State of the Art of the Theoretical Nuclear Physics

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Abstract. In this contribution, I summarizes the theoretical topics discussed at SOTANCP5, including various cluster states, clusters at the edges of the nuclear chart and in nuclear matter, and clustering aspects in reactions.

1 Introduction

This series of workshops was initiated in Strasbourg in 2008, followed by meetings in Brussels (2010), Yokohama (2014), and Galveston (2018). This workshop in Hvar continued the discussion on major topics of nuclear clusters, including clusters in stable and unstable nuclei, alpha condensates, clustering aspects in nuclear reactions and resonances, ab initio approaches in cluster physics, nuclear fission, superheavy elements, and cluster decay.

Here, I provides an overview of such discussions, reorganizing them to three sections; the various cluster structures, clusters at the edges of the nuclear chart and in nuclear matter, and clustering aspects in reactions.

2 Variety of Clusters

2.1 Hoyle state and its relatives

The Hoyle state was one of the central topics at SOTANCP. Since the prediction by F. Hoyle in 1954 [1], the Hoyle states has garnered interest in its structure. In 1955, Morinaga made an intriguing proposal [2] that the Hoyle state is a linear chain state of α clusters. Later, this idea was disproven as it could not explain the short lifetime of the Hoyle state [3]. Instead, the cluster model calculations [4–6] showed that the Hoyle state is a weakly bound state of three α clusters dominated by s-wave. Interestingly, at this point, people had already noticed the uniqueness of the Hoyle state. For example, Uegaki [5] stated the following: "In other words, the 0_2^+ state is the lowest state which belongs to the *new phase* and could be considered to be a finite system of α-boson gas."

In the 2000s, this idea was further advanced, and the Tohsaki-Horiuchi-Schuck-Röpke wave function [7], which asserts that the Hoyle state is a Bose-Einstein condensate of α particles (α condensation), achieved significant success. SOTANCP has played a crucial role in the study of alpha condensation. Today, many people discuss the existence of condensate of 4, 5 and more α particles [8–11]. The systems composed of α particles and excess neutrons (Boson-Fermion condensate) are also of interest [12].

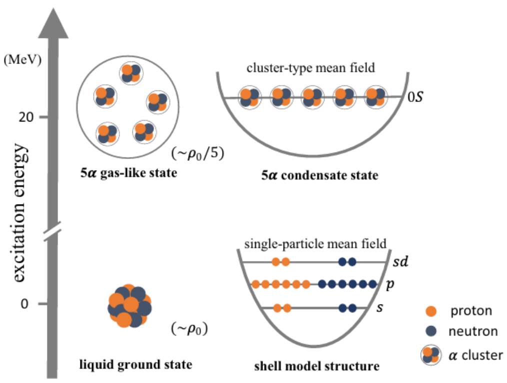

Figure 1. The snapshots of the intrinsic density for the ground and Hoyle state calculated by MCSM THis figure is taken from Ref. [11].

I must also address the remarkable progress in ab-initio calculations for the Hoyle state. In the 2000s, the No-Core Shell Model cannot describe the Hoyle state [13]. However, with advancements in computational power, this is becoming feasible [14]. The ab-initio methods, including GFMC [15] and Lattice EFT [16, 17], may explain why α condensate is formed based on nuclear forces. This represents a significant research direction in the next decade.

2.2 Algebraic models

Another important stream of the cluster study is provided by algebraic cluster models [18–23]. They assume the existence of clusters and impose some dynamical symmetries on the Hamiltonian to discuss various cluster systems including the Hoyle state. Let me examine the study by Bijker and Iachello [18, 19]. By assuming an equilateral triangular symmetry (D_{3h}) for the 3α system, they explain the excitation spectrum of ^{12}C, including the Hoyle state. Although this is a rather strong assumption, the results seem to be quite successful. However, we should answer the questions: Is this interpretation of the Hoyle state is

