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Pseudo-Conformal Field Theory

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Abstract. Starting from the observation that the 2-d Polyakov action is metric independent without being topological, while the 4-d Weyl-conformal action is not metric independent, I searched for a 4-d structure and field equation which would be metric independent. I found that the structure is the existence of a geodetic and shear-free Newman-Penrose (NP) null tetrad, which is a Frobenius integrable structure, called lorentzian Cauchy-Riemann (LCR) structure. The metric independent equation is a gauge field equation identified with the gluon field. Imposing the LCR-structure as the fundamental structure of nature (instead of the Einstein-metric) all the interactions (gravity, electroweak and gluonic) are derived. The three generations of the leptons are solitonic configurations with precise topological invariants. The quarks are confined sources of the gluonic field implied by the LCR-tetrad of the corresponding leptons. Quantum field theory emerges through the Bogoliubov causal approach and the standard model is derived through the Scharf procedure of the Kugo-Ojima BRS elimination technique of the unphysical modes of the spin-1 and spin-2 fields.

1 Introduction

The standard model (SM) actually provides an experimentally well established description of elementary particles, except the graviton. The discovery of the Higgs particle made the SM self-consistent and hence well defined in the context of quantum field theory (QFT). But supersymmetric particles have not been observed. Hence string theory has to be abandoned, and elementary particle physics has to look for other unifying quantum models, which do not need supersymmetry to incorporate SM and gravity. The present work provides such an alternative by simply replacing Einstein's metric with a special totally real (Cauchy-Riemann) CR-structure in the tangent space of a differential manifold (the spacetime). The first investigators (E. Cartan, Tanaka, Severi etc) of CR-structures used the term "pseudo-conformal transformations"[3], therefore the present model is called pseudo-conformal field theory (PCFT) and its fundamental CR-structure is called lorentzian Cauchy-Riemann (LCR) structure. In fact it generalizes the metric independence of the 2-dimensional Polyakov action into a four dimensional highly symmetric model which derives all the interactions and the fermionic particles of the SM as solitonic configurations. This 4-dimensional model started many years ago when I realized that the conventional 4-dimensional Weyl-conformal action is not metric independent to be the analogue of the Polyakov action. Hence I searched and found the 4-dimensional LCR structure[7], which essentially coincides with the Newman-Penrose (NP) null tetrad $(\ell, n, m, \overline{m})$[5] with vanishing NP spin-coefficients $\kappa = \sigma = 0 = \lambda = \nu$.

The four dimensional spacetime is assumed to be a differential manifold with a tangent space and its dual cotangent space. No metric is assumed. The fundamental geometric structure is a Frobenius integrable system defined on the tangent space in the first section. Its equivalent definition in the cotangent space is also given. If the LCR-structure is realizable we find the form of the LCR-structure implicit conditions which determine the spacetime LCR-structures as special totally real submanifolds in an ambient complex manifold. Nothing else is needed to derive in the successive sections the gravitational, electroweak, "Higgs" and gluonic fields. The three generations of leptons and their corresponding quarks are solitonic LCR-structures stabilized by topological invariants[8].

2 The geometric structure

The differential manifold is characterized by a well defined tangent space. The linear functionals on the tangent space define the cotangent space. In a coordinate chart we define the canonical basis ∂_μ in the tangent space at any point p. It is paired with its dual basis dx^ν of the cotangent space with $\partial_\mu \lrcorner dx^\nu = \delta^\nu_\mu$ implied by the linearity of the functionals on a simple vector space. These trivialities make clear that the operation $\lrcorner$ connects the two different tangent and cotangent vector spaces, while an inner product (metric) connects two vectors in the same vector space.

