{pmatrix} = \begin{pmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{pmatrix} \begin{pmatrix} x_0 \\ x_0' \end{pmatrix} \tag{1}$$

The rotation angle is given by $\arctan(R_{12}/R_{11})$. According to Liouville's theorem, the projection of the transformed phase space onto the x-direction is:

$$P = \sqrt{R_{11}^2 + R_{12}^2}\, P\left(x / \sqrt{R_{11}^2 + R_{12}^2}; z\right) \tag{2}$$

By altering the parameters of the beam transfer matrix (such as quadrupole magnet strength, drift section length, etc.), the corresponding transverse real space of the beam bunch can be measured, thus obtaining the transverse phase space projection at the corresponding rotation angle.

PARTICLE TRACKING SIMULATION RESULTS

Interpolation Method

During the transmission of high-brightness electron beams, space charge effects significantly influence the transverse emittance. Conventional methods for measuring emittance, such as the three-screen method or quadrupole scan method, often produce exaggerated results due to linear optical approaches. Typically, emittance measurements are conducted using the multi-slit, pepper-pot, or TEM grid method. To address this, a device based on the multi-slit method was developed by HUST-UED, evaluating the electron beam emittance at a specific location and roughly reconstructing its transverse phase space through interpolation techniques.

In the HUST-UED, an electron beam serving as a probe is precisely focused at the sample location to enhance the spatial resolution of electron diffraction. At the sample location, a multi-slit slices the electron beam into multiple sub-bunches, which then traverse drift sections before reaching a fluorescent screen. For a focused beam, the multi-slit method has a higher electron transmission efficiency than the pepper-pot method and a lower probability of sub-bunch overlap than the TEM grid method, making it more suitable for emittance measurement. The distribution of the electron beams just passing through the slit and on the detector obtained from ASTRA simulations is shown in Fig. 1. Through projection integration, essential parameters such as momentum and divergence angles of each sub-bunch can be calculated. The calculated emittance is 0.383 mm·mrad, closely aligning with the result of 0.371 mm·mrad obtained from beam tracking.

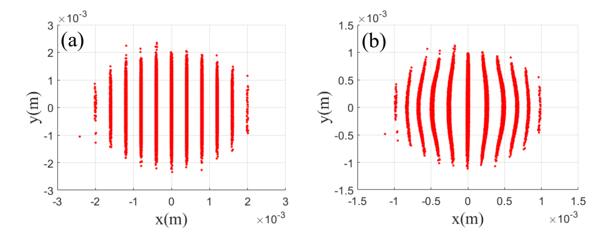

Figure 1: The electron distribution just after passing through the multi-slit (a) and on the detector (b).

The coordinates of the sub-bunches and the RMS divergence angle have been measured, and the transverse phase space distribution of the electron beam can be appropriately extrapolated through interpolation methods, as shown in Fig. 2.

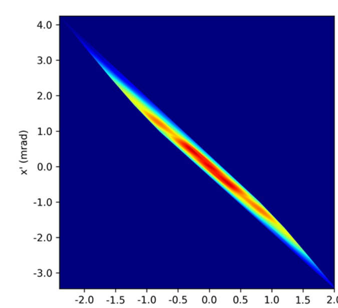

Figure 2: The transverse phase space of the electron beam reconstructed by interpolation method.

CT Method Based on ART Algorithm

The multi-slit method can accurately measure the emittance of high-charge electron beams. However, due to the limited effective information, it is challenging to fully reconstruct their transverse phase space distribution solely through interpolation methods. The CT imaging technique based on algorithms can better restore the details of the

ISBN: 978-3-95450-247-9 ISSN: 2673-5490 doi: 10.18429/JACoW-IPAC2024-WEPG44

phase space, which is of great significance for further investigating the mechanisms of emittance growth.

According to CT technology principles, reconstructing the electron beam phase space requires obtaining the projection values of the electron beam at different angles. To facilitate this process, a set of quadrupole magnets are integrated after the sample location.