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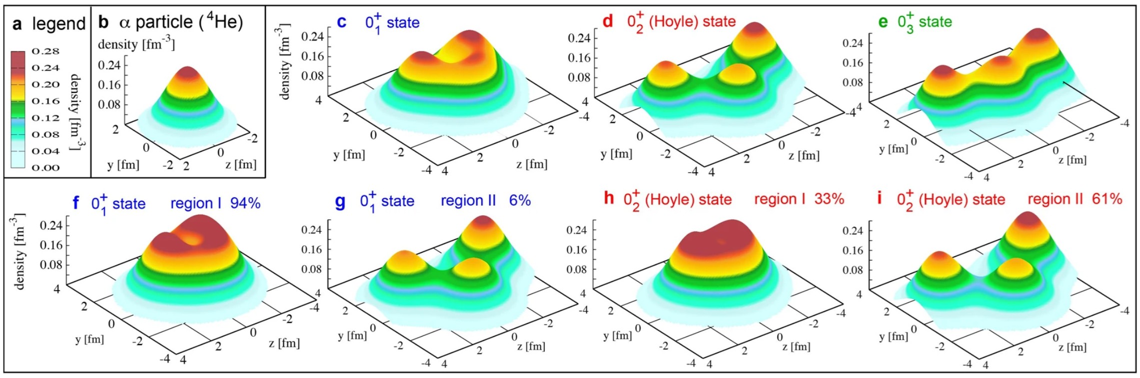

Figure 2. Illustration of the 5α condensate in the excited state of ^{20}Ne. THis figure is taken from Ref. [14].

consistent with the BEC scenario?; and How does the dynamical symmetry arise? Only the theories that does not introduce ad-hoc assumption on the cluster formations and symmetry can answer this question.

2.3 Revival of the linear chain

As mentioned above, the idea of a linear chain of alpha clusters was once abandoned but has recently been revived in the study of neutron-rich nuclei. In the study of unstable nuclei such as Be isotopes, it was discovered that excess neutrons act as a "glue" to stabilize clusters [24, 25]. Itagaki et al. [26] exploited this idea and investigated if excess neutrons stabilize linear chains in carbon isotopes. This early attempt has now evolved into more quantitative discussions. Theoretical calculations [27–30] predicted the energy and decay patterns of linear chain candidates. Based on comparisons with experiments [31–35], linear chain states are being identified.

In the next decade, we should uncover the impact of the large angular momentum on the linear chains. For instance, Afanasjev et al. [36, 37] suggested the emergence of the irrotational flows.

2.4 Clusters beyond drip line

Experimental techniques using RI-beam have advanced further, allowing us the discussion of clusters beyond the dripline. In the domain of near or beyond the dripline, nucleon correlations overwhelm the mean-field effects. Consequently, the multi-nucleon correlations (clusters in a broad sense) become clearly evident. One of the recent highlights is the discovery of the candidate of resonance in the $4n$ system [38, 39] and the resonance in the $^{24}\mathrm{O} + 4n$ (^{28}O) system [40].

The theoretical studies for these systems remain controversial. Some support the existence of athe $4n$ resonance [41–43], while others do not [44–46]. The calculations for the ground state energy and width of ^{28}O have not converged [47–50]. They seem to depend on both the employed nuclear fores and the methods of many-body calculations. Thus, the clusters beyond the drip line serves as a touchstone for our understanding of nuclear forces and multi-nucleon correlations.

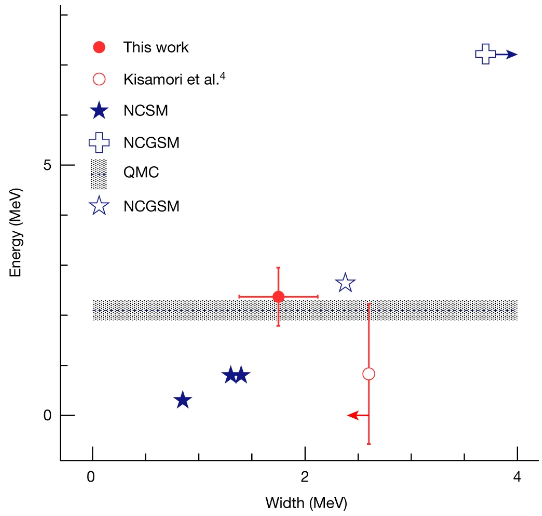

Figure 3. Comparison of the energy and width of the $4n$ resonance. This figure is taken from Ref. [39].