The four dimensional LCR-structure is defined as a frame with two real and one complex vector fields $(\ell^\mu\partial_\mu, m^\mu\partial_\mu;\ n^\mu\partial_\mu, \overline{m}^\mu\partial_\mu)$ on a smooth four dimensional manifold, which satisfy the involutive commutation relations

$$\begin{aligned}
&[(\ell^\mu\partial_\mu)\ ,\ (m^\nu\partial_\nu)] = h^{\widetilde{0}}_{\widetilde{0}}(\ell^\rho\partial_\rho) + h^{\widetilde{1}}_{\widetilde{0}}(m^\rho\partial_\rho) \\
&[(n^\mu\partial_\mu)\ ,\ (\overline{m}^\nu\partial_\nu)] = h^{0}_{0}(n^\rho\partial_\rho) + h^{1}_{0}(\overline{m}^\rho\partial_\rho)
\end{aligned} \tag{2.1}$$

where h^α_β and $h^{\widetilde{\alpha}}_{\widetilde{\beta}}$ are smooth functions and $\overline{m}^\rho$ is the complex conjugate of m^ρ. The involutive relations can be transcribed in the cotangent space with the existence of a coframe $(\ell'_\mu dx^\mu, m'_\mu dx^\mu; n'_\mu dx^\mu, \overline{m'}_\mu dx^\mu)$ of two real and one complex 1-forms determined via the non-vanishing duality relations

$$\begin{aligned}
&(\ell^\mu\partial_\mu)\lrcorner(n'_\nu dx^\nu) = (n^\mu\partial_\mu)\lrcorner(\ell'_\nu dx^\nu) = 1 \\
&(m^\mu\partial_\mu)\lrcorner(\overline{m}'_\nu dx^\nu) = (\overline{m}^\mu\partial_\mu)\lrcorner(m'_\nu dx^\nu) = -1
\end{aligned} \tag{2.2}$$

where the other contractions vanish. Notice that it is the NP tangent and cotangent dual frames, which formulate the NP formalism of general relativity, with the following vanishing spin-coefficients $\kappa = \sigma = 0 = \lambda = \nu$. If we contract the first condition of (2.1) with $(\ell'_\nu dx^\nu)$, $(m'_\nu dx^\nu)$ and the second condition (second line) with $(n'_\nu dx^\nu)$, $(\overline{m}'_\nu dx^\nu)$ we find the relations

$$\begin{aligned}
&(\ell^\mu m^\nu - \ell^\nu m^\mu)(\partial_\mu \ell'_\nu) = 0 \quad , \quad (\ell^\mu m^\nu - \ell^\nu m^\mu)(\partial_\mu m'_\nu) = 0 \\
&(n^\mu \overline{m}^\nu - n^\nu \overline{m}^\mu)(\partial_\mu n'_\nu) = 0 \quad , \quad (n^\mu \overline{m}^\nu - n^\nu \overline{m}^\mu)(\partial_\mu \overline{m}'_\nu) = 0
\end{aligned} \tag{2.3}$$

which are apparently metric-connection independent. In the present case, where there is no metric to relate the upper with the lower indices, it is more convenient to use the following equivalent relation in the closed formalism of the differential forms

$$\begin{aligned}
&d\ell' = Z_1 \wedge \ell' + i\Phi_1 m' \wedge \overline{m'} \\
&dm' = Z_3 \wedge m' + \Phi_3 \ell' \wedge n' \\
&dn' = Z_2 \wedge n' + i\Phi_2 m' \wedge \overline{m'} \\
&d\overline{m'} = \overline{Z_3} \wedge \overline{m'} + \overline{\Phi_3} \ell' \wedge n'
\end{aligned} \tag{2.4}$$

where Z_1, Z_2 are real 1-forms, Z_3 a complex 1-form, Φ_1, Φ_2 two real scalars and Φ_3 a complex scalar. A proof for the first two relations is found by performing the successive contractions $(\ell^\mu\partial_\mu)\lrcorner((m^\mu\partial_\mu)\lrcorner d\ell')$ and $(\ell^\mu\partial_\mu)\lrcorner((m^\mu\partial_\mu)\lrcorner dm')$. The other two relations are found by taking the $(n^\mu\partial_\mu)\lrcorner((\overline{m}^\mu\partial_\mu)\lrcorner dn')$ and $(n^\mu\partial_\mu)\lrcorner((m^\mu\partial_\mu)\lrcorner d\overline{m}')$ contractions. The distinction of the dual frames in the tangent and cotangent spaces of a manifold is simple therefore I will remove the accents on the 1-forms.