By adjusting the strength of these quadrupole magnets, the electron beam phase space is rotated, yielding diverse electron beam projection distributions on the detector. Notably, this distribution represents the real space of the electron beam, i.e., the (x, y) distribution. Additionally, the beam distribution needs to be projected onto the x-direction, equivalent to projecting the phase space (x, x') onto the x-direction, as shown in Fig. 3.

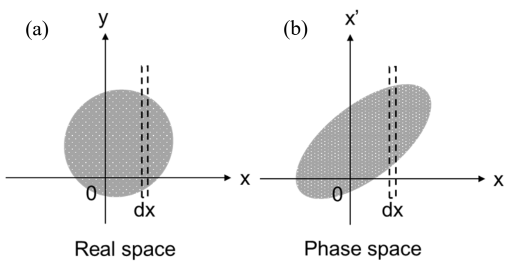

Figure 3: The relationship between electron beam real space (a) and phase space (b).

Due to parameter limitations, it is unfeasible to fully reconstruct the phase space at all angles. This necessitates adjusting the quadrupole magnet strength. According to the HUST-UED beamline design, the gradient of the quadrupole magnet ranges from 0.0025 to 0.173 T/m, corresponding to angles from 22° to 60°, and 39 sets of projection data collected with a spacing angle of 1°. For incomplete data reconstruction, the FBP algorithm is unsuitable, while the ART algorithm is more appropriate. The ART algorithm iteratively solves algebraic equations to approximate the true image gradually, thus demonstrating robustness and convergence in reconstructing incomplete data.

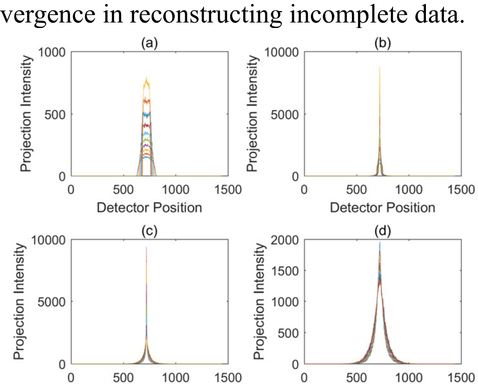

Figure 4: The 39 projections of the electron beam transverse distribution along the x-axis, where (a) corresponds to 22°-31°, (b) 32°-41°, (c) 42°-51°, (d) 52°-60°.

The ART algorithm was originally used to solve the system of equations. For image reconstruction, it is equivalent to solving the equation $\boldsymbol{b} = \boldsymbol{A}\boldsymbol{x}$, where $\boldsymbol{x}=[x_1, x_2 \ldots x_N]^T$ represents the image to be estimated, $\boldsymbol{b}=[b_1, b_2 \ldots b_M]^T$ represents the projection data, and $\boldsymbol{A}$ is the system matrix with dimensions $M\times N$. When both the matrix $\boldsymbol{A}$ and vector $\boldsymbol{b}$ are known, the Kaczmarz iteration algorithm can be used to compute the vector $\boldsymbol{x}$:

$$\mathbf{x}^{k+1} = \mathbf{x}^k + \lambda_k \frac{b_i - \langle \mathbf{a}_i, \mathbf{x}^k \rangle}{|\mathbf{a}_i|^2} \mathbf{a}_i^T \quad (3)$$

where λ is the relaxation factor; k is the number of iterations; i represents the i-th ray; $\boldsymbol{x}^k$ denotes the grayscale value of the pixel; b_i represents the projection data of the i-th ray; $\boldsymbol{a}_i$ denotes the length at which the i-th ray intersects the pixel.

Figure 4 shows the projection curves of the electron beam profile in the x-direction under different gradients of the quadrupole magnet. Fig. 5 compares the reconstructed results using the ART algorithm with the electron beam phase space obtained from beam tracking. Notably, even with reconstruction angles less than 40, satisfactory reconstruction results can still be achieved. Compared to the interpolation method used with the multi-slit method, the ART algorithm better restores the details of the phase space distribution.