Table 1. Short summary of the theoretical studies for the $4n$ resonance.

model	NN int.	$4n$ resonance (MeV)
QMC [41]	χEFT (NNLO)	$E_r \simeq 2.1$
GSM [42]	χEFT (N^{3}LO)	$E_r \simeq 7$, $\Gamma \simeq 3.5$
NCSM [43]	JISP16	$E_r = 0.8$, $\Gamma = 1.4$
FY [44]	χEFT (NLO)	False
FY [46]	AV8	False
HH [45]	AV8, AV18	False

Another intriguing domain is the proton drip line where the Coulomb force competes with the nuclear forces. Furthermore, the Coulomb barrier allows unbound correlated protons to have a finite lifetime, which is sufficiently long to be examined experimentally. A typical example of this is the $2p$-decay, where two correlated protons are emitted simultaneously [51]. The time-dependent models [52, 53] discuss the tunneling decay of correlated $2p$ cluster. Note that the possible $4p$-decay has also been reported experimentally [54]. Understanding the time-evolution of such proton clusters is an important challenge. Creating a map of $2p$ and $4p$ decays in the nuclear chart

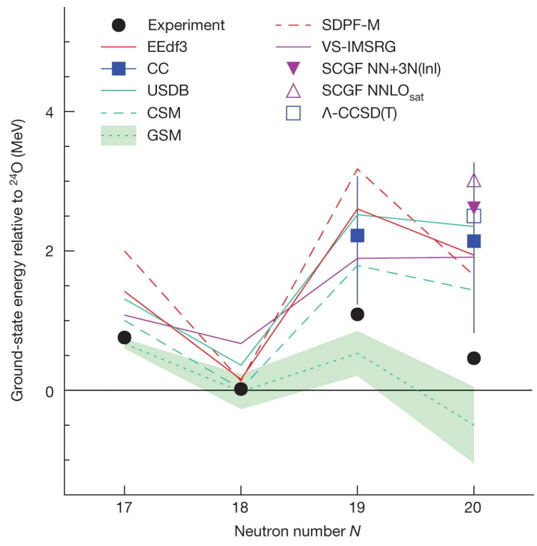

Figure 4. Comparison of the energy and width of ^{28}O. Tis figure is taken from Ref. [40].

is also a crucial task for theoretical research. Additionally, I also remark that comparing neutron systems with proton systems like $4n$ v.s. $4p$ is quite interesting.

3 Clusters formation and decay in finite nuclei and infinite nuclear matter

3.1 Clusters formation and decay in finite nuclei

One of the exciting developments in cluster research is the observation of clusters on the nuclear surface through α knockout reactions. Tanaka et al. [55] showed that the cross-sections for the α known-out from Sn isotopes are reduced as function of the neutron excess. This experiment proposed a new method for observing clusters on the nuclear surface and suggests that the formation of clusters strongly depends on density and the proton/neutron ratio. These are strong motivation for theoretically investigating cluster formation on the surfaces of various stable and unstable nuclei [57–59].

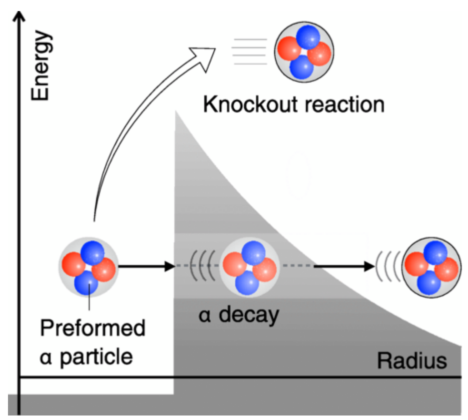

Figure 5. Illustration for the α knockout reaction from a finite nucleus. This figure is taken from Ref. [56].

For example, a recent study [59] based on DFT calculation showed that the α formation probability increases by the pairing correlation, and reduced by the neutron excess, which potentially explains the experimental trend. However, the calculated α formation probability is much smaller than that deduced from the observed cross-sections. The significant underestimation is also reported by an AMD calculation [57]. Thus, the current nuclear models seem unable to describe cluster formation due to fluctuations beyond the mean field. Solving this problem will be an important task in understanding nuclear many-body systems.

Research on cluster formation at the nuclear surface naturally leads to studies of alpha and cluster decays. In addition to the studies based on the potential model, there have been significant progress in the studies based on mean-field models [60–63]. For example, in Ref. [61], the α decay path was calculated and the decay probability was estimated. Similar approaches have also been applied to the exotic cluster decays such as ^{14}C decay and 2α decay [63]. Although the microscopic model approaches are currently less quantitative, they are expected to make significant progress in the next decade. Beyond this, a complete microscopic understanding of nuclear fission and fusion awaits us [64]. These are the most challenging issue in the next era.

3.2 Clusters in nuclear matter

The α-knockout experiment provides an opportunity to investigate cluster formation in nuclear matter. In fact, in Ref. [67], the α cluster formation on the nuclear surface was investigated by combining the binding energy of alpha particles in nuclear matter [65, 66] with DFT calculations of finite nuclei. These results explain the measured cross-sections to some extent. However, calculating cluster formation in nuclear matter is not straightforward. Many approximations are introduced to evaluate the many-body Green's function, and the nuclear forces are also simplified. Novel approaches have been proposed to solve these problems [68], and future developments are expected. Additionally, cluster formation affects the equation of state of nuclear matter, which has implications for related fields such as supernova explosions [69].