In general relativity the symmetry of a frame compatible with the metric is the local $SO(1,3)$ group[5].

In the present case of the LCR-structure fundamental form, the symmetry is

$$\begin{aligned}
&\ell'_\mu = \Lambda \ell_\mu \quad , \quad \ell'^\mu = \tfrac{1}{N}\ell^\mu \\
&n'_\mu = N n_\mu \quad , \quad n'^\mu = \tfrac{1}{\Lambda} n^\mu \\
&m'_\mu = M m_\mu \quad , \quad m'^\mu = \tfrac{1}{\overline{M}} m^\mu \\
&\Lambda \neq 0 \ , \ N \neq 0 \ , \ M \neq 0
\end{aligned} \tag{2.5}$$

which I call tetrad-Weyl transformation. The auxiliary fields transform as follows

$$\begin{aligned}
&Z'_{1\mu} = Z_{1\mu} + \partial_\mu \ln \Lambda \ , \quad Z'_{2\mu} = Z_{2\mu} + \partial_\mu \ln N \ , \quad Z'_{3\mu} = Z_{3\mu} + \partial_\mu \ln M \\
&\Phi'_1 = \tfrac{\Lambda}{M\overline{M}} \Phi_1 \quad , \quad \Phi'_2 = \tfrac{N}{M\overline{M}} \Phi_2 \quad , \quad \Phi'_3 = \tfrac{M}{\Lambda N} \Phi_3
\end{aligned} \tag{2.6}$$

Notice that the 1-forms Z_1, Z_2 and Z_3 transform as abelian gauge potentials, while the scalar fields Φ_1, Φ_2 and Φ_3 transform multiplicatively. That is, after a tetrad-Weyl transformation these scalar fields may locally take the values either zero or one, because the tetrad-Weyl symmetry cannot change these discrete values to each other. Hence they act as topological invariants, which may stabilize solitonic configurations.

The LCR-structure conditions (2.1) or (2.4) are simply the necessary hypothesis to apply the Frobenius theorem. But the involutive pairs are not both real vector fields. Therefore we cannot apply the real Frobenius theorem. We must first complexify the variables x^μ to $r^\mu = x^\mu + iy^\mu$ and after we can use the holomorphic version of the Frobenius theorem. The application of this theorem implies the existence of a generally complex coordinate system $(z^\alpha, z^{\widetilde{\beta}}) \ : \ \alpha, \beta = 0, 1$, called structure coordinates, such that

$$\begin{aligned}
&dz^\alpha = f_0^\alpha \ \underline{\ell}_\mu dr^\mu + f_1^\alpha \ \underline{m}_\mu dr^\mu \quad , \quad dz^{\widetilde{\alpha}} = f_{\widetilde{0}}^{\widetilde{\alpha}} \ \underline{n}_\mu dr^\mu + f_{\widetilde{1}}^{\widetilde{\alpha}} \ \underline{\widetilde{m}}_\mu dr^\mu \\
&\Updownarrow \\
&\underline{\ell} = \underline{\ell}_\alpha dz^\alpha \ , \ \underline{m} = \underline{m}_\alpha dz^\alpha \quad , \quad \underline{n} = \underline{n}_{\widetilde{\alpha}} dz^{\widetilde{\alpha}} \ , \ \underline{\widetilde{m}} = \underline{\widetilde{m}}_{\widetilde{\alpha}} dz^{\widetilde{\alpha}}
\end{aligned} \tag{2.7}$$