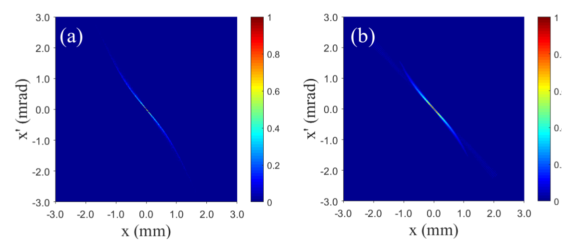

Figure 5: The transverse phase space distribution of the electron beam: Astra simulation results (a) and the distribution reconstructed by CT (b).

CONCLUSION

This study investigated the method of reconstructing electron beam transverse phase space distribution based on the CT algorithm. Unlike traditional methods, this approach eliminates the need for assumptions regarding the initial electron beam phase space distribution, thereby enhancing accuracy. It excels in capturing intricate phase space details and lays a solid foundation for subsequent emittance compensation efforts. Considering the electron beam properties in the HUST-UED system and aligning them with beamline design, we chose appropriate CT reconstruction angles. This led to notably superior reconstruction results compared to the multi-slit interpolation method.

ACKNOWLEDGMENTS

This work is supported by the National Natural Science Foundation of China (NSFC 12235005) and the National Key Research and Development Program of China (2022YFA1602202).

ISBN: 978-3-95450-247-9 ISSN: 2673-5490 doi: 10.18429/JACoW-IPAC2024-WEPG44

REFERENCES

[1] D. Filippeto *et al.*, "Ultrafast electron diffraction: Visualizing dynamic states of matter", *Reviews of modern physics,* vol. 94, Oct. 2022. doi:10.1103/RevModPhys.94.045004

[2] J. C. Williamson *et al.*, "Ultrafast electron diffraction. Velocity mismatch and temporal resolution in crossed-beam experiments", *Chem. Phys. Lett.*, vol. 209, p. 10-16, 1993. doi:10.1063/0009-2614(93)87193-7

[3] A. Odate *et al.*, "Brighter, faster, stronger: ultrafast scattering of free molecules", *Advances in Physics: X,* vol. 8, No. 1, 2126796, 2023. doi:10.1080/23746149.2022.2126796

[4] J. Maxson *et al.*, "Adaptive electron beam shaping using a photoemission gun and spatial light modulator", *Phys. Rev. Accel. Beams*, vol. 18, p. 023401, 2015. doi:10.1103/PhysRevSTAB.18.023401

[5] S. Li *et al.*, "Ultraviolet laser transverse profile shaping for improving x-ray free electron laser performance", *Phys. Rev. Accel. Beams*, vol. 20, p. 080704, 2018. doi:10.1103/PhysRevAccelBeams.20.080704

[6] P. Zhu *et al.*, "Femtosecond time-resolved MeV electron diffraction", *New Journal of Physics*, vol. 17, p. 063004, 2015. doi: 10.1088/1367-2630/17/6/063004

[7] B. Hermann *et al.*, "Electron beam transverse phase space tomography using nanofabricated wire scanners with submicrometer resolution", *Phys. Rev. Accel. Beams*, vol. 24, p. 022802, 2021. doi:10.1103/PhysRevAccelBeams.24.022802

[8] D. Stratakis *et al.*, "Tomography as a diagnostic tool for phase space mapping of intense particle beams", *Phys. Rev. Accel. Beams*, vol. 9, p. 112801, 2006. doi:10.1103/PhysRevAccelBeams.9.112801

[9] A. H. Andersen *et al.*, "Simultaneous algebraic reconstruction technique (SART): a superior implementation of the ART algorithm", *Ultrasonic Imaging*, vol. 6, p. 81-94, 2007. doi:10.1177/016173468400600107

[10] K. M. Hock, *et al.*, "A study of the maximum entropy technique for phase space tomography", *Journal of Instrumentation*, vol. 8, p. p02003. doi:10.1088/1748-0221/8/02/P02003