4 Clusters and nuclear reactions

As suggested by the Ikeda diagram, the entrance and exit channels of reactions are seamlessly connected with the clusters. In other words, when we consider clusters, it is necessary to treat structure and reaction problems simultaneously. Cluster knockout and transfer reactions are typical and traditional examples of this and have been discussed by many authors [70–72].

In addition to them, there have been attempts to access the scattering problems with full-microscopic and ab-initio theories. For example, in Ref. [73], the asymptotic normalization constants for the $\alpha+^{12}$C and $\alpha+^{16}$O systems from the the ab-initio shell model calculations. These

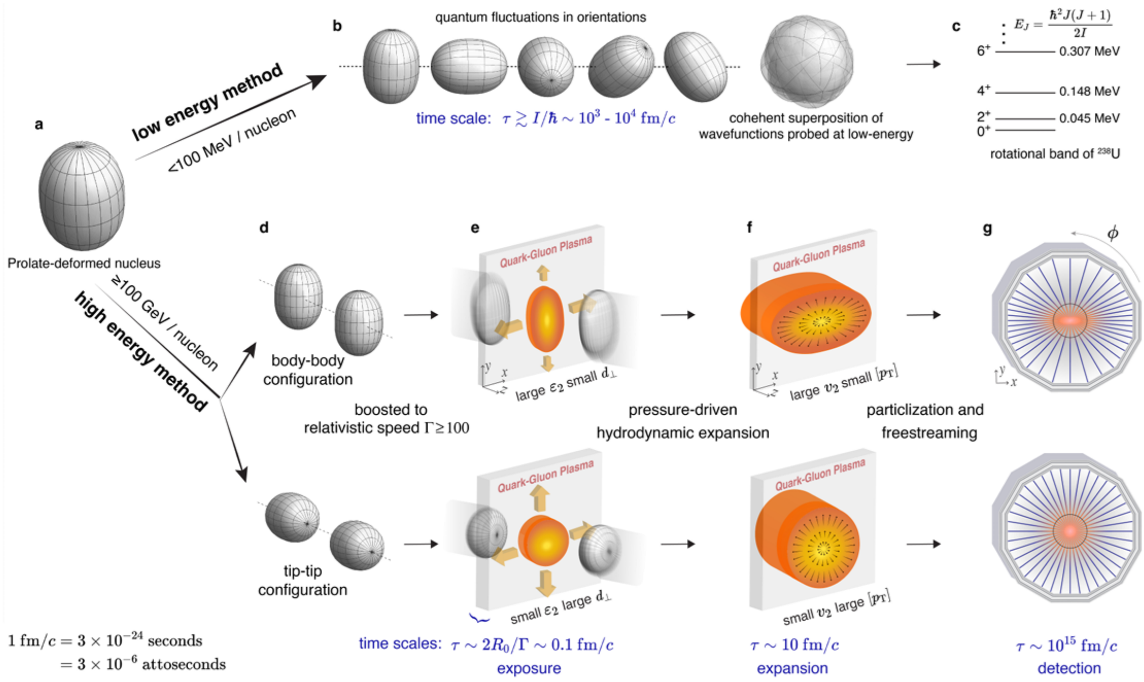

Figure 6. Illustration for heavy-ion collision. This figure is taken from Ref. [76].

are key to understanding radiative capture reactions in the stellar environment. Furthermore, more complex fusion reactions with rearrangement of many nucleons are now being described microscopically. And, the $^{12}C + ^{12}C$ reaction rates at low-temperatures were derived [74, 75]. The synergy between reaction calculations using these microscopic cluster models and recent advances in experimental research will be significant.

Finally, I would like to comment on a new trend that relates reactions and clusters. A new trend is emerging in high-energy heavy-ion collisions, such as those at RHIC and LHC, where nuclear clusters are being observed. In these experiments, the hadron flow generated by the collisions is observed, and the anisotropy and composition of the flow depend on the fireball created by the collision, and ultimately on the shape and momentum distribution of the nuclei before the collision.