where $(\underline{\ell}, \underline{m})$ and $(\underline{n}, \underline{\widetilde{m}})$ are the pairs of the cotangent tetrad after the necessary complexification of the coordinates x^μ to $r^\mu = x^\mu + iy^\mu$. By construction, the coordinate functions $(z^\alpha(r^\mu), z^{\widetilde{\beta}}(r^\mu))$ determine a holomorphic transformation in a patch of $\mathbb{C}^4$ outside the real surface $y^\mu = 0$, which is the LCR-manifold M, viewed as a real submanifold of the ambient complex manifold[1] $\underline{M}$. When we return back in M, the following generally complex functions $(z^\alpha(x^\mu), z^{\widetilde{\beta}}(x^\mu)) := (z^\alpha(r^\mu), z^{\widetilde{\beta}}(r^\mu))|_M$ determine an embedding of M in $\mathbb{C}^4$, which takes the form

$$\begin{aligned}
&\rho_{11}(\overline{z^\alpha}, z^\alpha) = 0 \quad , \quad \rho_{12}\left(\overline{z^\alpha}, z^{\widetilde{\alpha}}\right) = 0 \quad , \quad \rho_{22}(\overline{z^{\widetilde{\alpha}}}, z^{\widetilde{\alpha}}) = 0 \\
&(\rho_{ij}) = \begin{pmatrix} \rho_{11}(\overline{z^\alpha}, z^\alpha) & \rho_{12}\left(\overline{z^\alpha}, z^{\widetilde{\alpha}}\right) \\ \overline{\rho_{12}\left(\overline{z^\alpha}, z^{\widetilde{\alpha}}\right)} & \rho_{22}(\overline{z^{\widetilde{\alpha}}}, z^{\widetilde{\alpha}}) \end{pmatrix} \quad , \quad \tfrac{\partial \rho_{ij}}{\partial z^b} \neq 0 \neq \tfrac{\partial \rho_{ij}}{\overline{\partial z^b}}
\end{aligned} \tag{2.8}$$

with two real functions ρ_{11} , ρ_{22} and a complex one ρ_{12}. Notice that these embedding functions have a special dependence on the structure coordinates and (ρ_{ij}) is a hermitian matrix. Besides, they are defined up to non-vanishing factors $\rho'_{ij} = f_{ij}\rho_{ij}$ with the same dependence on the structure coordinates. According to the conventional terminology, the manifold is locally (in every patch of a covering atlas)

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a totally real submanifold of $\mathbb{C}^4$. The precise dependence of the defining functions on the structure coordinates characterizes the LCR-structure from the general definition of a totally real submanifold of $\mathbb{C}^4$. The four functions $z^b := (z^\alpha,\ z^{\widetilde{\alpha}})$, $\alpha = 0,\ 1$ are the structure coordinates of the LCR-structure in the corresponding coordinate chart. The holomorphic transformations in the intersection of the charts (of the LCR-atlas), which preserve the LCR-structure, are $z'^\alpha = f^\alpha(z^\beta)$, $z'^{\widetilde{\alpha}} = f^{\widetilde{\alpha}}(z^{\widetilde{\beta}})$ called LCR-transformations. I point out that the general holomorphic transformations $z'^b = f^b(z^c)$ do not preserve the LCR-structure!

The Cartan approach of the CR-structures starts by first considering the projectivization of their real conditions[6]. In the present case the proper projectivization of (2.8) is

$$\begin{gathered}\rho_{11}(\overline{Y^{n1}}, Y^{n1}) = 0 \quad , \quad \rho_{12}\left(\overline{Y^{n1}}, Y^{n2}\right) = 0 \quad , \quad \rho_{22}(\overline{Y^{n2}}, Y^{n2}) = 0 \\ K(Y^{ni}) = 0\ , \quad i = 1,2 \quad \Longleftrightarrow \quad Y^{n1} := Y^{n1}(z^\alpha), \quad Y^{n2} := Y^{n2}(z^{\widetilde{\alpha}})\end{gathered} \tag{2.9}$$

where $K(Y^n)$ is a homogeneous function. Y^{n1} is a projectivization of z^α and Y^{n2} is related with the $z^{\widetilde{\beta}}$ complex coordinates. Considering $K(Y^n)$ as a surface in the projective space $CP(3)$, we essentially assume that Y^{ni} are two points of an irreducible or reducible surface of $CP(3)$. This projectivization implies that the LCR-structure admits the general linear symmetry group $SL(4, \mathbb{C})$. It opens up the possibility to consider the LCR-manifold as a smooth deformation of the characteristic (Shilov) boundary of the $SU(2, 2)$ symmetric classical domain, which is necessary to apply the Cartan classification. Hence the conformal group $SU(2, 2)$ becomes the fundamental linear symmetry group of the LCR-structures and its Poincaré subgroup will be identified with the observed group in nature. Notice that we do not need any metric to fix the observed Poincaré symmetry.

The bounded and unbounded realizations of the $SU(2, 2)$ symmetric classical domain is the set of points of the grassmannian $G(4, 2)$ defined by the relations

$$\begin{gathered}\begin{pmatrix} Y_1^\dagger & Y_1^\dagger w^\dagger \end{pmatrix} \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \begin{pmatrix} Y_1 \\ wY_1 \end{pmatrix} > 0 \quad \Longleftrightarrow \quad I - w^\dagger w > 0 \\ \begin{pmatrix} X_1^\dagger & iX_1^\dagger r^\dagger \end{pmatrix} \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix} \begin{pmatrix} X_1 \\ -irX_1 \end{pmatrix} > 0 \quad \Longleftrightarrow \quad -i(r - r^\dagger) > 0\end{gathered} \tag{2.10}$$

with $\det Y_1 \neq 0$ and $\det X_1 \neq 0$. The first line defines the bounded realization and the second line defines the unbounded realization of the classical domain. They correspond to the "disc" and the "upper half-plane" realizations of the $SU(1, 1)$ symmetric classical domain in the complex plane. Because of its spinorial character the boundary $U(2)$ of the bounded realization needs two $\mathbb{R}^4$ sheets of the unbounded realization to be covered.

The Schwarzschild type Einstein metrics admit two geodetic and shear-free null congruences and precisely the KN spacetime is of special interest, because it has all the characteristics to be identified as the solitonic "electron" LCR-structure. After its discovery by Newman, Carter[4] computed its electromagnetic and gravitational moments and he realized that it has $g = 2$ gyromagnetic ratio. But introducing the electron mass and rotation parameters he found that it has a naked singularity. Recall that the existence of the essential singularity makes all the Schwarzschild type metrics unphysical (because of the Hawking-Penrose singularity theorem), except if they have the appropriate horizons to hide their essential singularity at a finite point. This mathematical fact forced general relativists to reject the interpretation of the electron as a gravitational soliton. But as an LCR-structure defined on a deformation of the boundary of the $SU(2, 2)$ classical domain (2.10), there is no problem, because a simple ray tracing of the retarded ℓ^μ and the advanced n^μ integrable curves pass through the ring singularity from the one $\mathbb{R}^4$ sheet to the second $\mathbb{R}^4$ sheet[8].

The flat "KN" LCR-tetrad (in the unbounded realization) is

$$\begin{aligned}
&L_\mu dx^\mu = dt - dr - a\sin^2\theta d\varphi \\
&N_\mu dx^\mu = \tfrac{r^2+a^2}{2(r^2+a^2\cos^2\theta)}[dt + \tfrac{r^2+2a^2\cos^2\theta-a^2}{r^2+a^2}dr - a\sin^2\theta\ d\varphi] \\
&M_\mu dx^\mu = \tfrac{-1}{\sqrt{2}(r+ia\cos\theta)}[-ia\sin\theta\ (dt-dr) + (r^2+a^2\cos^2\theta)d\theta + \\
&\qquad +i\sin\theta(r^2+a^2)d\varphi]
\end{aligned} \tag{2.11}$$