In recent years, the reliability of transport models has improved, allowing for discussions on the shape of nuclei before collisions based on event-by-event analysis. For example, significant differences in elliptic flow have been found between highly deformed U+U collisions and less deformed Ag+Ag collisions [76]. Building on this idea, discussions have begun on detecting clusters in nuclei before collisions from the flow of heavy-ion collisions. For example, Ref. [77] proposes detecting parity-asymmetric clusters in Ne by comparing the flow of O+O collisions with Ne+Ne collisions. This is a new direction in the study of nuclear clusters to which the community should pay attention, I believe.

5 Summary

I have overview the theoretical topics discussed at SOTANCP5. Despite the disruption by COVID-19, theoretical research has steadily progressed. Traditional cluster studies, such as α-condensation, clusters in neutron-rich nuclei, and cluster transfer reactions, have continued to advance. Significant progress was also reported for the ab-initio calculations for clusters and full-microscopic studies of α and cluster decays paving the way for a microscopic understanding of nuclear fusion and fission. Additionally, new probes for clusters, such as α-knockout reactions and high-energy heavy-ion collisions, have been proposed. These developments are expected to stimulate theoretical research and bring about new advancements.

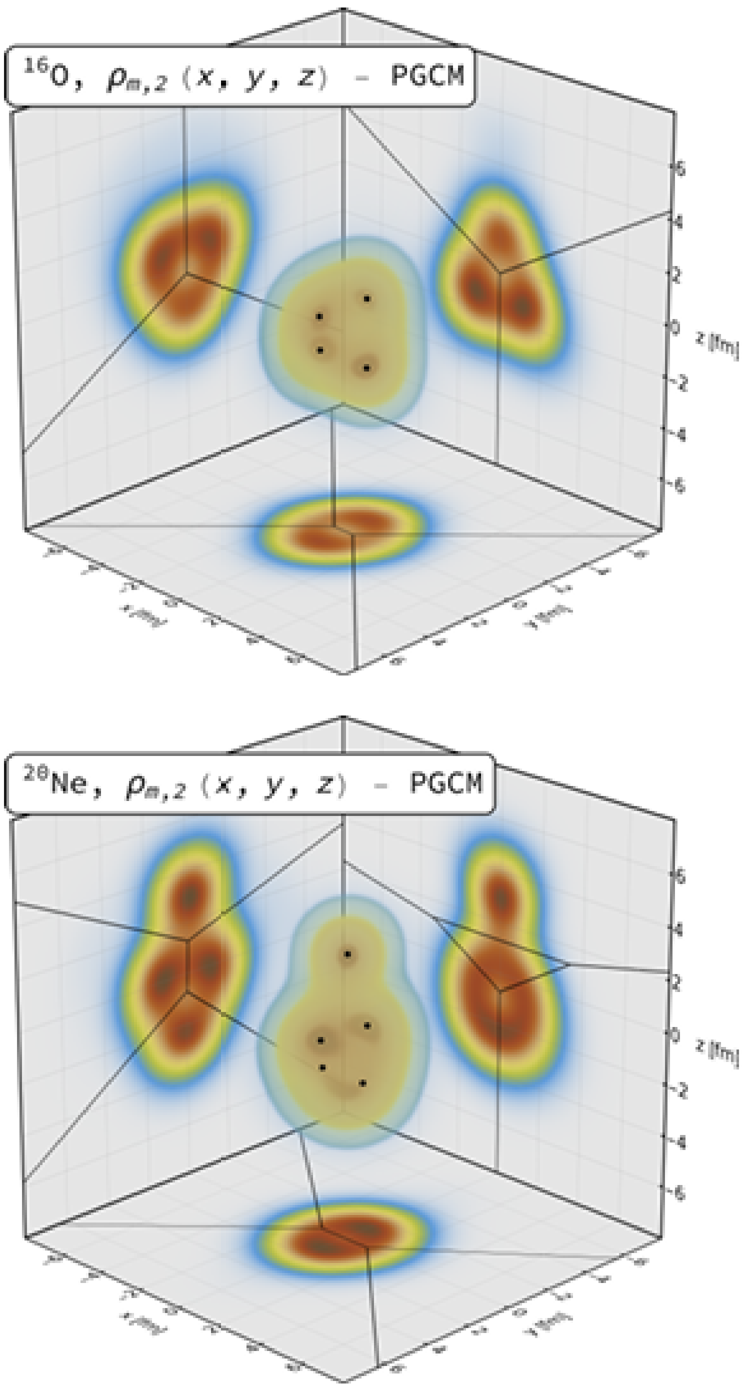

Figure 7. The clustered densities of ^{16}O and ^{20}Ne. This figure is taken from Ref. [41].

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