Using the "Kerr-Schild" ansatz adapted to the LCR-structure formalism, we find

$$\ell_\mu = L_\mu \quad , \quad m_\mu = M_\mu \quad , \quad n_\mu = N_\mu + \tfrac{h(r)}{2(r^2+a^2\cos^2\theta)}\ L_\mu \tag{2.12}$$

where $h(r)$ is an arbitrary function of the coordinate r. Then the structure coordinates have the form

$$\begin{aligned}
&z^0 = t - r + ia\cos\theta \quad , \quad z^1 = e^{i\varphi}\tan\tfrac{\theta}{2} \\
&z^{\widetilde{0}} = t + r - ia\cos\theta - 2f_1 \quad , \quad z^{\widetilde{1}} = \tfrac{r+ia}{r-ia}\ e^{2iaf_2}\ e^{-i\varphi}\tan\tfrac{\theta}{2} \\
&f_1(r) := \int \tfrac{h}{r^2+a^2+h}\ dr \quad , \quad f_2(r) := \int \tfrac{h}{(r^2+a^2+h)(r^2+a^2)}\ dr
\end{aligned} \tag{2.13}$$

Because of its terad-Weyl symmetry (2.5), the LCR-structure defines a class of metrics. Recall that a metric may not be compatible with any LCR-tetrad. But if it does admit a LCR-tetrad, the ℓ and n vectors are principal null directions of its corresponding conformal tensor. The fact that the conformal tensor may have at most four principal null directions implies that the possible solitonic sectors are topologically restricted to those with conformal tensors with 2, 3 and 4 principal vectors. The conformal tensor of the above "KN" LCR-tetrad, identified with the static electron, has only two principal vectors. This suggests that the sectors with 3 and 4 principal null directions should be identified with the muon and tau particles. But I have not yet found their explicit form.

In the context of general relativity Newman observed that a complex trajectory defines two geodetic and shear-free null congruences. In the present algebraic terminology the LCR-structures implied by a ruled surface of $CP(3)$ are characterized by a complex trajectory $\xi^b(\tau)$ in $G(4,2)$. The free electron LCR-structure is characterized by the linear trajectory $\xi^b = v^b\tau + c^b$ with v^b real. There are two kinds of ruled surfaces, the scrolls with $\overset{\cdot}{\xi}{}^a \overset{\cdot}{\xi}{}^b \eta_{ab} \neq 0$ and the developable with $\overset{\cdot}{\xi}{}^a \overset{\cdot}{\xi}{}^b \eta_{ab} = 0$. Hence the electron corresponds to the massive case and its neutrino to the massless one, as observed in nature.

3 Electroweak and gluonic fields

The electromagnetic potential

$$A = \tfrac{qr}{4\pi(r^2+a^2\cos^2\theta)}(dt - dr - a\sin^2\theta d\varphi) \tag{3.1}$$

of the Kerr-Newman manifold is proportional to the retarded null vector ℓ. On the other hand the embedding LCR-conditions (2.8) form a 2×2 hermitian matrix implying that the cotangent vectors

$$\widehat{e}' = i(\partial - \overline{\partial})\begin{pmatrix} \rho_{11} & \rho_{12} \\ \overline{\rho_{12}} & \rho_{22} \end{pmatrix} =: \begin{pmatrix} \ell' & \overline{m'} \\ m' & n' \end{pmatrix} \tag{3.2}$$

belong to the $u(2)$ algebra. But (2.4) shows that its curvature (gauge field strength) may not always vanish. We will make the precise calculations in the case of the "KN" LCR-tetrad (2.12), which has the following relative invariants

$$\begin{aligned}
&\Phi_1 = \tfrac{-2a\cos\theta}{r^2+a^2\cos^2\theta} \\
&\Phi_2 = -\tfrac{(r^2+a^2+h)a\cos\theta}{(r^2+a^2\cos^2\theta)^2} \\
&\Phi_3 = \tfrac{2iar\sin\theta}{\sqrt{2}(r+ia\cos\theta)^2(r-ia\cos\theta)}
\end{aligned} \tag{3.3}$$

We first make the tetrad-Weyl transformation (2.6) to reach the (normalization) conditions $\Phi_1' = 1 = -\Phi_2'$, i.e. with

$$\begin{aligned}
&\Lambda = \tfrac{qr}{4\pi(r^2+a^2\cos^2\theta)} \\
&N = -\tfrac{qr}{2\pi(r^2+a^2+h)} \\
&M\overline{M} = -\tfrac{qra\cos\theta}{2\pi(r^2+a^2\cos^2\theta)^2}
\end{aligned} \tag{3.4}$$

where the factor Λ gives the proper electromagnetic potential (3.1) of the KN manifold. Identifying the $U(2)$ connection with the electroweak potentials, we find that they have the form

$$\begin{aligned}
&A = \tfrac{qr}{4\pi(r^2+a^2\cos^2\theta)}(dt - dr - a\sin^2\theta d\varphi) \\
&Z = \tfrac{-qr}{4\pi(r^2+a^2\cos^2\theta)}(dt + \tfrac{r^2+2a^2\cos^2\theta-a^2-h}{r^2+a^2+h}dr - a\sin^2\theta d\varphi) \\
&W = \tfrac{-M}{\sqrt{2}(r+ia\cos\theta)}[-ia\sin\theta\ (dt-dr) + (r^2+a^2\cos^2\theta)d\theta + \\
&\qquad +i\sin\theta(r^2+a^2)d\varphi]
\end{aligned} \tag{3.5}$$

in the "electron" LCR-structure sector. It is the electroweak dressing of the static "electron" LCR-structure (2.13) with the proper electromagnetic gauge potential.

The gluonic gauge field is implied as a unique field equation which is invariant under the tetrad-Weyl symmetry as follows. The tetrad-Weyl transformation (2.5) is the symmetry of the fundamental LCR-structure, which corresponds to the ordinary Weyl symmetry of the Polyakov action. Looking for invariant PDEs, we found

$$\begin{aligned}
&\tfrac{1}{2ie}(D_\mu)_{ij}(e(\Gamma^{\mu\nu\rho\sigma} - \overline{\Gamma^{\mu\nu\rho\sigma}})F_{j\rho\sigma}) = 0 \quad \text{or its dual} \\
&\Gamma^{\mu\nu\rho\sigma} := \tfrac{1}{2}[(\ell^\mu m^\nu - \ell^\nu m^\mu)(n^\rho \overline{m}^\sigma - n^\sigma \overline{m}^\rho) + \\
&\qquad +(n^\mu \overline{m}^\nu - n^\nu \overline{m}^\mu)(\ell^\rho m^\sigma - \ell^\sigma m^\rho)] \\
&(D_\mu)_{ij} := \delta_{ij}\partial_\mu - \gamma f_{ikj}A_{k\mu}, \quad F_{j\mu\nu} := \partial_\mu A_{j\nu} - \partial_\nu A_{j\mu} - \gamma\, f_{jik}A_{i\mu}A_{k\nu} \\
&e := \tfrac{-i}{4}\epsilon^{\mu\nu\rho\sigma}\ell_\mu m_\nu n_\rho \overline{m}_\sigma = \sqrt{-g}
\end{aligned} \tag{3.6}$$

where $A_{j\rho}$ is a $SU(N)$ gauge field. Its invariance under the tetrad-Weyl transformation does not permit any other term with second order derivatives. The first observation is that the simple 26-dimensional field $X^\mu(\tau,\sigma)$, which string theory interprets as the embedding of the 2-dimensional space in the ambient 26-dimensional physical space, is now replaced with a gauge field. Hence the 4-dimensional analogue of the Polyakov action is not a string theory. It is a $SU(N)$ gauge field theory which contains only the real (or the imaginary part) of the two null self-dual 2-forms $\Gamma^{\mu\nu\rho\sigma}F_{j\rho\sigma}$. In the case of the "electron" LCR-tetrad (2.11) the field equation (3.6) takes the form of the ordinary Yang-Mills PDE

$$\begin{aligned} &\tfrac{1}{\sqrt{-g}}(D_\mu)_{ij}\{\sqrt{-g}[(\ell^\mu m^\nu-\ell^\nu m^\mu)(n^\rho\overline{m}^\sigma F_{j\rho\sigma})+ \\ &+(n^\mu\overline{m}^\nu-n^\nu\overline{m}^\mu)(\ell^\rho m^\sigma F_{j\rho\sigma})]\}=-k_i^\nu \end{aligned} \tag{3.7}$$

where $k_i^\nu(x)$ is a real gluonic vector current with compact support.

Recall that the triangle anomaly of the standard model is canceled with $N=3$. Restricted to the Cartan sublgebra of $su(3)$, with $j=3,8$, the above PDE gives the gluon field strength solution

$$\begin{aligned} F_j &= \tfrac{-\gamma_j}{4\pi a}[\tfrac{a}{r^2+a^2}dt\wedge dr+dr\wedge d\varphi]= \\ &= d[\tfrac{\gamma_j}{4\pi a}(\tan^{-1}\tfrac{r}{a}dt-rd\varphi)] \end{aligned} \tag{3.8}$$

The corresponding chromoelectric and chromomagnetic fields in cartesian coordinates are

$$\begin{aligned} \overrightarrow{E_j^{(g)}} &= \tfrac{-\gamma_j r}{4\pi(r^2+a^2)(r^4+a^2(x^3)^2)}[r^2x^1,\ r^2x^2,\ (r^2+a^2)x^3] \\ \overrightarrow{B_j^{(g)}} &= \tfrac{\gamma_j r}{4\pi a\rho^2(r^4+a^2(x^3)^2)}[(r^2+a^2)x^1x^3,\ (r^2+a^2)x^2x^3,\ -\rho^2r^2] \end{aligned} \tag{3.9}$$

where $\rho^2:=(x^1)^2+(x^2)^2$. The potentials are

$$\begin{aligned} A_{i0}^{(g)} &= \tfrac{\gamma_i}{4\pi a}(\arctan\tfrac{r}{a}) \\ \overrightarrow{A^{(g)}}_j &= \tfrac{\gamma_i r}{4\pi a}[-\tfrac{x^2}{(x^1)^2+(x^2)^2}\ ,\quad \tfrac{x^1}{(x^1)^2+(x^2)^2}\ ,\quad 0] \\ &= \tfrac{\gamma_i r}{4\pi a}\overrightarrow{\nabla}\arctan\tfrac{x^2}{x^1} \end{aligned} \tag{3.10}$$

The vector potential $\overrightarrow{A^{(g)}}_j$ has a line singularity at the z-axis and a singularity along the negative part of the x^1 axis, which is the characteristic singularity implied by the gap $\varphi+2\pi$ chosen to be the negative x^1 axis. Notice that both the gluonic magnetic potential and its field strength are singular, while they do not have magnetic charge. These line singularities are an indication of quark confinement. The source of this gluonic potential is identified with the quark corresponding to the electron, because we started from the static electron LCR-tetrad. This should be the origin of the observed lepton-quark correspondence.

Concluding the present short note, I want to point out that QFT emerges through the Bogoliubov[2], Epstein-Glaser and Scharf[9] causal approach. The Bogoliubov causal approach derives the Dyson formula from a definition of causality in the rigged Hilbert-Fock space, which is identified with the observed "free" particles. The Epstein-Glaser remark properly treats the non-permitted product of the step-function distribution with the other emerging distributions. The final contribution of Scharf and coworkers uses the Kugo-Ojima BRS technique to eliminate order by order the unphysical modes of the spin-1 and spin-2 free vector fields. This simple technique derives the well known relations between the coupling constants and the masses, usually attributed to the "spontaneous symmetry breaking of the Higgs field". The entire procedure makes the standard model a kind of harmonic expansion in the rigged Hilbert-Fock space of the Poincaré representations, which appear in the fundamental background geometry as described before[8]. In this general unifying Einstein's picture implied by the present model, there are some points not well understood. The most important is the geometric origin of the mass and charge transmutation in the lepton-quark correspondence.

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